MODELLING
Aristotle (384-322 BC)

- First to relate symbols more explicitly to the external world and to successively clarify the nature of the symbol-world relation.
  - Student of Plato, educated Alexander the Great
  - First to consider specific observable factors which determine motion.
- In Physics
  - He recognized (mathematical) rules which could describe the relation between an object's weight, the medium's density and the consequent rate of motion (fall):
    1. For freely falling or freely rising bodies, speed is proportional to the density of the medium.
    2. In forced motion, speed is proportional to the force applied and inversely proportional to the mass of the body moved.
  - First time that observable quantities had been expressed in symbolic (numerical) form allowing the results of observations to be used in calculations.
    - The nature of causation
    - [http://classics.mit.edu/Aristotle/physics.html](http://classics.mit.edu/Aristotle/physics.html)

Modeling!

Raphael's “Plato and Aristotle”
Hertzian modeling paradigm

“The most direct and in a sense the most important problem which our conscious knowledge of nature should enable us to solve is the *anticipation of future events*, so that we may arrange our present affairs in accordance with such anticipation”. (Hertz, 1894)
The Antikythera Mechanism

- 2,000-year-old astronomical calculator
  - bronze mechanical analog computer
    - discovered more than 100 years ago in a Roman shipwreck, was used by ancient Greeks to display astronomical cycles.
    - built around the end of the second century BC to calculate astronomical positions
    - With imaging and high-resolution X-ray tomography to study how it worked.
      - complicated arrangement of at least 30 precision, hand-cut bronze gears housed inside a wooden case covered in inscriptions.
      - technically more complex than any known device for at least a millennium afterwards.

Not a universal Turing machine, but an analog computer
Let’s Observe Nature!

- What do you see?
  - Plants typically **branch** out
  - How can we **model** that?
    - Observe the distinct parts
      - Color them
      - Assign **symbols**
    - Build Model
      - Initial State: b
      - b -> a
      - a -> b
      - a -> ab
    - Doesn’t quite Work!

Psilophyta/Psilotum
Our First Model

- **Rewriting** production rules
  - Initial State: b
  - b -> a
  - a -> ab
    - n=0 : b
    - n=1 : a
    - n=2 : ab
    - n=3 : aba
    - n=4 : abaa
    - n=5 : ababa
    - n=6 : abababa
    - n=7 : abababaababaababaababaababaababa

- The length of the string is the Fibonacci Sequence
  - 1 1 2 3 5 8 13 21 34 55 89 ...

- Fibonacci numbers in Nature
  - Romanesco: [http://alt.venus.co.uk/weed/fractals/romanesco.htm](http://alt.venus.co.uk/weed/fractals/romanesco.htm)
Mathematics Is The Language Of Nature

http://pithemovie.com

When I was a kid my mother told me never to stare into the centre of the sun.
So once, when I was 6, I did.
organized complexity

Warren Weaver’ classes of systems and problems

- organized simplicity
  - very small number of variables
    - Deterministic
  - classical mathematical tools
    - Calculus

- disorganized complexity
  - very large number of variables
    - Randomness, homogenous
  - statistical tools

- organized complexity
  - sizable number of variables which are interrelated into an organic whole
  - study of organization
    - whole more than sum of parts
    - Massive combinatorial searches need for new mathematical and computational tools

examples

Disorganized complexity

Organized Complexity

Most relevant to problems in biology, medicine, society, and technology

Organized simplicity

Randomness

Complexity
organized complexity
- study of organization
  - whole is more than sum of parts
  - Systemhood properties
- Need for new mathematical and computational tools
  - Massive combinatorial searches
  - Problems that can only be tackled with computers
    - Computer as lab
- Interdisciplinary and collaborative science
  - Thrives in problem-driven environments
    - Los Alamos, Santa Fe, all new computing centers.

thinghood and systemhood
- Integration of empirical science with general systems
  - Interdisciplinarity coupled with computational modeling
- Understanding structure and function
  - Of multi-level wholes
    - Systems biology, Evolutionary thinking, Systems thinking
  - Emergence (or collective behavior)
    - How do elements combine to form new unities?
    - Micro- to macro-level behavior
key roots

- Mathematics
- Computer Technology
- Systems Thinking
  - Cybernetics
    - Looking at mind, life, society with control, computation, information, networks
  - Functional equivalence
    - General principles and modeling

Organized Complexity

- Study of organization
- "Whole is more than some of parts", nonlinearity, interaction, communication
- Interdisciplinary outlook
  - Not just math and computing, modeling requires understanding of focus domain
  - Bio-inspired mathematics and computing
  - Computing/Mechanism-inspired biology and social science

1965: Society for the Advancement of General Systems Theory

- Kenneth Boulding
- Ludwig von Bertalanffy
- Ralph Gerard
- Anatol Rapoport
Systemhood properties of nature

- Robert Rosen
  - Systems depend on a specific adjective: **thinghood**
  - **Systemhood**: properties of arrangements of items, independent of the items
    - Similar to “setness” or cardinality

- George Kliir
  - **Organization** can be studied with the mathematics of **relations**
  - $S = (T, R)$
    - $S$: a System, $T$: a set of things (thinghood), $R$: a (or set of) relation(s) (Systemhood)
  - Examples
    - Collections of books or music files are sets of things
    - But organization of such sets are systems (alphabetically, chronologically, typologically, etc.)
what is a system?

more formally

- **S = (T, R)**
  - a System
- **T = \{A_1, A_2, \ldots, A_n\}**
  - A set (of sets) of things: *thinghood*
- **Cartesian Product**
  - Set of all possible associations of elements from each set
    - All n-tuples
    - \(\{A_1 \times A_2 \times \ldots \times A_n\}\)
- **R: a relation (systemhood)**
  - Subset of cartesian product on T.
    - Many relations R can be defined on the same T
  
- **\(R \subseteq A^2 = A \times A\)**
  - \(R \subseteq A^n = (A \times A \times \ldots \times A)\)
  - \(R \subseteq (A \times A) \times A\)
  - \(R \subseteq A \times (A \times A)\)
  - \(R \subseteq (A \times A) \times (A \times A)\)
  
- **\(R \subseteq A \times B\)**
  - \(R \subseteq (A \times B) \times (A \times B)\)
  - \(R \subseteq (A \times A \times A) \times B\)
  - \(R \subseteq (A \times A \times A) \times (B \times B)\)
  - \(R \subseteq (A \times B) \times (A \times B) \times (A \times B)\)

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informatics.indiana.edu/rocha
Equivalence classes

Example

Table 2.1. Set of Students with Four Characteristics

<table>
<thead>
<tr>
<th>Student</th>
<th>Grade</th>
<th>Major</th>
<th>Age</th>
<th>Full-time/part-time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alan</td>
<td>B</td>
<td>Biology</td>
<td>19</td>
<td>Full-time</td>
</tr>
<tr>
<td>Bob</td>
<td>C</td>
<td>Physics</td>
<td>19</td>
<td>Full-time</td>
</tr>
<tr>
<td>Cliff</td>
<td>C</td>
<td>Mathematics</td>
<td>20</td>
<td>Part-time</td>
</tr>
<tr>
<td>Debby</td>
<td>A</td>
<td>Mathematics</td>
<td>19</td>
<td>Full-time</td>
</tr>
<tr>
<td>George</td>
<td>A</td>
<td>Mathematics</td>
<td>19</td>
<td>Full-time</td>
</tr>
<tr>
<td>Jane</td>
<td>A</td>
<td>Business</td>
<td>21</td>
<td>Part-time</td>
</tr>
<tr>
<td>Lisa</td>
<td>B</td>
<td>Chemistry</td>
<td>21</td>
<td>Part-time</td>
</tr>
<tr>
<td>Mary</td>
<td>C</td>
<td>Biology</td>
<td>19</td>
<td>Full-time</td>
</tr>
<tr>
<td>Nancy</td>
<td>B</td>
<td>Biology</td>
<td>19</td>
<td>Full-time</td>
</tr>
<tr>
<td>Paul</td>
<td>B</td>
<td>Business</td>
<td>21</td>
<td>Part-time</td>
</tr>
</tbody>
</table>

Table 2.2. Equivalence Relation $R_g$ Defined on the Set of Students Listed in Table 2.1 with Respect to Their Grades

<table>
<thead>
<tr>
<th>$R_g$</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>G</th>
<th>J</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>J</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>L</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>M</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ R \subseteq A \times B \times C \times D \]
Equivalence classes

example

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<td>Paul</td>
<td>B</td>
<td>Business</td>
<td>21</td>
<td>Part-time</td>
</tr>
</tbody>
</table>

Table 2.2. Definition of the Relation R

<table>
<thead>
<tr>
<th>R_{xy}</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
</tr>
<tr>
<td>J</td>
<td>0</td>
</tr>
<tr>
<td>L</td>
<td>1</td>
</tr>
<tr>
<td>M</td>
<td>1</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
</tr>
<tr>
<td>P</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ R \subseteq A \times B \times C \times D \]
Uncovering hierarchical organization

From genetic interaction maps (in yeast)

\[ R \subseteq A \times B, \]
\[ R \subseteq (A \times A) \times B, \]
\[ R \subseteq (A \times B) \times (A \times B), \]
\[ R \subseteq (A \times A \times A) \times B, \]
\[ R \subseteq (A \times A \times A) \times (B \times B), \]

general (complex) systems theory

Models of organized complexity

- Systemhood properties
  - Search for a language of *generalized circuits*
  - Isomorphisms of concepts, laws and models across fields
  - Minimize duplication of efforts across fields
  - Unity of science

- Not mathematics
  - Kenneth Boulding
    - “in a sense, because mathematics contains all theories it contains none; it is the language of theory, but it does not give us the content”
    - “body of systematic theoretical construction which will discuss general relationships of the empirical World”.
    - “somewhere between the specific that has no meaning and the general that has no content there must be, for each purpose an at each level of abstraction, an optimum degree of generality”.
  - Empirical and problem-driven

- Other relevant areas
  - Mathematical theories of control and generalized circuits
  - Optimal scheduling and resource allocation (operations research)
  - dynamical systems, chaos, AI, Alife, machine learning, network science, etc.
What about our plant?

Branching as a general system

- An Accurate Model
  - Requires
    - Varying angles
    - Varying stem lengths
    - Randomness
  - The Fibonacci Model is similar
    - Initial State: b
    - b -> a
    - a -> ab
  - sneezewort

Psilophyta/Psilotum

13 8 5 3 2 1
Branches
example of general principle of organization

**Barabasi-Albert Model**: leads to power-law node degree distributions in networks

**Amaral et al**: Most real networks have a cut-off distribution for high degree nodes which can be computationally modeled with vertex aging.
(complex) systems science

Study of “systemhood” separated from “thinghood”

- **Study of “systemhood” properties**
  - Classes of isomorphic abstracted systems
  - Search of **general principles of organization**
    - Weaver’s organized complexity (1948)

- **Systemhood properties**
  - preserved under suitable transformation from the set of things of one system into the set of things from the other system
    - Divides the space of possible systems (relations) into equivalent classes

- **Devoid of any interpretation!**
  - General systems
    - Canonical examples of equivalence classes

---

**Figure 2.6.** Two ways of classifying systems and the role of general systems. From Klar [2001]
Cybernetics and systems science

The language lives on

- Learning and cognition as information transmission
  - Brain and mind as mechanism
  - Computer as prevalent analogy/model for understanding life and cognition
- Feedback has come to mean information about the outcome of any process or activity
  - No word existed previously in English to convey that concept
- Interaction everywhere
  - Attention shifted from individualism and cause & effect, to circular causation and social interaction
  - "Programmed" behavior
- Society and organisms as systems
- Wiener's prediction of a second industrial revolution centered on communication, control, computation, information, and organization was correct
  - Abundance of technology and mass production of communication devices
    - Grew out of the ideas first reported by the cyberneticians
- Informatics is an offspring of cybernetics

The Google Books Ngram Viewer graph illustrates the trend of various terms over time, showing how the language of cybernetics and systems science has evolved.
The complexity worldview
  - Interdisciplinary and collaborative
    - Integration of empirical sciences with general-purpose modeling
    - Thrives in problem-driven environments
      - Los Alamos, Santa Fe, new computing centers
  - Data-driven, computational and mathematical modeling
    - Massive combinatorial searches
    - Networks, feedback, statistics, machine learning, dynamical systems
  - Study of organization
    - Whole is more than sum of parts
  - Nonlinear thinking
    - Counterintuitive system-level properties

Small changes in micro-level rules can change macro-level behavior dramatically

- Intuition can be a poor guide to predicting the behavior of a complex system.
- Simulation is a powerful tool for harnessing the dynamics of complex systems.
Training to see the world differently

The complexity worldview
- Nonlinear thinking
- Counterintuitive system-level properties

How can world function when “everything is connected”?

Kenneth Arrow

Stuart Kauffman

NK Boolean Network (N=13, K=3)

- When mean number of links greater than 2, dynamics is chaotic (with lower probability of “on”, better)

- 3 or more choices lead to unstable collective political choices

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