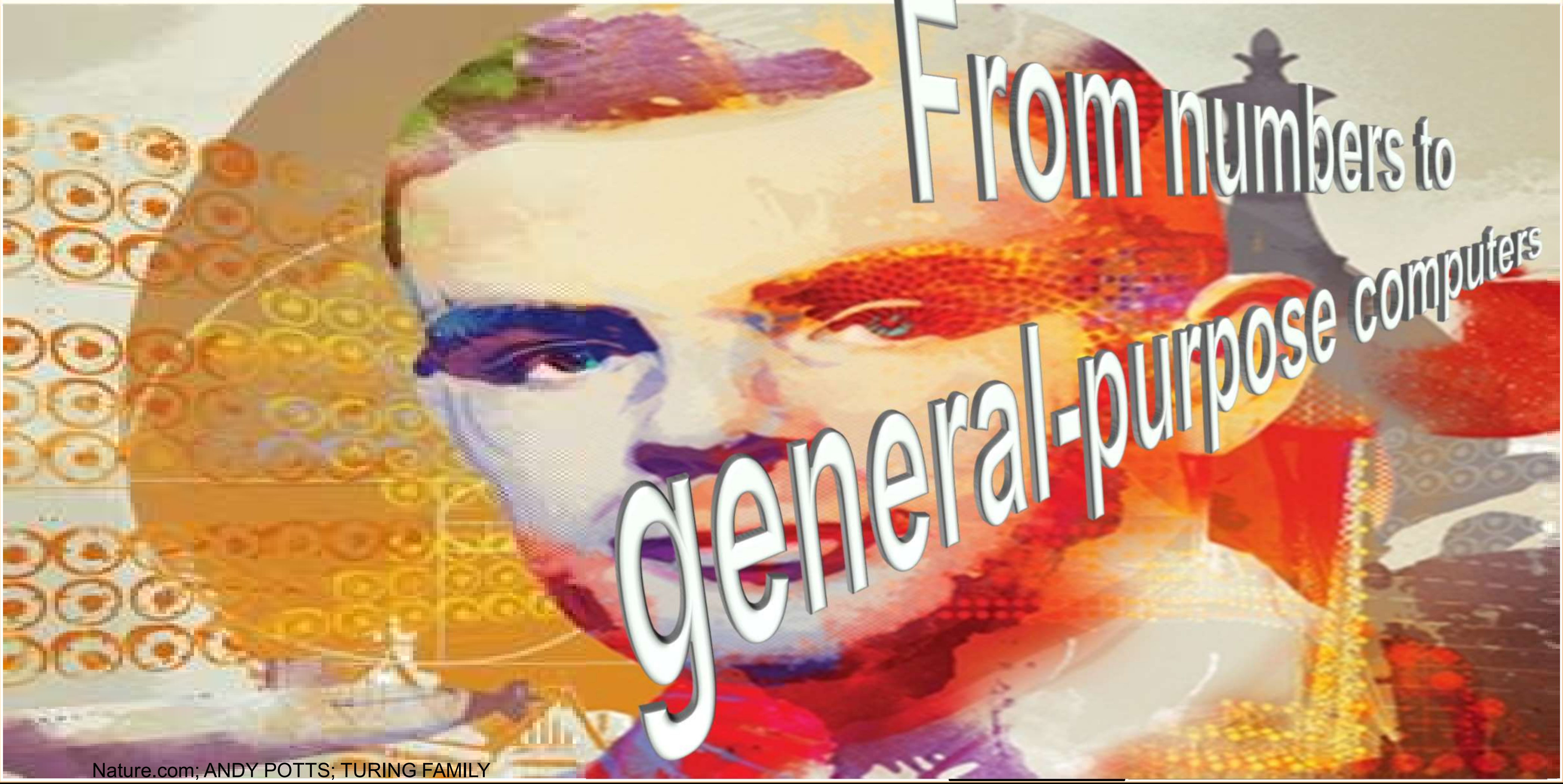
lecture 2

From numbers to general-purpose computers



Nature.com; ANDY POTTS; TURING FAMILY


How did we get here?
MEDIA
NETWORK SEARCH
WORLD
BUSINESS

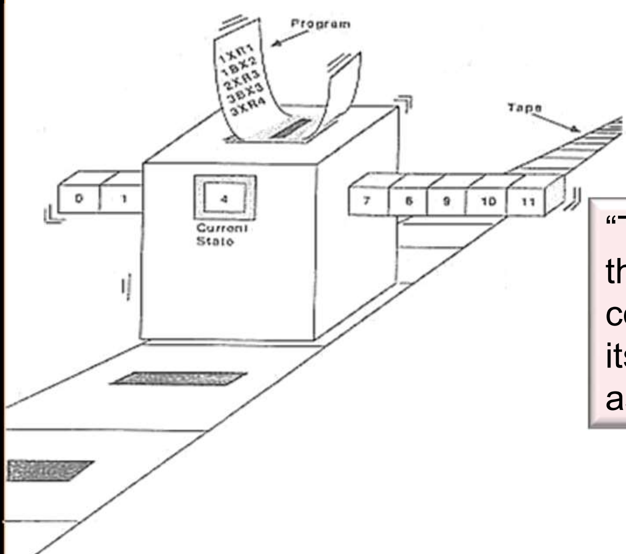
key contributions (most relevant to biocomplexity)

- “The chemical basis of morphogenesis”
  - Turing, A. M. *Phil. Trans. R. Soc. Lond. B* **237**, 37–72 (1952).
    - Reaction-diffusion systems
- “Computing machinery and intelligence”
  - Turing, A. M. *Mind* **49**, 433–460 (1950).
    - The “Turing Test”
- “On computable numbers with an application to the *Entscheidungsproblem*”
  - Turing, A. M. *Proc. Lond. Math. Soc.* **s2–42**, 230–265 (1936–37).
    - Turing machine, universal computation, decision problem

Brenner, Sydney. [2012]. “Life’s code script.” *Nature* **482** (7386): 461-461.

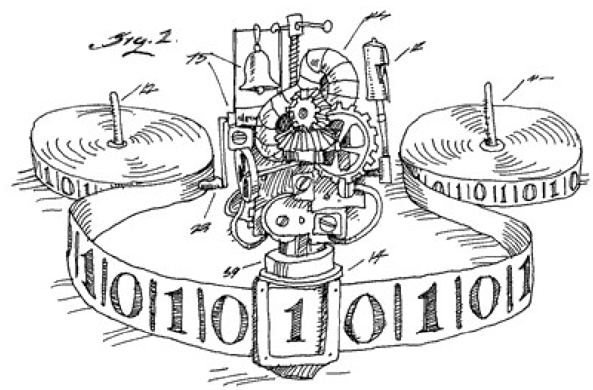
A fundamental principle of computation

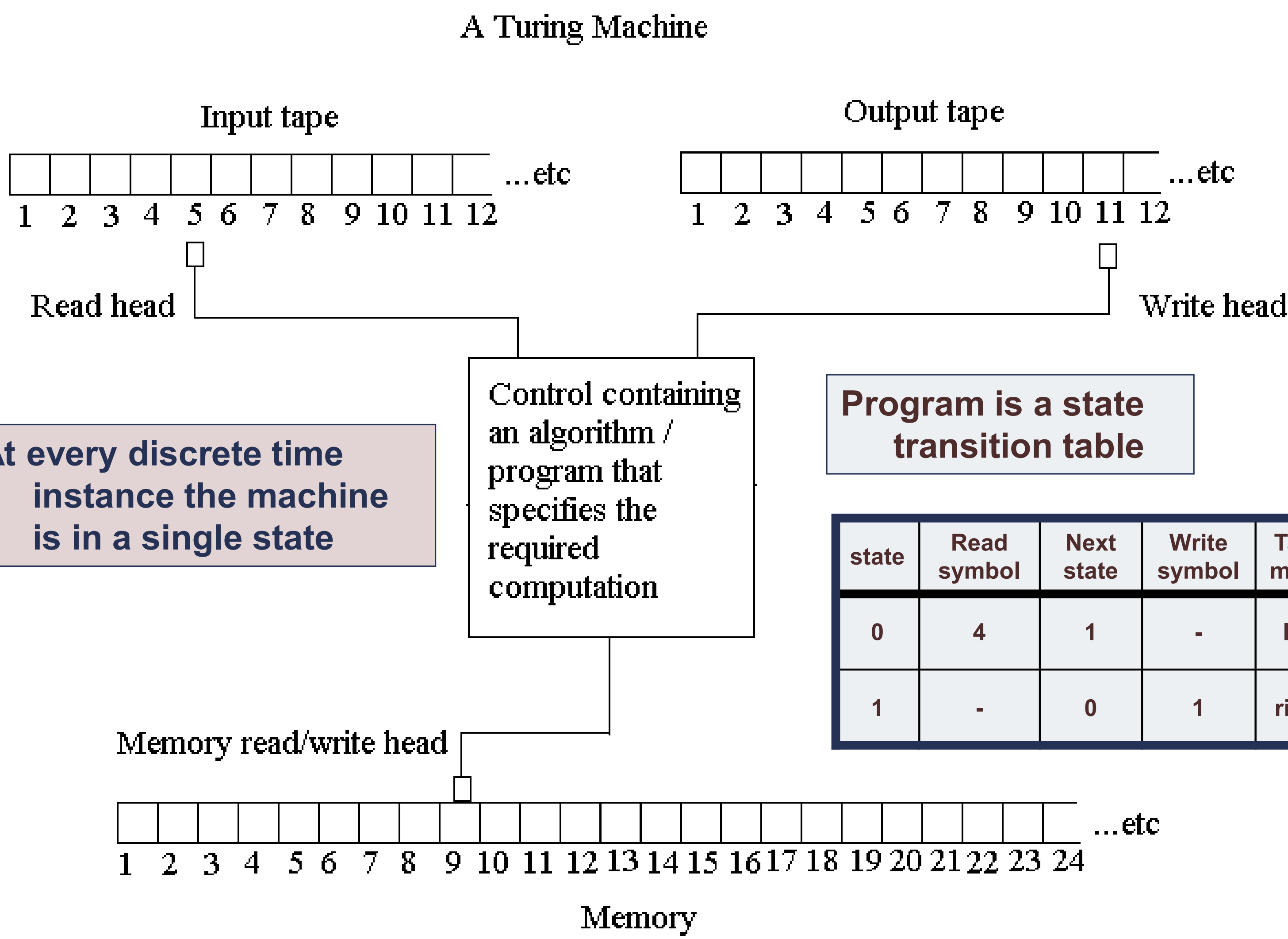
- "On computable numbers with an application to the *Entscheidungsproblem*"
  - Turing, A. M. *Proc. Lond. Math. Soc.* **s2–42**, 230–265 (1936–37).
    - **Turing machine**, universal computation, decision problem
  - **Machine**'s *state* is *controlled* by a *program*, while *data* for program is on limitless **external tape**
    - every machine can be described as a **number** that can be stored on the tape (for itself or another machine)
      - Including a Universal machine
    - **distinction** between *numbers that mean things* (data) and *numbers that do things* (program)



code

"The fundamental, indivisible unit of information is the bit. The fundamental, indivisible unit of digital computation is the transformation of a bit between its two possible forms of existence: as [memory] or as [code]. George Dyson, 2012.

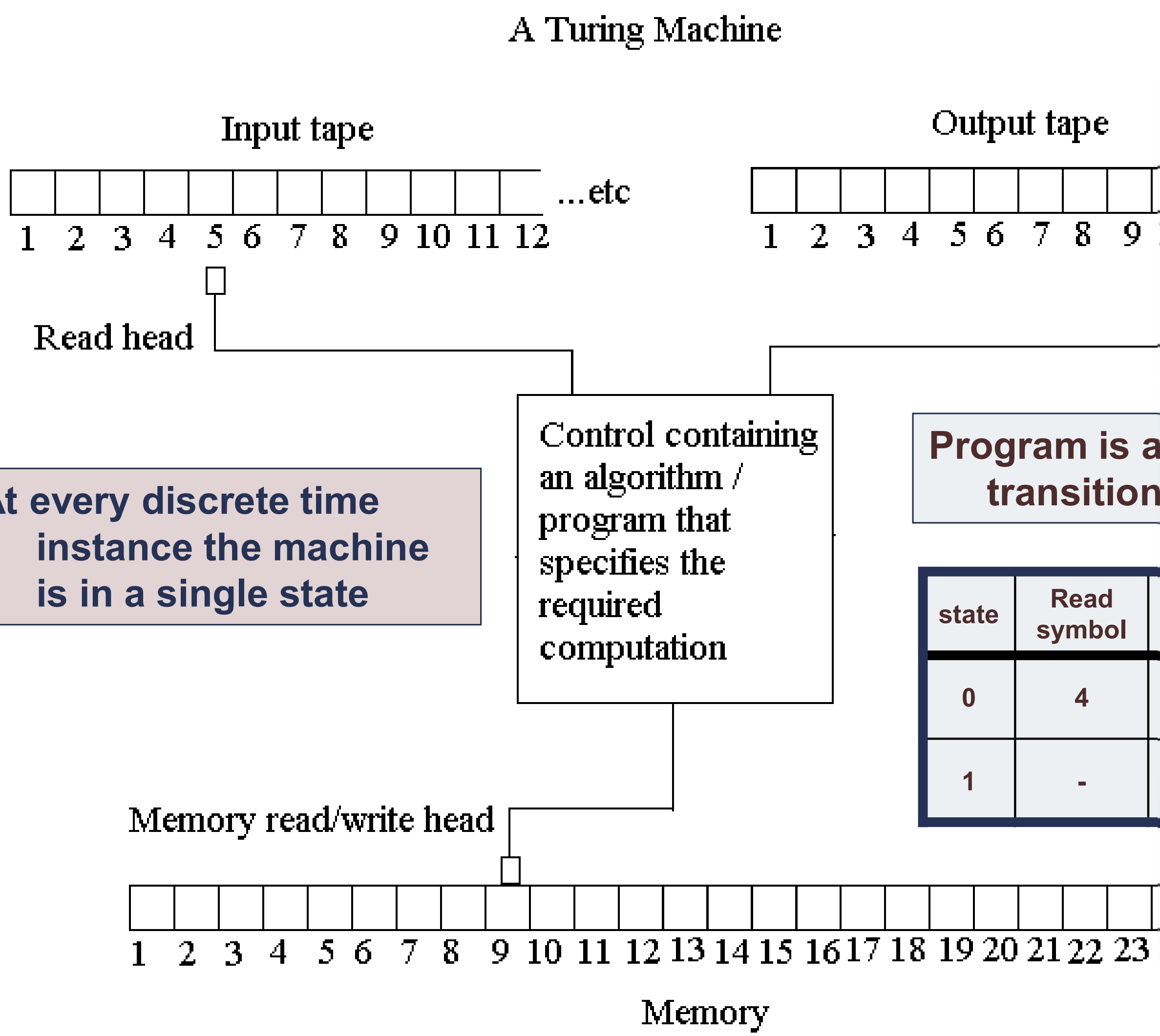




At every discrete time instance the machine is in a single state

Program is a state transition table

state	Read symbol	Next state	Write symbol	Tape move
0	4	1	-	left
1	-	0	1	right



At every discrete time instance the machine is in a single state

Program is a transition

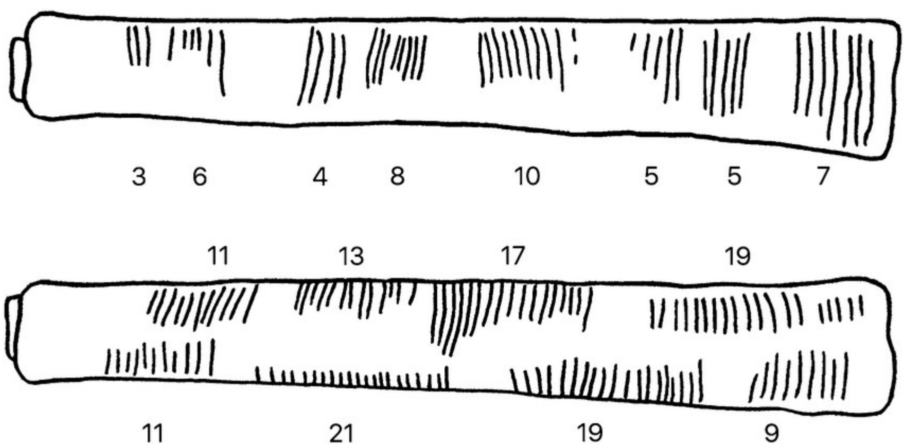
state	Read symbol
0	4
1	-

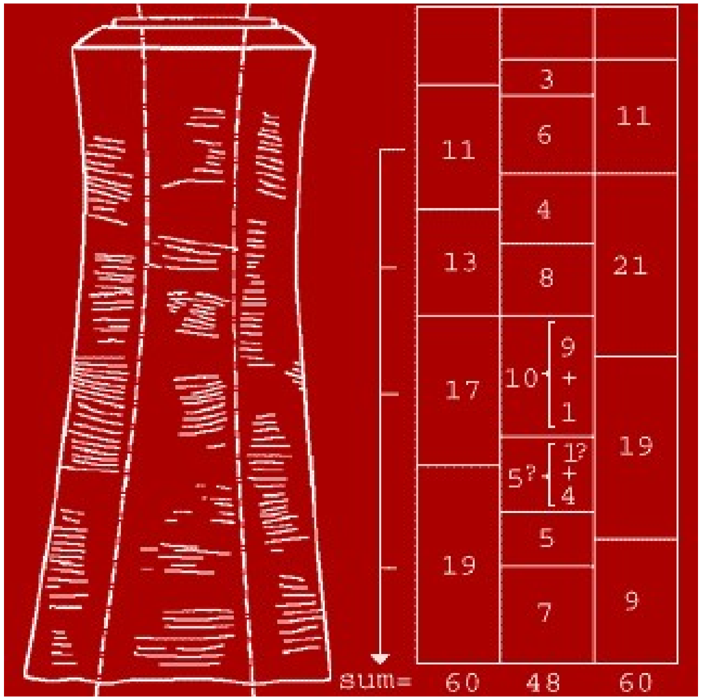
where do numbers come from?

- Number Perception
  - Recognition of a discrete quantity of objects distinct from a continuous quantity
    - Exists even in animals, birds, and insects
- Counting
  - A measurement process from a physical system to a symbol
    - E.g. notches on a bone
    - First symbols were probably numbers
- ***Lebombo bone***
  - Oldest counting tool is a piece of baboon fibula with 29 notches from 35,000 BC, discovered in the mountains between South Africa and Swaziland
    - Probably representing the number of days in a Moon Cycle
  - "Wolf Bone" from Czech Republic
    - with 55 notches in groups of 5, from 30,000 BC.

earliest examples

- The ***Ishango Bone***
  - Oldest Mathematical Artefact?
    - 20,000 BC, border of Zaire and Uganda
  - Used as a counting tool?
    - 9,11,13,17,19, 21: odd numbers
    - 11, 13, 17, 19: prime numbers
    - 60 and 48 are multiples of 12





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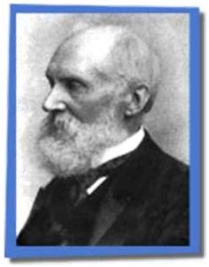
abstracting symbol mappings

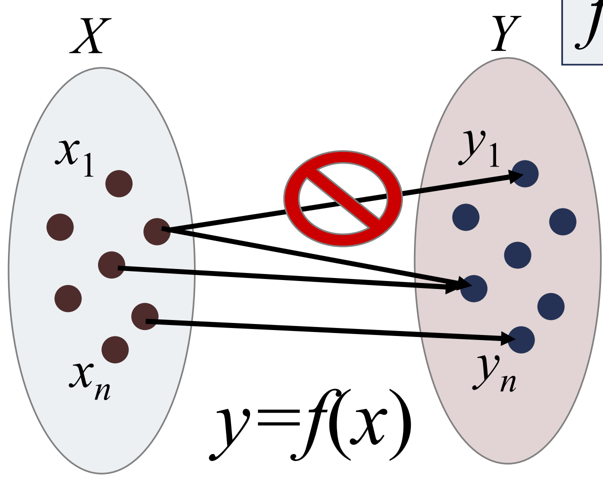
- Counting
  - A measurement process from a physical system to a symbol
    - A mapping between discrete objects and symbols
    - First numbers were not completely abstract
      - Specific attributes of concrete objects
- Computation
  - Abstract concept of one-to-one pairing of symbols
  - Mathematical concept of ***function***
- Formalization
  - To completely abstract away the significance of measuring observables from real objects

"*When you can measure what you are speaking of and express it in numbers you know that on which you are discoursing. But if you cannot measure it and express it in numbers. your knowledge is of a very meagre and unsatisfactory kind*". Lord Kelvin



producing symbols from symbols

$$f : X \to Y$$



Function: a complete and unambiguous mapping between sets of symbols

Computation: automatic process or method of implementing a function

Leibniz introduced the word in 1694

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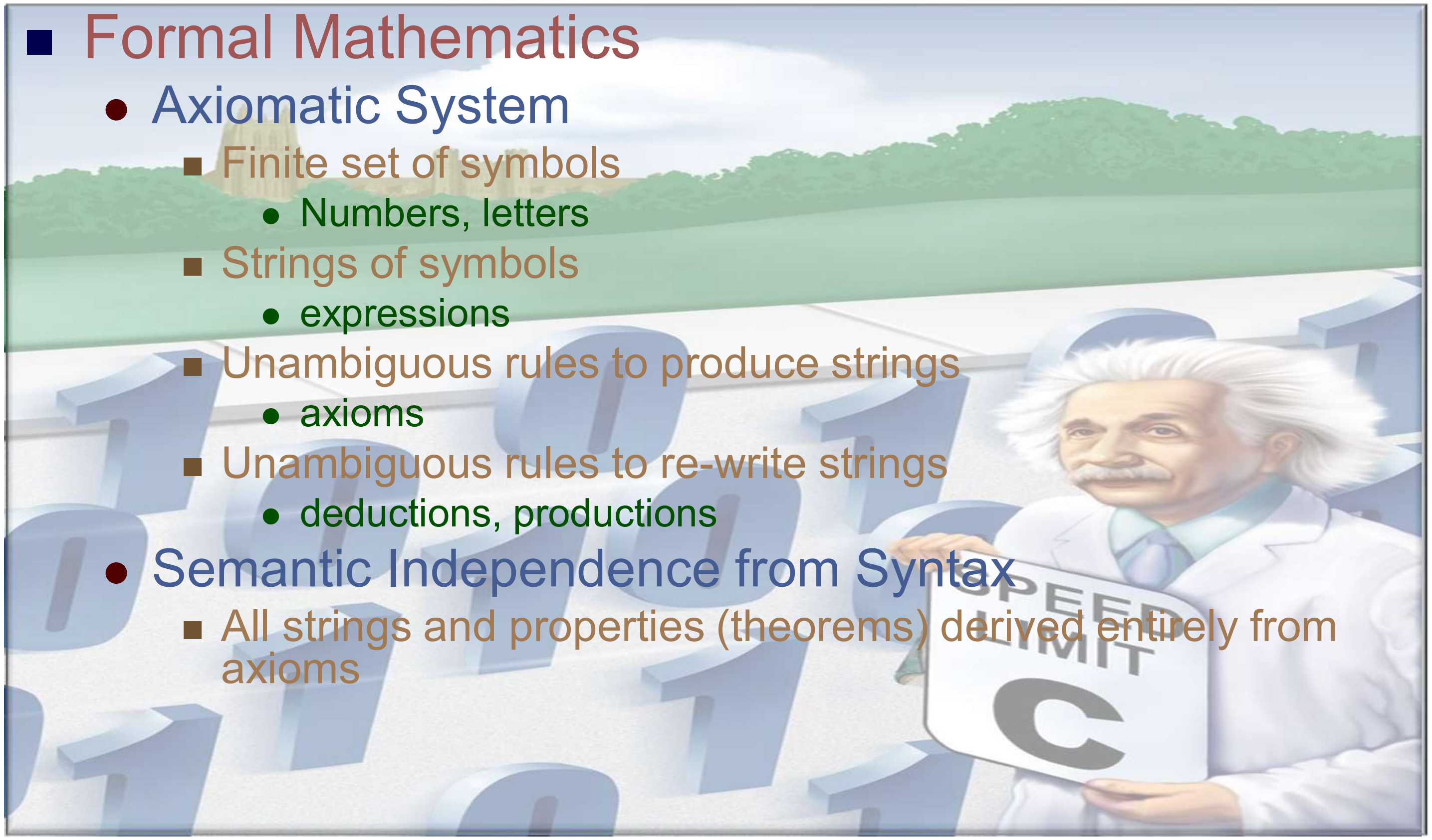
abstracting symbol mappings

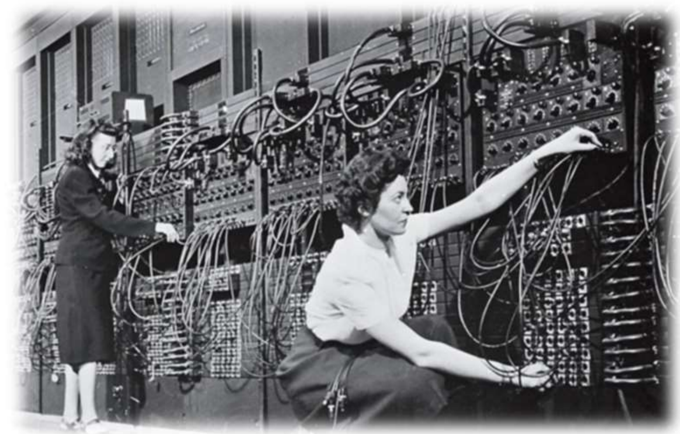
- **Formal Mathematics**
  - Axiomatic System
    - Finite set of symbols
      - Numbers, letters
    - Strings of symbols
      - expressions
    - Unambiguous rules to produce strings
      - axioms
    - Unambiguous rules to re-write strings
      - deductions, productions
  - Semantic Independence from Syntax
    - All strings and properties (theorems) derived entirely from axioms

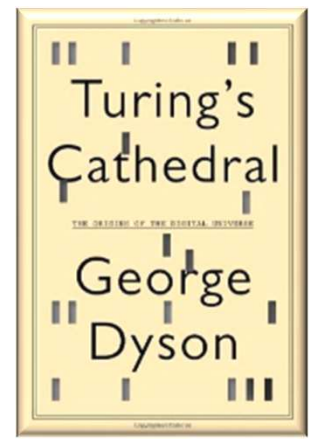


"*Insofar as the propositions of mathematics are certain they do not refer to reality; and insofar as they refer to reality, they are not certain*". Albert Einstein

from mathematical generality to physical implementation constraints

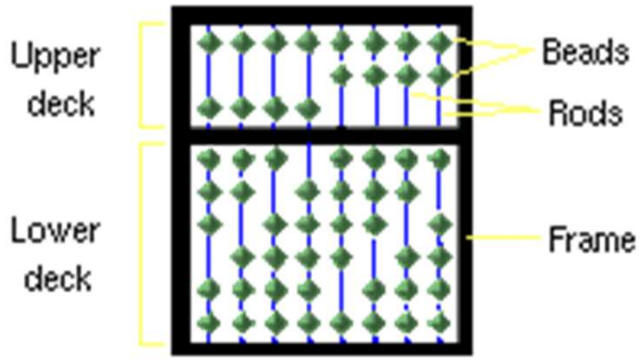
- Process of rewriting strings in a formal system according to a program of rules
  - Operations and states are syntactic
  - Symbols follow syntactical rules
  - Rate of computation is irrelevant
    - Program determines result, not speed of machine
  - Physical implementation is irrelevant for result
- Computer
  - **Physical device** that can reliably execute/approximate a formal computation
    - Errors always exist
    - Design aims to make **rate** and **dynamics** irrelevantD

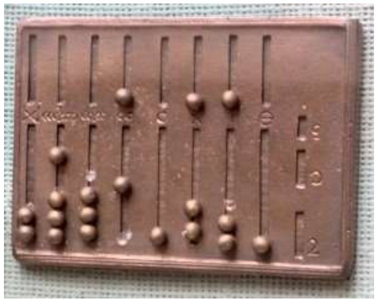




"[…] essential elements in the machine are of a binary […] nature. Those whose state is determined by their history and are time-stable are memory elements. Elements of which the state is determined essentially by the existing amplitude of a voltage or signal are called 'gates'". Bigelow et al, 1947

- Abacus
  - A counting aid, may have been invented in Babylonia in the fourth century B.C.
    - Not automatic: memory aid for intermediate calculations
  - Very used in China and Japan
    - Each bead on the upper deck has a value of 5,
    - Each bead on the lower deck has value of 1
      - Beads are considered counted, when moved towards the beam that separates the two decks.

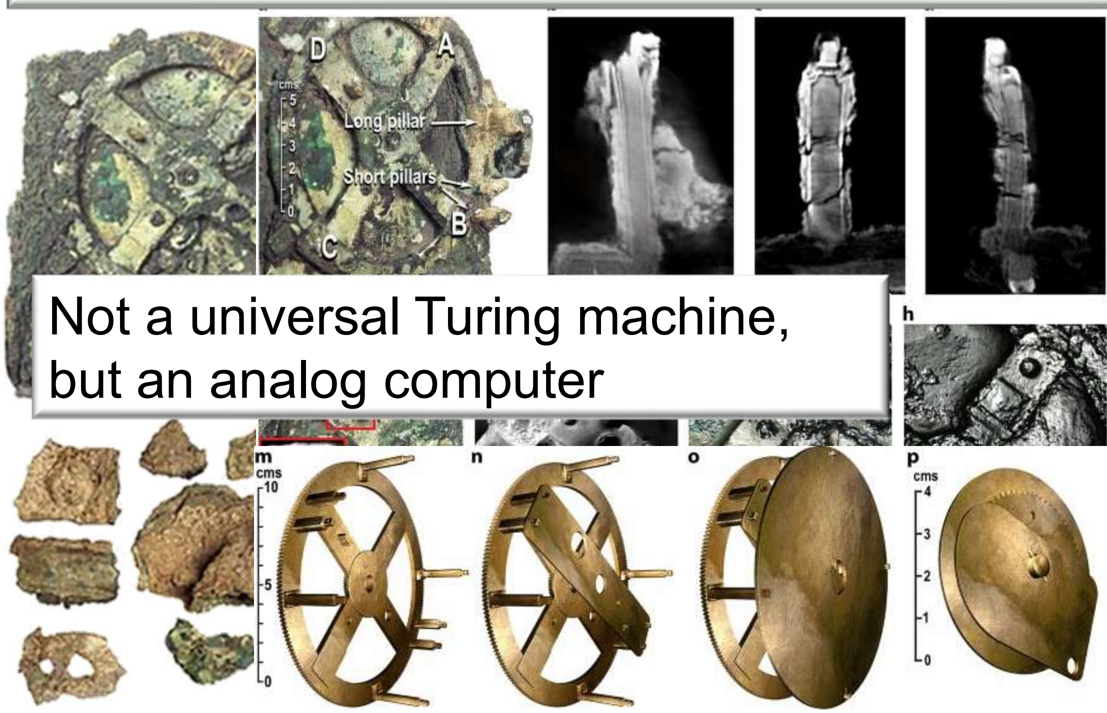




Reconstruction of a Roman abacus in the *Cabinet des Médailles, Bibliothèque nationale*, Paris.

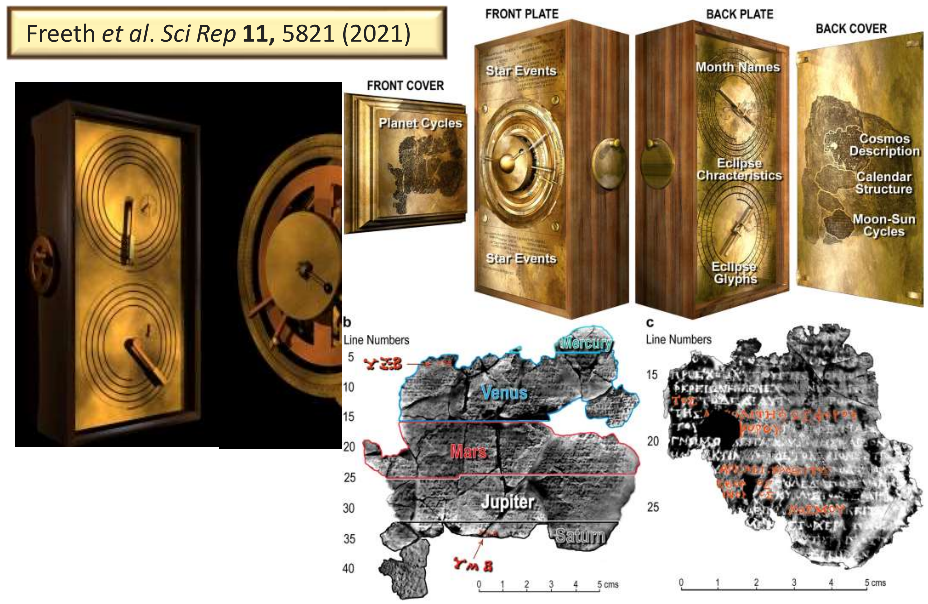
2,000-year-old astronomical calculator

- bronze mechanical **analog** computer
  - discovered more than 100 years ago in a Roman shipwreck, was used by ancient Greeks to display astronomical cycles.
- built around the end of the second century BC to calculate astronomical positions
- With imaging and high-resolution X-ray tomography to study how it worked.
  - complicated arrangement of at least 30 precision, hand-cut bronze gears housed inside a wooden case covered in inscriptions.
  - technically more complex than any known device for at least a millennium afterwards.



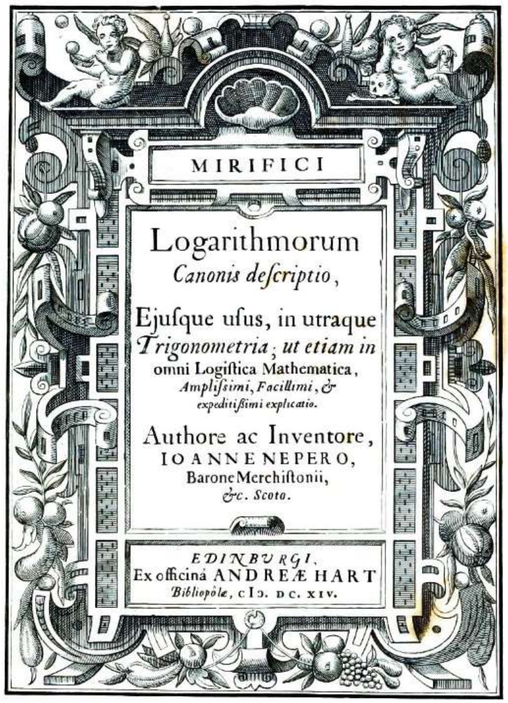
Not a universal Turing machine, but an analog computer

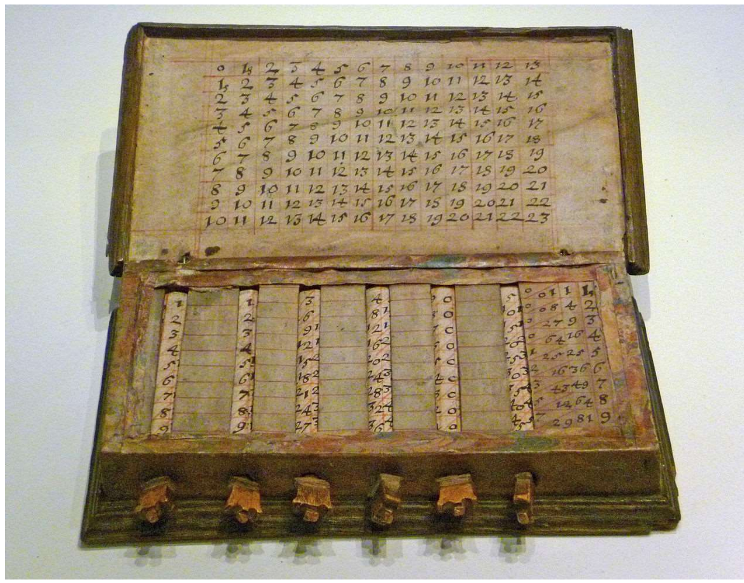
Freeth *et al*. *Sci Rep* **11,** 5821 (2021)



are people (and tables) too!

need to efficiently compute numerical tables, used in math, ballistics, astronomy, etc.





Briggs (1561-1630): decimal algorithm, logs of 30,000 numbers to 14 decimal places and logs/tans of 1/100 of every degree, 14 decimal places

John Napier's (1550-1617)

1614: logarithm, "bones" and tables convert multiplication/division to addition/subtraction computers

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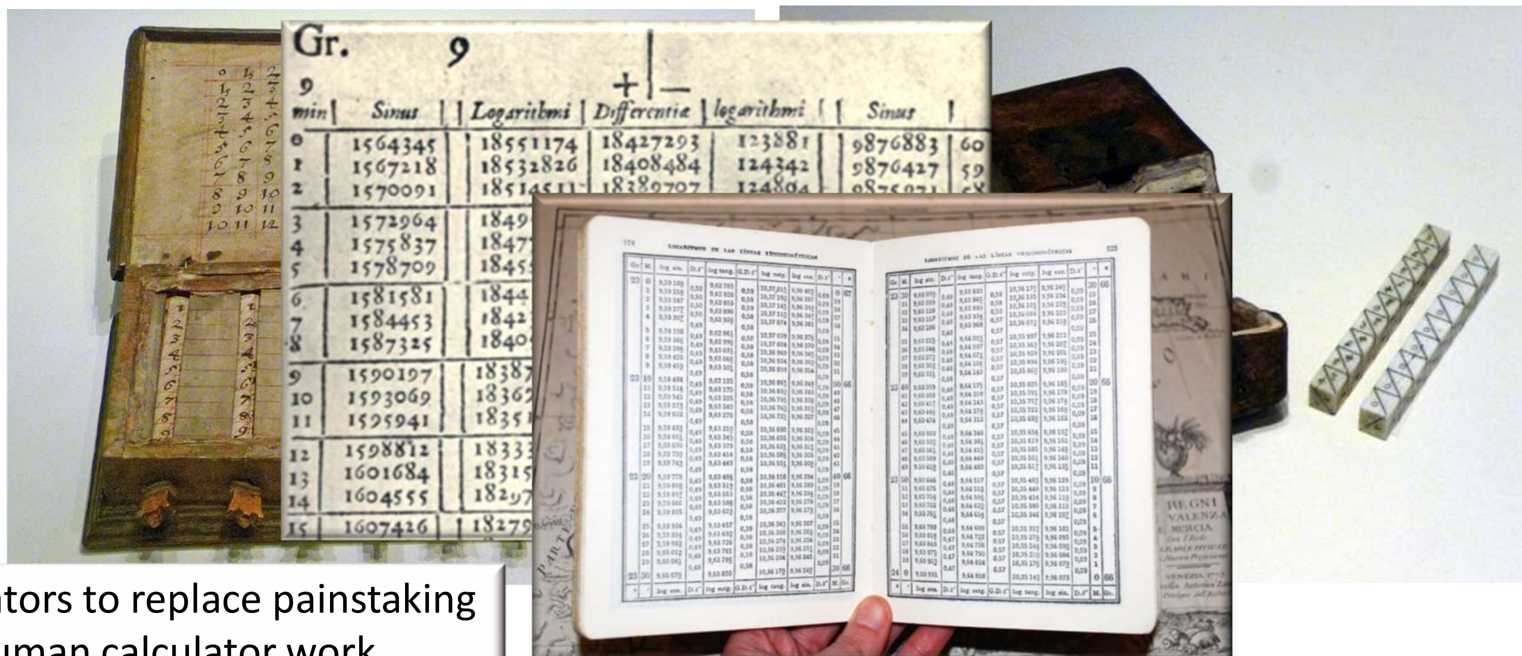

mechanical calculators to replace painstaking
and error-prone human calculator work

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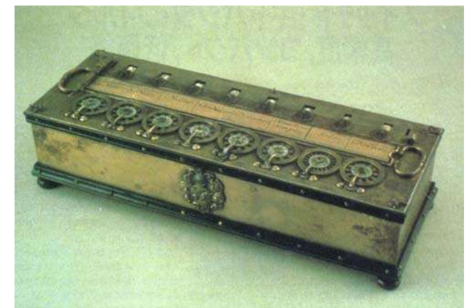
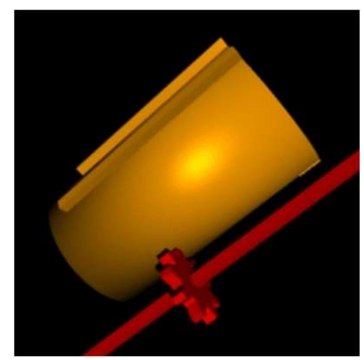
analog machines

- Wilhelm Schickard (1592- 1635)
  - In 1623 built the first mechanical calculator
    - can work with six digits, and carries digits across columns. It works, but never makes it beyond the prototype stage.
- Blaise Pascal (1623-1662)
  - built a mechanical calculator in 1642
    - It has the capacity for eight digits, but has trouble carrying and its gears tend to jam.
    - 10-teeth gears
- Gottfried von Leibniz (1614-1716)
  - built a mechanical calculator in 1670 capable of multiplication and division
    - (shift) registers for binary arithmetic
    - Credited Chinese for Binary arithmetic (I-Ching)
- Closer to abacus
  - Passive register (memory) of states



"The human race will have a new kind of instrument which will increase the power of the mind much more than optical lenses strengthen the eyes... One could carry out the ***description of a machine***, no matter how complicated, in characters which would be merely the letters of the alphabet, and so provide the mind with a method of knowing the machine and all its parts." Leibniz, 1679.



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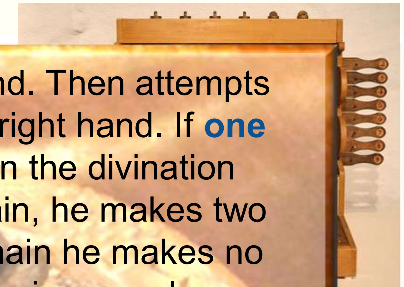

a priest holds six sacred palm nuts in his left hand. Then attempts to grab all of them out at the same time with his right hand. If **one** nut remains in his left hand , he makes a mark on the divination board which represents **a zero**. If **two** nuts remain, he makes two marks which represent **one**. If none or more remain he makes no marks at all. This is continued until four pairs of unique marks are left on the board which generate a 8-bit binary code.

Ifá (intangible cultural heritage of humanity by UNESCO): system of divination is a binary code to access oracular literary body made up of 256 volumes (signs).

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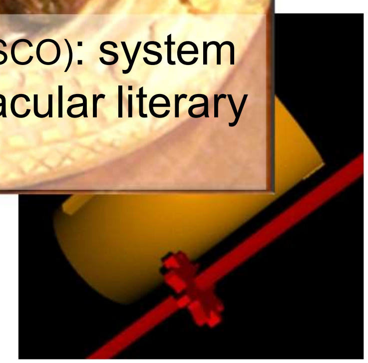
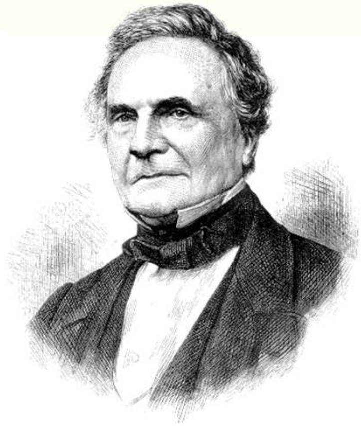
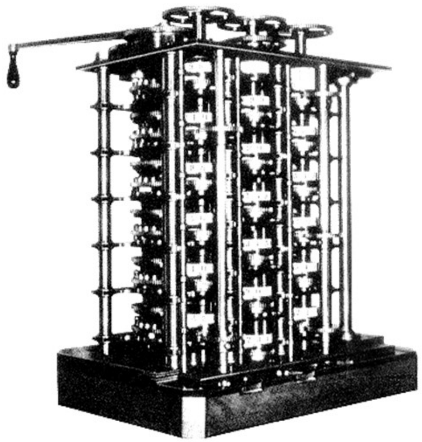


difference engine

- Special-purpose digital computing machine for the **automatic** production of mathematical tables.
  - logarithm tables, tide tables, and astronomical tables
  - Steam-driven, consisted entirely of mechanical components - brass gear wheels, rods, ratchets, pinions, etc.
  - Numbers were represented in the decimal system by the positions of 10-toothed metal wheels mounted in columns.
- Never completed the full-scale machine
  - Completed several fragments. The largest is on display in the London Science Museum. In 1990, it was built (London Science Museum)
- The Swedes Georg and Edvard Scheutz (father and son) constructed a modified version of Babbage's Difference Engine.
- For an interesting "what-if" scenario read "The Difference Engine" by Bruce Sterling and William Gibson





Not a universal Turing machine, but an analog computer difference engine

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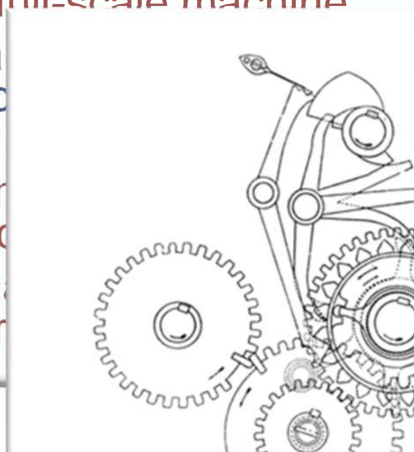

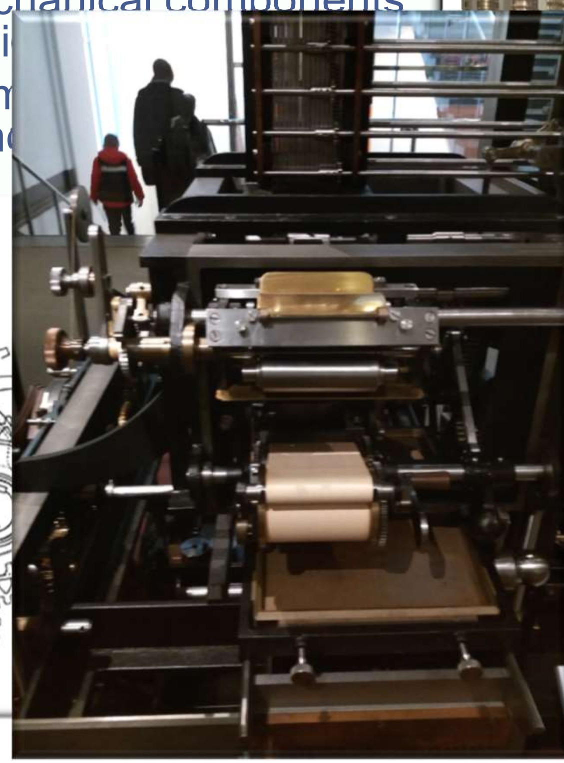

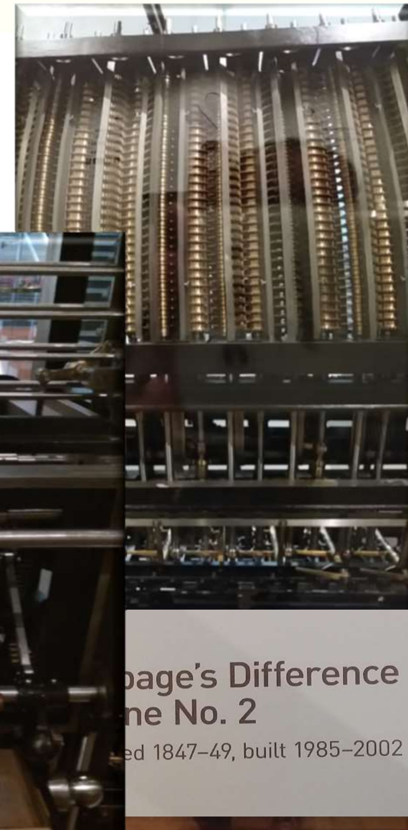

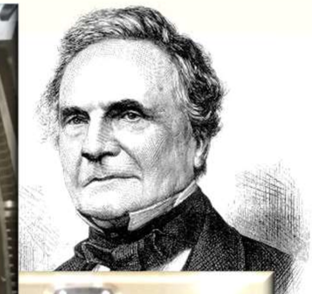

Not a universal Turing machine, but an analog computer

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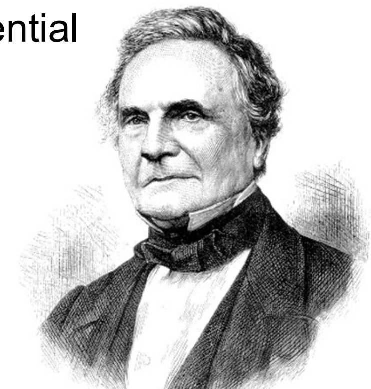
The analytical engine had an "external tape"

Turing on programs (numbers as instructions) : "[Babbage] had all the essential ideas [and] planned such a machine, called the ***Analytical Engine***. […]

- general-purpose mechanical digital computer.
  - Separated **memory store** from a **central processing unit** (or 'mill')
  - able to select from among **alternative actions** consequent upon the outcome of its previous actions
    - Conditional branching: Choice, information
  - Mechanical cogs not just numbers
    - **Variables** (states/configurations)
- Programmable
  - Data and instructions on distinct **punched cards**



"It is only a question of cards and time, […] and there is no reason why (twenty thousand) cards should not be used if necessary, in an Analytical Engine for the purposes of the mathematician". Henry Babbage (1888)

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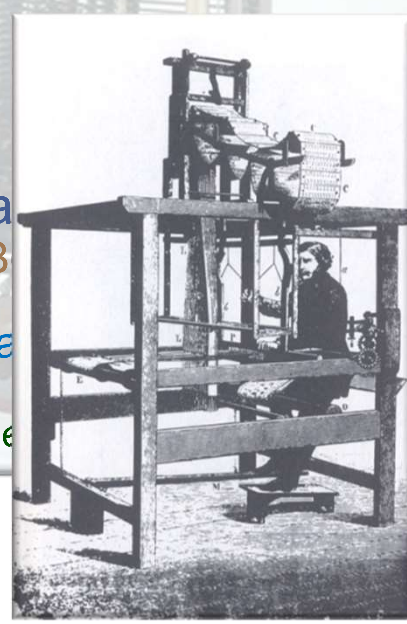
The external tape as a general design principle (system) of universal computation

- Analytical engine
  - Separated **memory store** from a **central processing unit** (or 'mill')
  - Cogs not just numbers
    - **variables**
- Programmable
  - instructions on **punched cards**
    - Inspired by the Jacquard Loom
  - Ada Lovelace: the science of opera
    - Set of (recursive) rules for producing B program)
    - **Separation** of variable and operationa
    - would punch out cards for later use
      - "the Engine eating its own tail." (Babbage



code

distinction between *numbers that mean things* and *numbers that do things*.

The external tape as a general design principle (system) of universal computation

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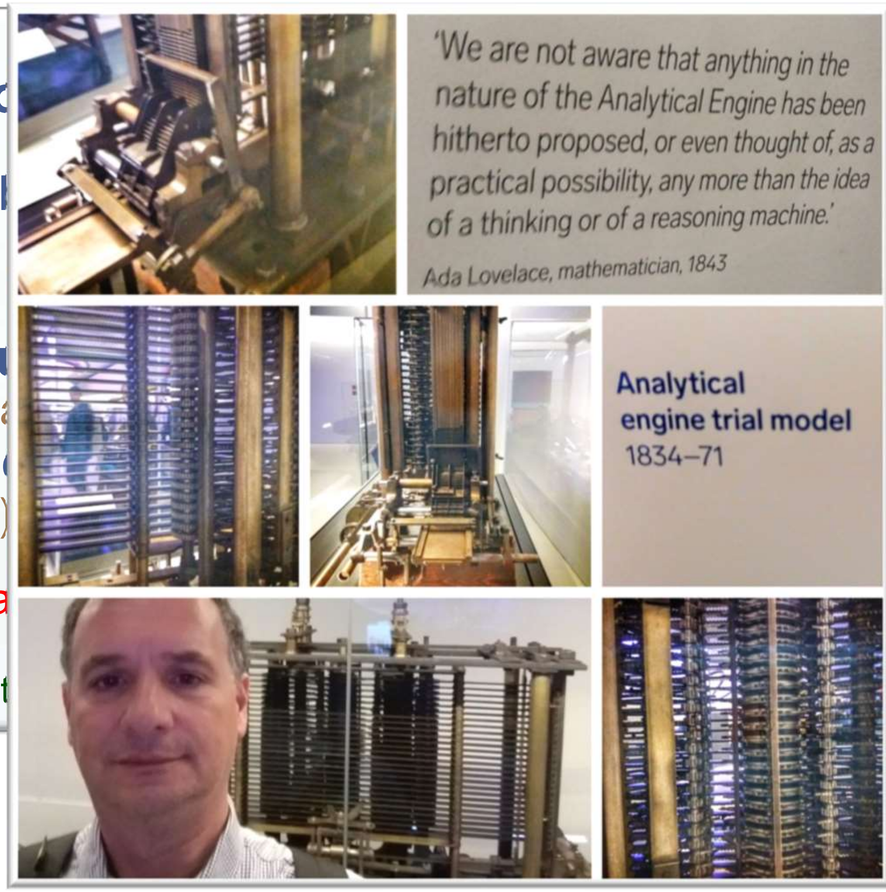

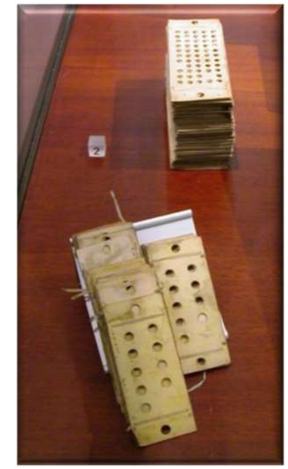

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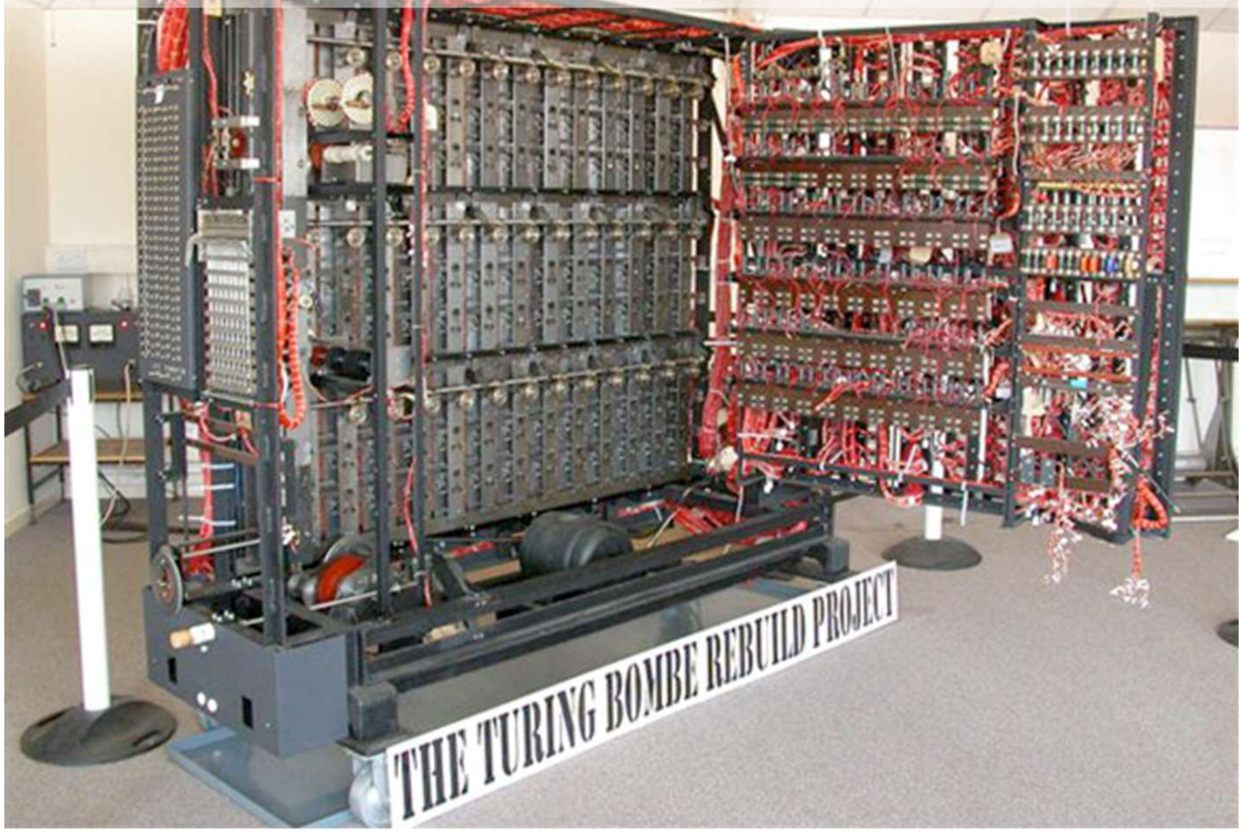
early computers

not electronic, not digital, not general-purpose

Turing bombe: Enigma Cracker at Bletchley Park (1940-1945)
Electro-mechanical, hundreds produced in UK and US



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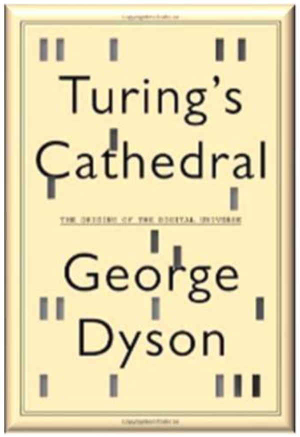
ENIAC (1945)

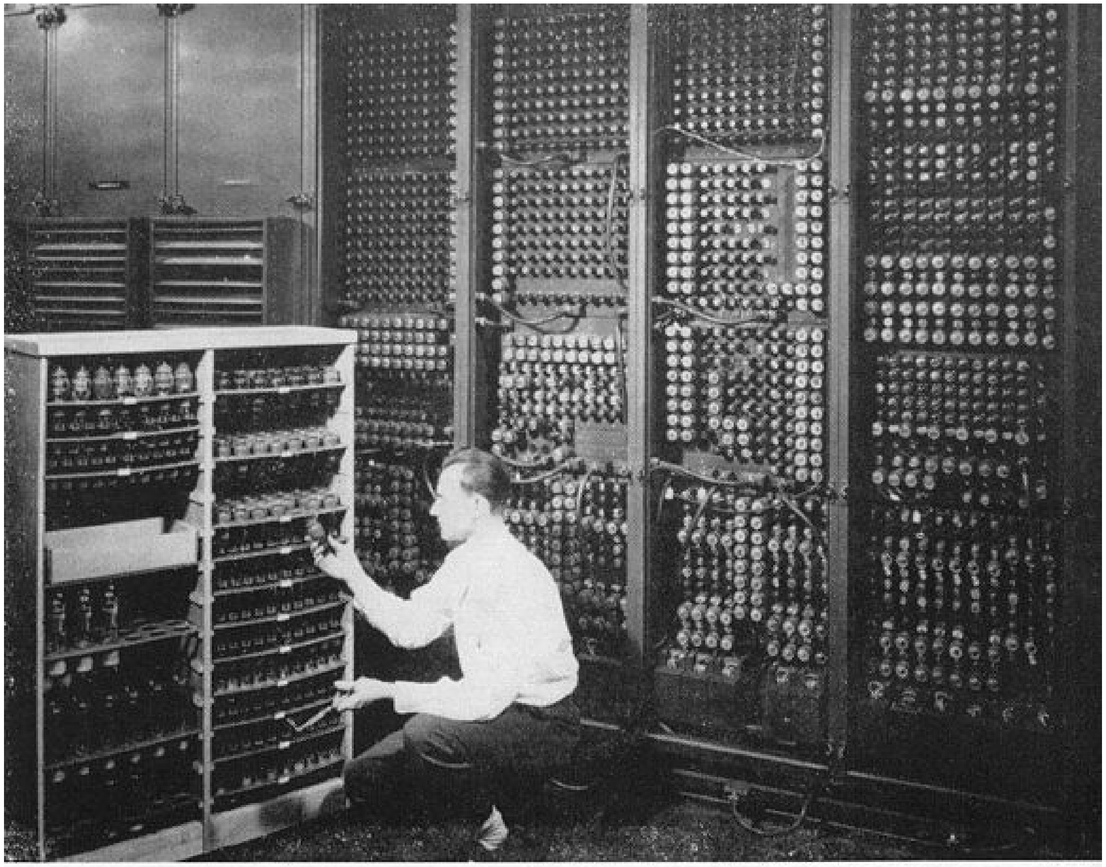
Electronic Numerical Integrator And Computer



- First fully functioning electronic digital computer to be built in the U.S.
  - Electrical Numerical Integrator and Computer
  - University of Pennsylvania, for the Army Ordnance Department, by J. Presper Eckert and John Mauchly.
    - Far from general-purpose: The primary function was calculation of tables used in aiming artillery.
    - ENIAC was not a stored-program computer, and setting it up for a new job involved reconfiguring the machine by means of plugs and switches.
    - Used decimal digits instead of binary ones
    - Nearly 18,000 ***vacuum tubes for switching.***
      - Storage of all those vacuum tubes and the machinery required to keep the cool took up over 167 square meters (1800 square feet) of floor space.
      - invented by American physicist Lee De Forest in 1906.
      - worked by using large amounts of electricity to heat a filament inside the tube. the presence of current represented a one.
    - punched-card input and output

ENIAC (1945)

Electronic Numerical Integrator And Computer (decimal)



Replacing a bad tube meant checking among ENIAC's 19,000 possibilities.

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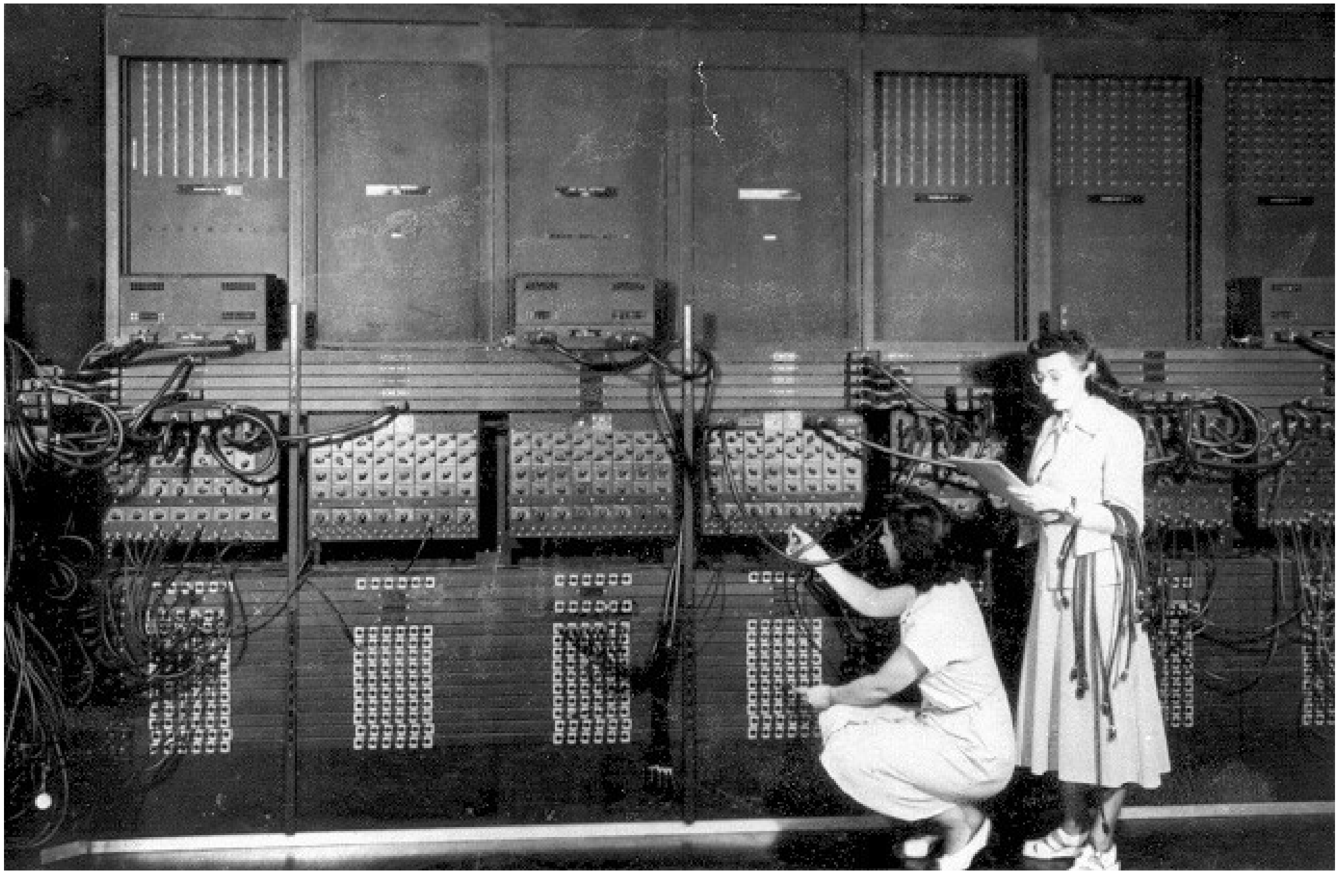
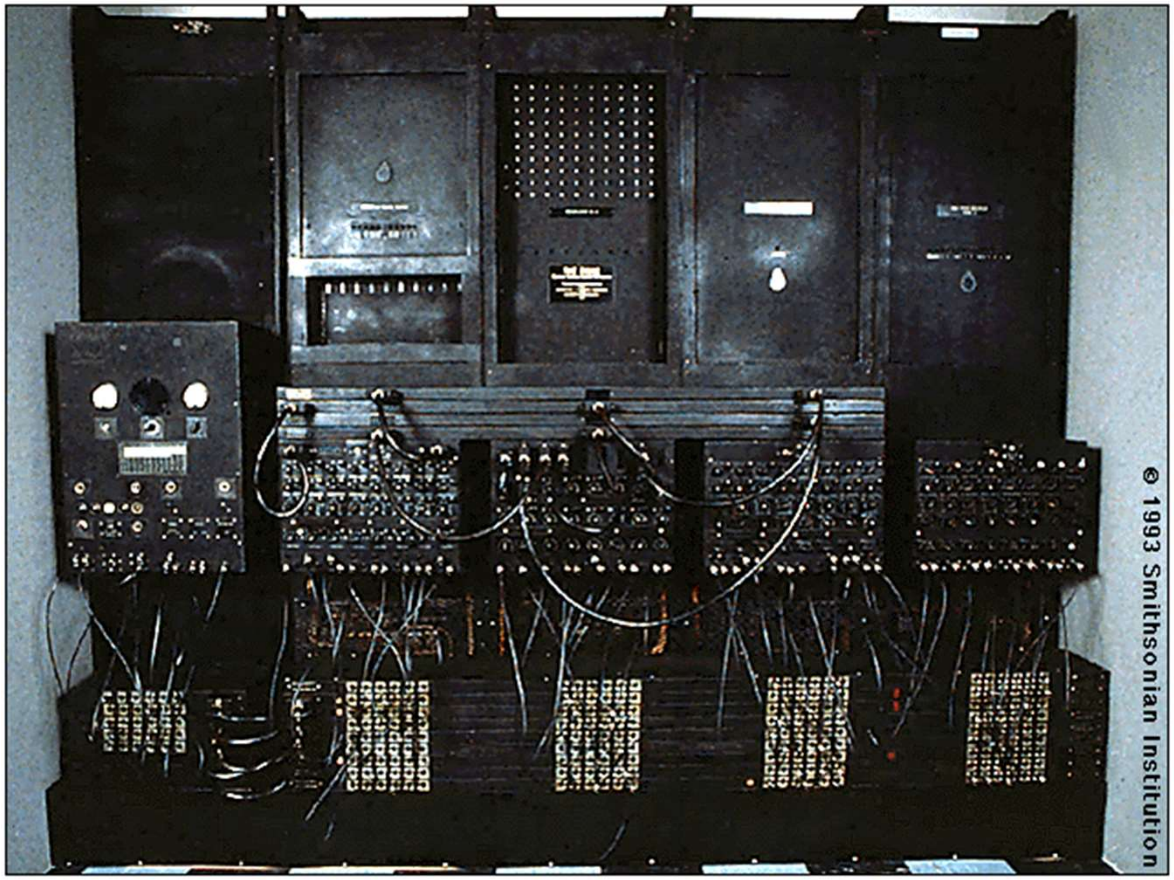


ENIAC (1945)

Electronic Numerical Integrator And Computer

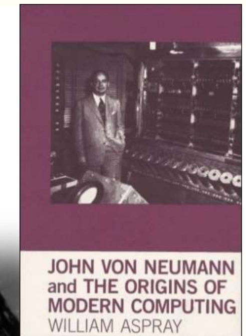


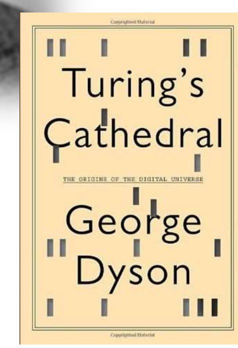
John Von Neumann (1903-1957)

Turing machines beyond the decision problem

" 'Words' coding the orders are handled in the memory just like numbers" --- distinction between *numbers that mean things* and *numbers that do things*.

- realizing the power of Turing's tape
  - physical (electronic) computers
  - emphasized the importance of the ***stored-program computer*** concept (the external tape)
    - EDVAC (1951), IAS Machine (1952) - binary
  - allows machine to modify its own program
    - von Neumann architecture: The functional **separation** of storage from the processing unit.
      - programs can exist as data (two roles)
    - Converts tape to fixed-address memory (random-access memory)
  - Ultimate ***general-purpose*** machines





"Let the whole outside world consist of a long paper tape".
—John von Neumann, 1948

Turing machines beyond the decision problem

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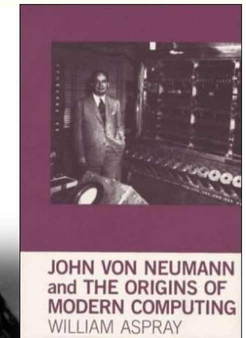
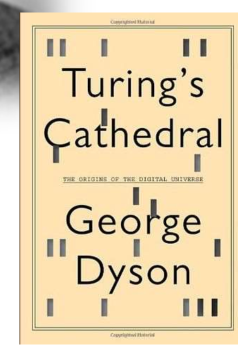
    - (random-access memory)
  - Ultimate ***general-purpose*** machines

"Since Babbage's machine was not electrical, and since all digital computers are in a sense equivalent, we see that this use of electricity cannot be of theoretical importance.… The feature of using electricity is thus seen to be only a very superficial similarity." (Alan Turing)





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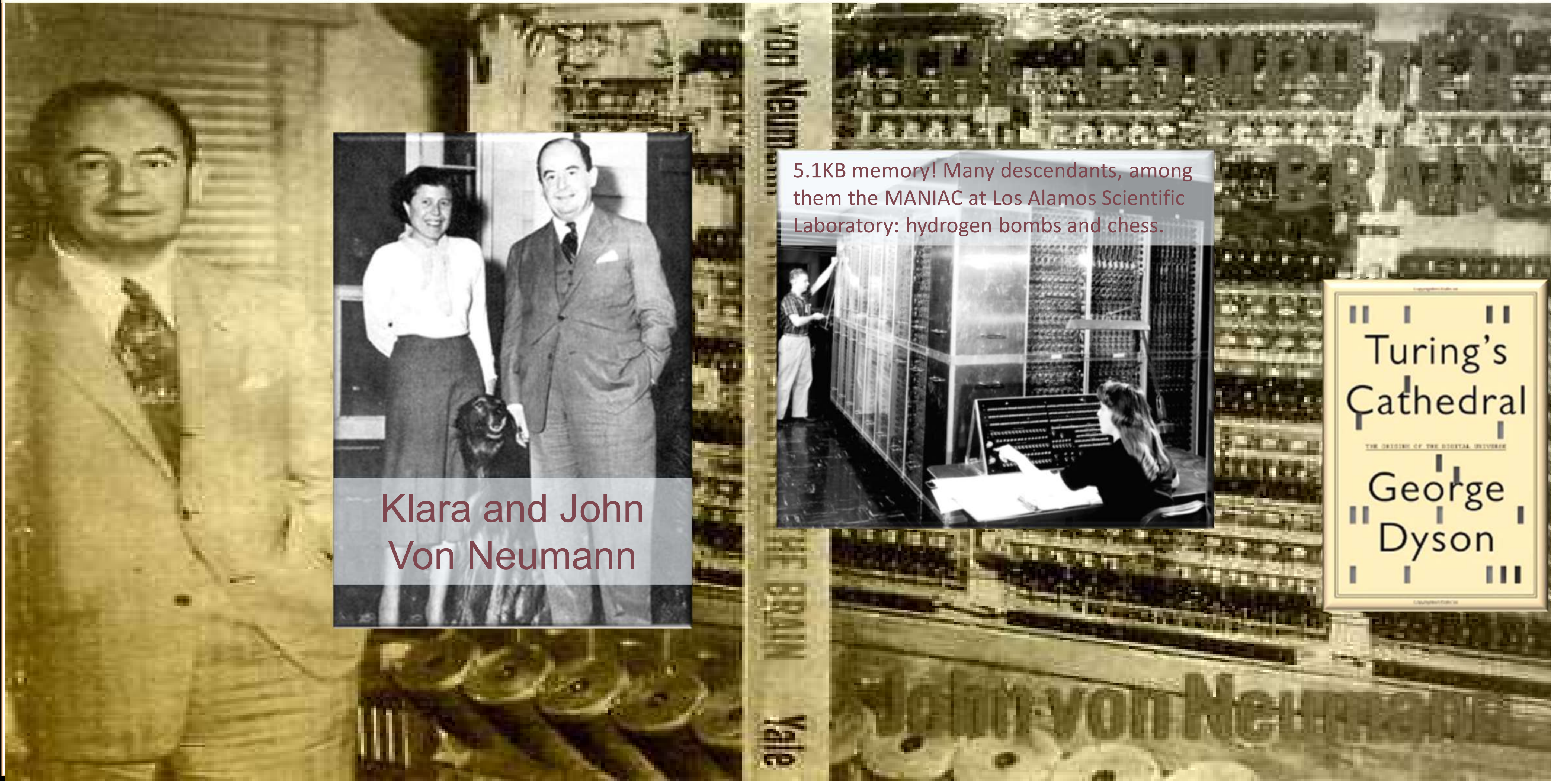
IAS Machine (1952)

electronic digital (stored-program) computer with 40 bit word (IAS, Princeton)



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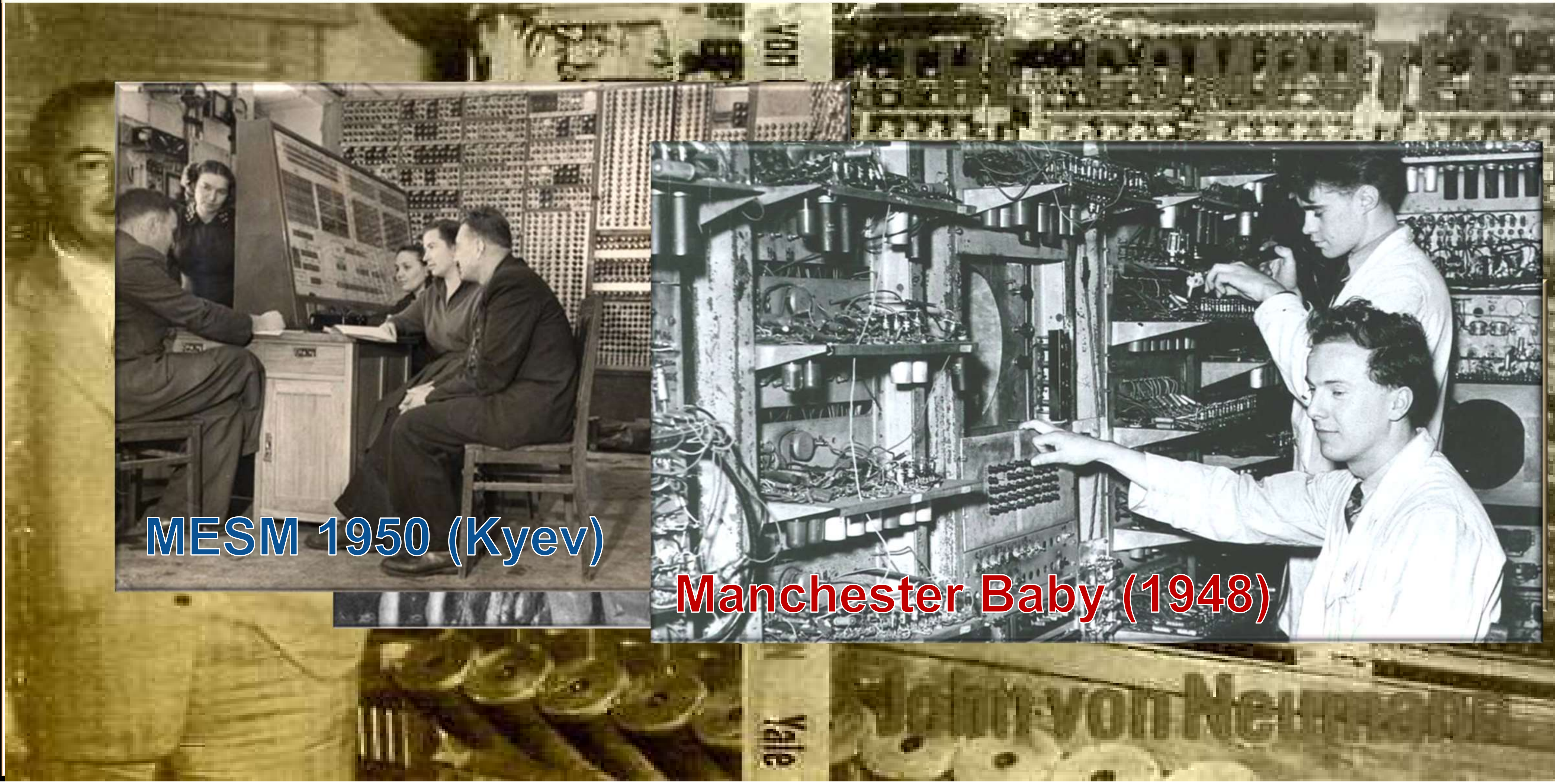
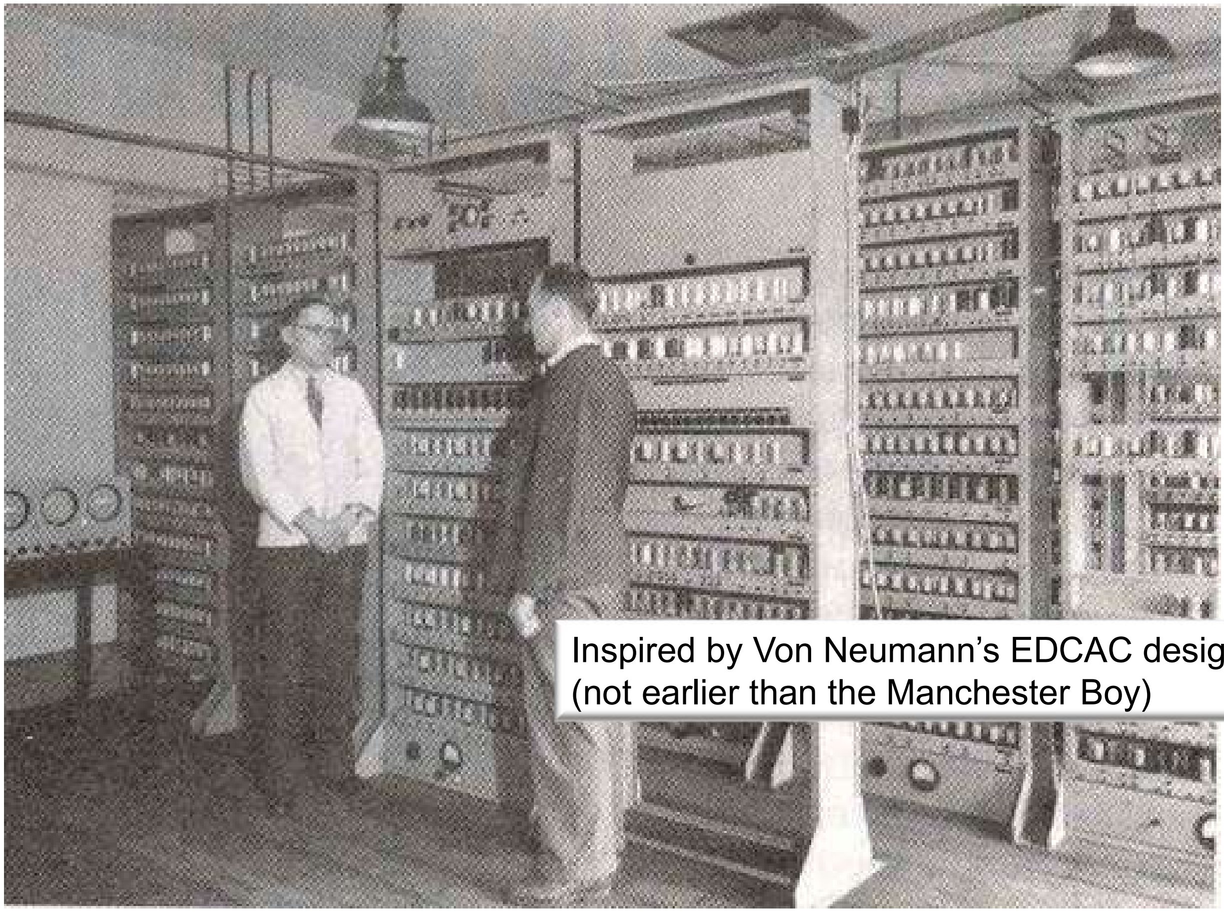


EDSAC (1949)

Electronic Delay Storage Automatic Calculator (Cambridge)



Inspired by Von Neumann's EDCAC design, but built earlier (not earlier than the Manchester Boy)

Babbage/Lovelace first to try to build it (before Turing)

distinction between *numbers that mean things*
and *numbers that (do things) move matter*