Integer Representations and Arithmetic

Yan Huang

Decimal Numbers

- Examples:

  5

  123

  8920

- Base-10

The Concept of Bases

- Base-16 (Hexadecimal)
  - Ounces and Pounds
  - How many ounces is 32 pounds? 16 x 32
- Base-12
  - Feet and Inches
  - How many inches are there in 6 feet? 12 x 6
- Base-8
  - Gallons and Pints
- Base-4
  - Dollars and Quarters, Gallons and Quarts
- Base-2 (Binary)

Base-10 Representation

- Ten states: 0, 1, …, 9.
- Need more than one digit to denote numbers greater than or equal to 10.
  - e.g., 10, 11
- 139
- 8901

Binary Numbers

- $(0)_2 = (0)_{10}$
- $(1)_2 = (1)_{10}$
- $(10)_2 = (2)_{10}$
- $(11)_2 = (3)_{10}$
- $(1001)_2 = (9)_{10}$
- $(1100)_2 = (12)_{10}$

$0111 = 7$

$1111 = (15)_{10}$

Octal Numbers

0 ~ 7

- $(0)_8 = (0)_{10}$
- $(1)_8 = (1)_{10}$
- $(10)_8 = (8)_{10}$
- $(11)_8 = (9)_{10}$
- $(1001)_8 = (513)_{10}$
- $(2671)_8 = 1465$

$2\times512 + 6\times64 + 7\times8 + 1$

$1024 + 384 + 56 + 1$

$1408 + 57$

$1\times8^3 + 0\times8^2 + 0\times8^1 + 1\times8^0$

$= 1\times8^3 + 1\times8^0 = 513$

Hexadecimal

- 16 tokens: 0, 1, …, 9, A, B, C, D, E, F
- $(A)_{16}$ $(B)_{16}$ $(C)_{16}$
- $(10)_{16} = (16)_{10}$
- $(11)_{16} = (17)_{10}$
- $(20)_{16} = (32)_{10}$
- $(100)_{16} = (1 \times 16^2)_{10} = (256)_{10}$
- $(3A1)_{16} = (3 \times 16^2 + 10 \times 16 + 1) = (\quad)_{10}$

Base-b Numbers to Decimal Numbers

$$(d_{n-1}d_{n-2}\cdots d_1d_0)_b$$

$$= d_0 + d_1 \times b + d_2 \times b^2 + \cdots + d_{n-1} \times b^{n-1}$$

$$= \sum_{i=0}^{n-1} d_i \times b^i$$

Base-b Numbers to Decimals (b<10)

- In Haskell

```
toDecimal :: Int -> [Int] -> Int
```

Base-b Numbers to Decimals (b<10)

Decimals to Base-b Numbers

$$d_0 + d_1 \times b^1 + d_2 \times b^2 + \cdots + d_{n-1} \times b^{n-1}$$

$$= (d_{n-1} d_{n-2} \cdots d_1 d_0)_b$$

$\lfloor (123 - d_0) / b \rfloor \bmod b$

$d_0 = 123 \bmod 8$

$= 3$

$120 / 8 = 15$

$15 \bmod 8 = 7$

$15 - 7 = 8 / 8 = 1$

$(123)_{10} = (173)_8$

$1 \times 8^2 + 7 \times 8 + 3$

$1 \times 6^2 + 3 \times 6 + 4 = 36 + 18 + 4 = 58$

$(58)_{10} = (\ ?\ )_6$ $\qquad (34)_6$

$d_0 = 58 \bmod 6 = 4$

$d_1 = \left[(58 - 4)/6\right] \bmod 6 = 9 \bmod 6 = 3$

$d_2 = \left[(9 - 3)/6\right] \bmod 6 = 1$

$d_3 = 0$

$\vdots \quad 0$

Decimals to Base-b Numbers (b<10)

```
toBaseb  :: Int -> Int -> [Int]
```

Decimals to Base-b Numbers (b<10)

General Cross Base Translations

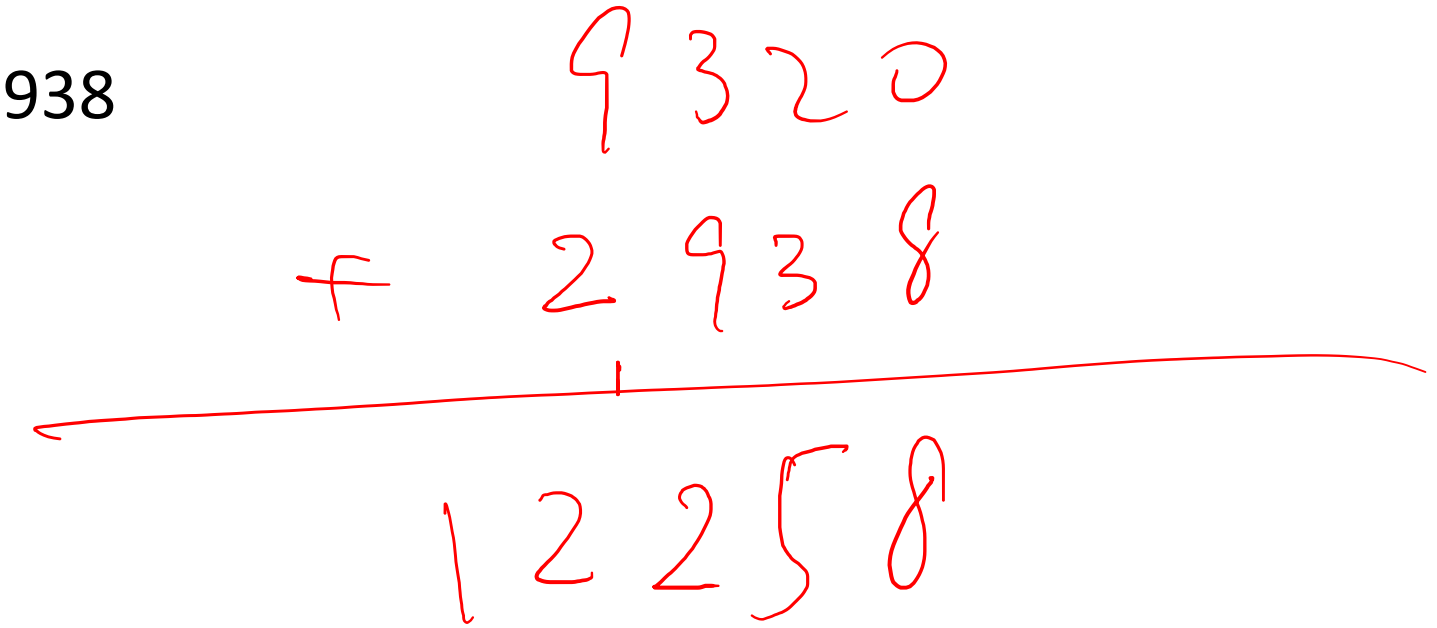
- $(72)_9 = (?)_4$

$(72)_9 = (7 \times 9 + 2)_{10} = (65)_{10}$

$= (1001)_4$

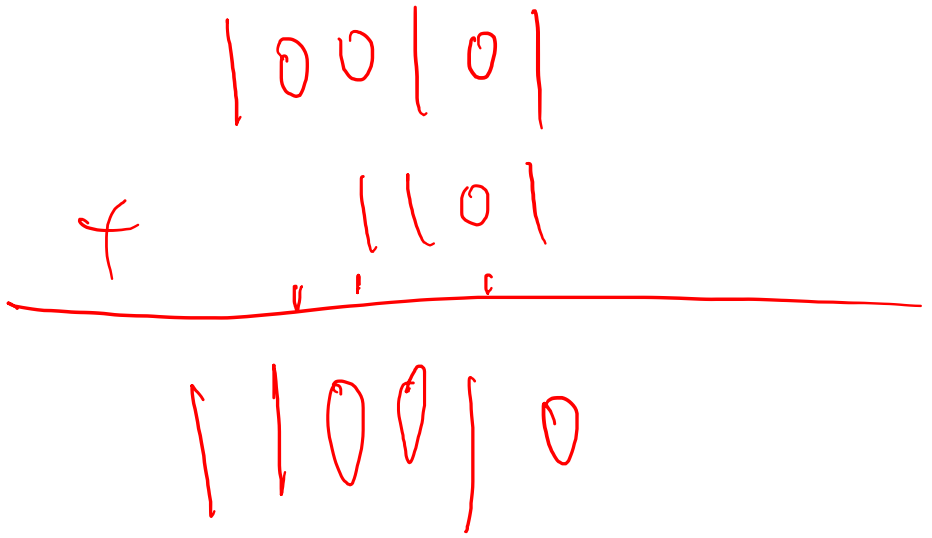
Arbitrary Precision Arithmetic (Addition)

- 9320+2938

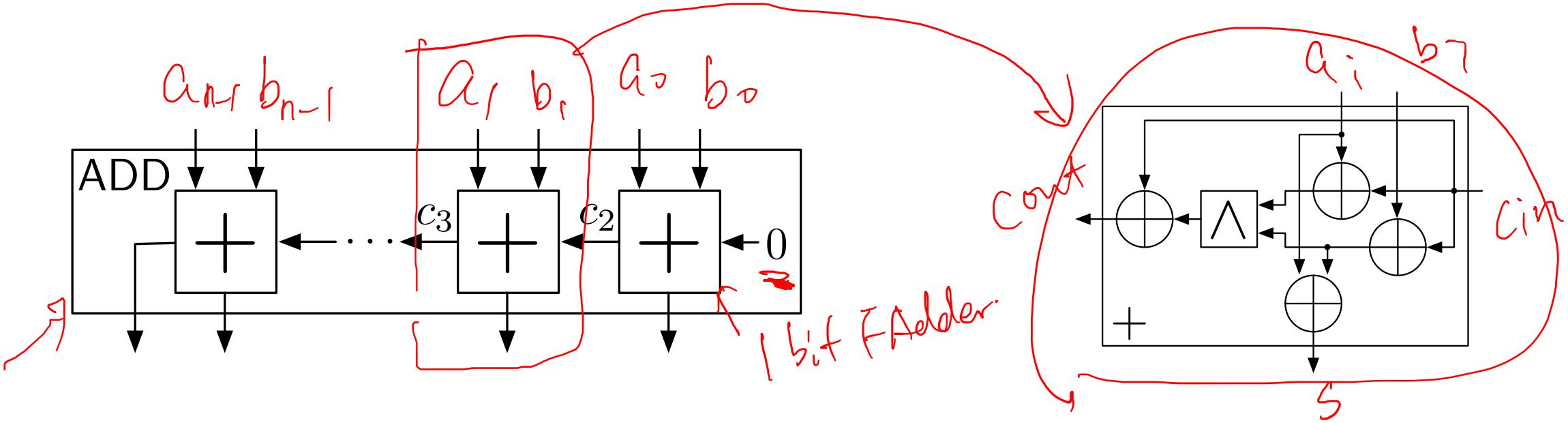


Adding Binary Numbers

- $(100101)_2 + (1101)_2$



```
add :: [Int] -> [Int] -> [Int]
```

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```