lecture 8

chaos and the Logistic Map

key events coming up

- Labs: 35% (ISE-483)
  - Complete 5 (best 4 graded) assignments based on algorithms presented in class
    - Lab 2 : February 19th
      - *L-Systems* (Assignment 2)
        - Delivered by SSIE583 Group 1
        - Due: February 26th
    - Lab 3: March 11th
      - Cellular Automata and Boolean Networks (Assignment 3)
        - Delivered by SSIE583 Group 3
        - Due: March 18th
- SSIE – 583 -Presentation and Discussion: 25%
  - Present and lead the discussion of an article related to the class materials
    - Enginet students post/send video or join by Zoom
  - February 26th
    - Kauffman, S.A. [1969]. "Metabolic stability and epigenesis in randomly constructed genetic nets". *Journal of Theoretical Biology* **22**(3):437-467.
      - Yoshiaki Fujita
  - Dates TBA
    - Conrad, M. [1990]. "The geometry of evolution.“ *Biosystems* **24**: 61-81.
      - Mario Franco
    - Stanley, Kenneth O., Jeff Clune, Joel Lehman, and Risto Miikkulainen. “Designing Neural Networks through Neuroevolution.” *Nature Machine Intelligence* **1**, no. 1 (January 2019): 24–35.
      - Jessica Lasebikan
    - Discussion by all

bit.ly/atBIC

until now

- **Class Book**
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press. Preface, **Sections 4.1, 4.2, Chapter 2.**
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall. **Chapter 1**, pp. 1-23. Chapter 7, sections **7.1-7.4**, **Appendix B.3.1**, **Chapter 2**, Chapter 8, sections **8.1, 8.2, 8.3.10**
- **Lecture notes**
  - Chapter 1: What is Life?
  - Chapter 2: The logical Mechanisms of Life
  - Chapter 3: Formalizing and Modeling the World
    - posted online @ http://informatics.indiana.edu/rocha/i-bic

- **Papers and other materials**
  - Dynamical Systems
    - Kauffman, S.A. [1969]. "Metabolic stability and epigenesis in randomly constructed genetic nets". Journal of Theoretical Biology 22(3):437-467.
  - Optional
    - Prusinkiewicz and Lindenmeyer [1996] *The algorithmic beauty of plants*.
      - Chapter 1
    - Flake's [1998], *The Computational Beauty of Life*. MIT Press.
      - Chapters 10, 11, 14 – Dynamics, Attractors and chaos

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until now

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constructed genetic nets".

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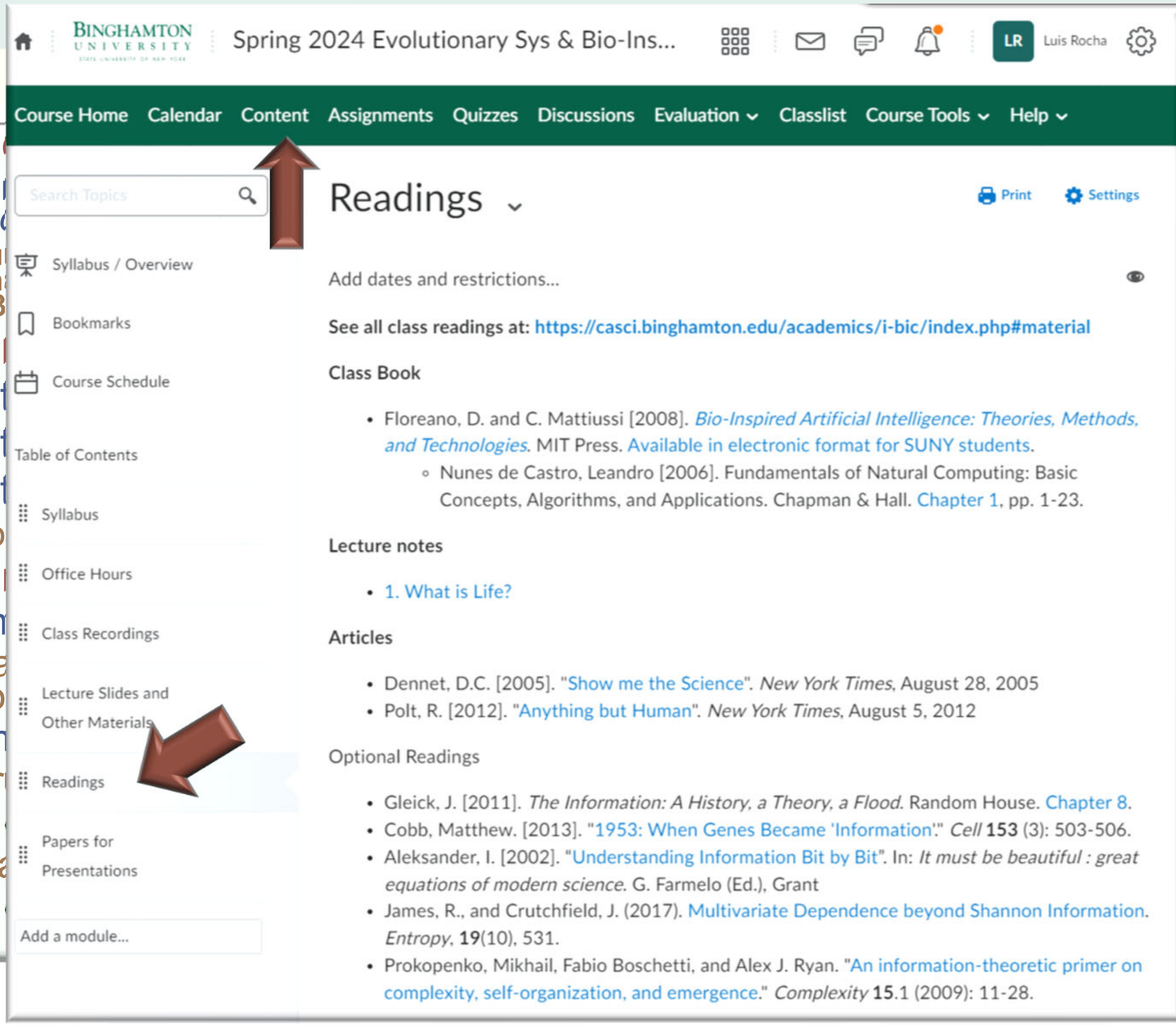
BINGHAMTON UNIVERSITY | Spring 2024 Evolutionary Sys & Bio-Ins... | LR Luis Rocha

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See all class readings at: https://casci.binghamton.edu/academics/i-bic/index.php#material

Class Book

- Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press. Available in electronic format for SUNY students.
  - Nunes de Castro, Leandro [2006]. Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications. Chapman & Hall. Chapter 1, pp. 1-23.

Lecture notes

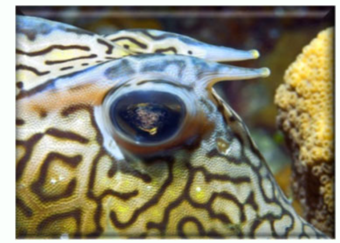
- 1. What is Life?

Articles

- Dennet, D.C. [2005]. "Show me the Science". *New York Times*, August 28, 2005
- Polt, R. [2012]. "Anything but Human". *New York Times*, August 5, 2012

Optional Readings

- Gleick, J. [2011]. *The Information: A History, a Theory, a Flood*. Random House. Chapter 8.
- Cobb, Matthew. [2013]. "1953: When Genes Became 'Information'." *Cell* **153** (3): 503-506.
- Aleksander, I. [2002]. "Understanding Information Bit by Bit". In: *It must be beautiful : great equations of modern science*. G. Farmelo (Ed.), Grant
- James, R., and Crutchfield, J. (2017). Multivariate Dependence beyond Shannon Information. *Entropy*, **19**(10), 531.
- Prokopenko, Mikhail, Fabio Boschetti, and Alex J. Ryan. "An information-theoretic primer on complexity, self-organization, and emergence." *Complexity* **15**.1 (2009): 11-28.



- Projects
  - Due by May 6th in Brightspace, “Final Project Paper” assignment
    - ALIFE 2023
      - Not to submit to actual conference due date (April 3rd , 2024)
      - https://2024.alife.org/
      - 8 pages, author guidelines:
      - https://2024.alife.org/call_paper.html
      - MS Word and Latex/Overleaf templates
    - Preliminary ideas **by March 15**
      - Submit to “Project Idea” assignment in Brightspace.
  - Individual or group
    - With very definite tasks assigned per member of group

ALIFE 2024

Tackle a real problem using bio-inspired algorithms, such as those used in the labs.

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exploring similarities across nature

- self-similar structures
  - Trees, plants, clouds, mountains
    - morphogenesis
  - Mechanism
    - Iteration, recursion, feedback
- dynamical systems and unpredictability
  - From limited knowledge or inherent in nature?
  - Mechanism
    - Chaos, measurement
- self-organization, collective behavior, emergence
  - Complex behavior from collectives of many simple units or agents
    - cellular automata, dynamical networks, morphogenesis, swarms, brains, social systems
  - Mechanism
    - Parallelism, multiplicity, multi-solutions, redundancy
- evolution
  - Adaptation, learning, social evolution
  - Mechanism
    - Reproduction, transmission, variation, selection, Turing's tape
- Network causality (heterogenous complexity)
  - Behavior derived from many inseparable sources
    - Immune system, anticipatory systems, brain-body-environment-culture, embodiment, epigenetics, culture
  - Mechanism
    - Modularity, control, hierarchy, connectivity, stigmergy, redundancy

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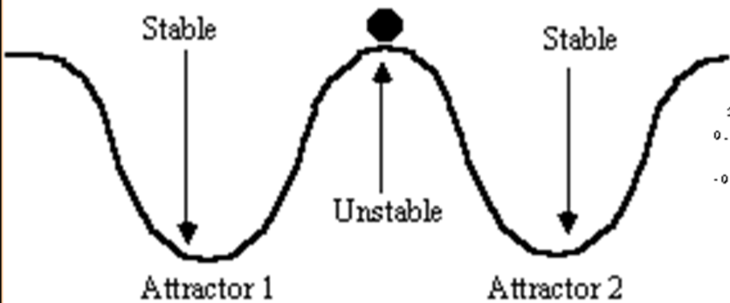
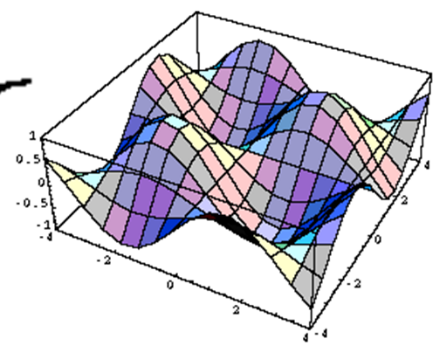
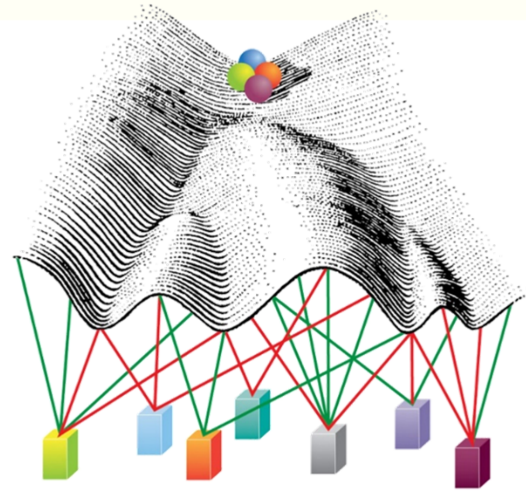
(energy) landscape metaphor

- Phase-space as landscape
  - State of the system as a drop of water released in hills and valleys

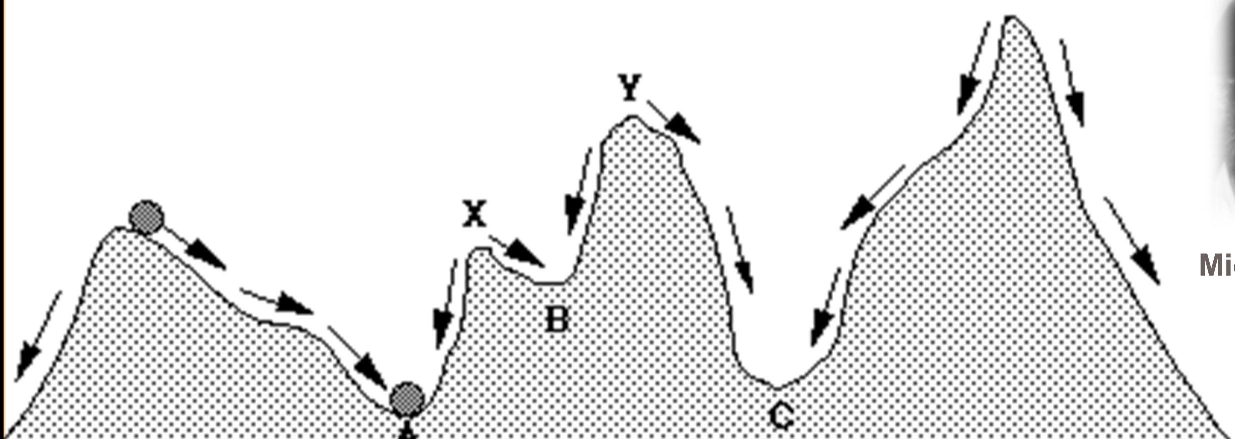




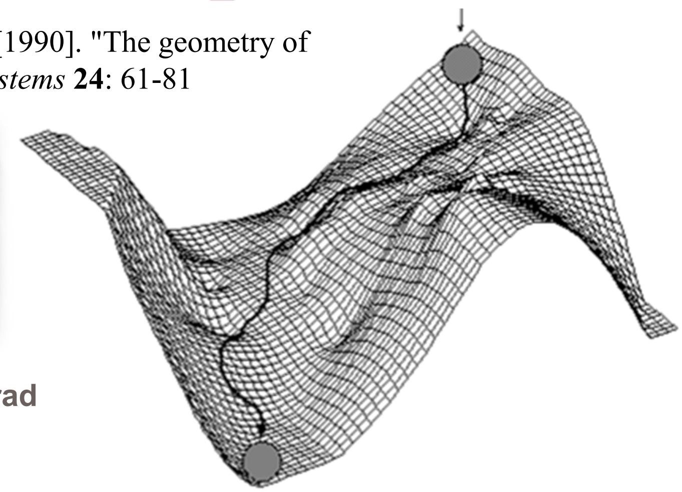


See: Conrad, M. [1990]. "The geometry of evolution.“ *Biosystems* **24**: 61-81

Waddington CH (1942). *Nature*.**150** (3811):563–565



Michael Conrad



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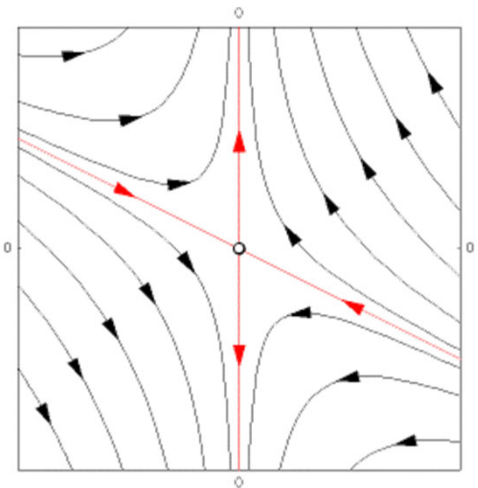
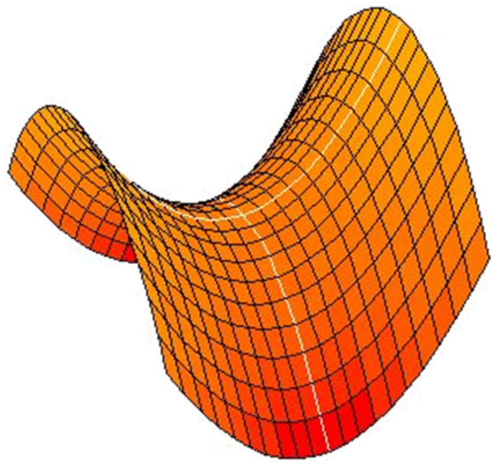
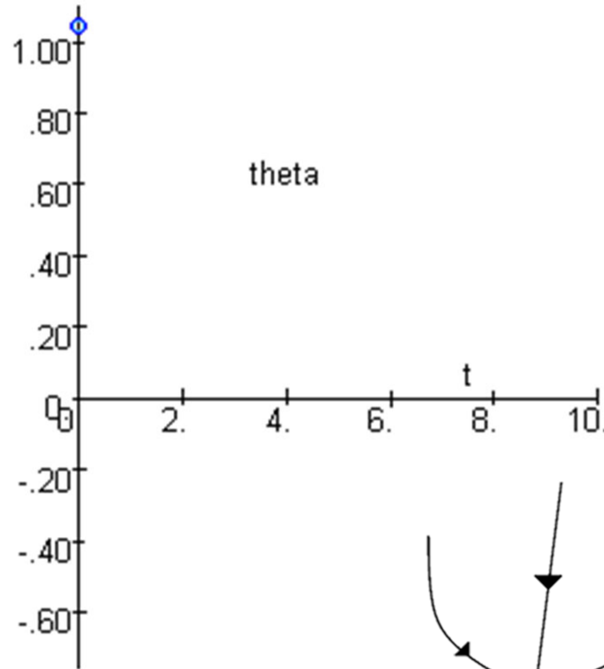
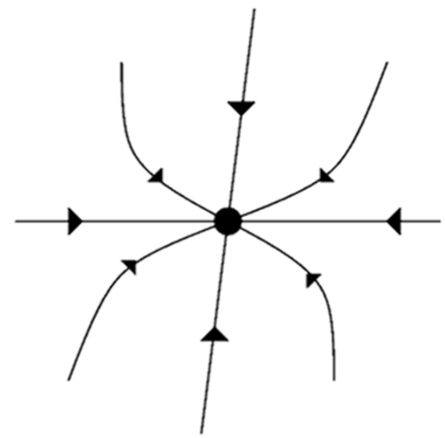
from simple…

- Attractors
  - Phase space volume to where dynamical system converges asymptotically over time
- Fixed point
  - Steady-state
  - Saddle
    - Stable in a dimension and unstable on another
    - When basins of attraction meet









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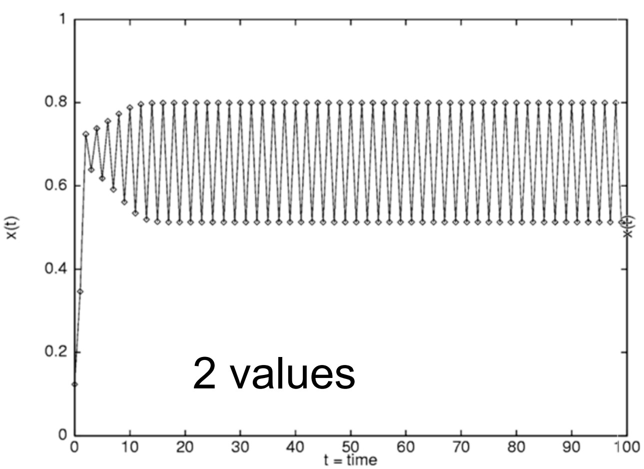
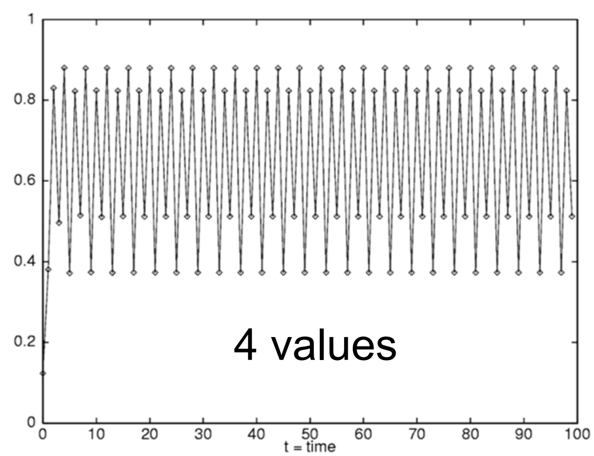
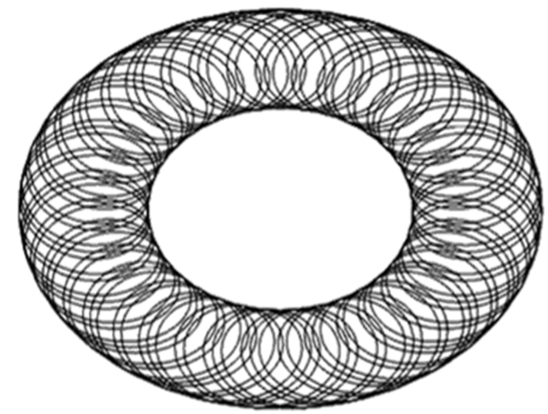
types of attractors

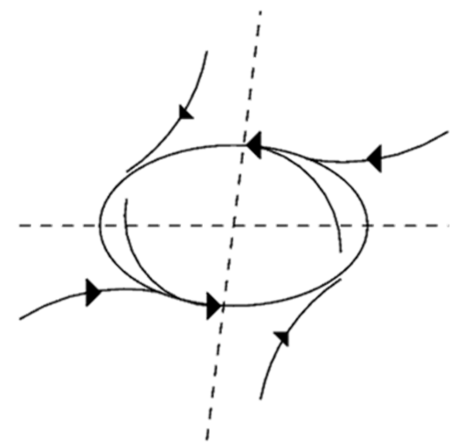
…to more complex

- **Limit cycle**
  - Periodic motion
  - Repetitive oscillation among a number of states
    - Loop
- **Quasiperiodic attractor**
  - Several independent cyclic motions
  - Toroidal attractors
  - Never quite repeat themselves









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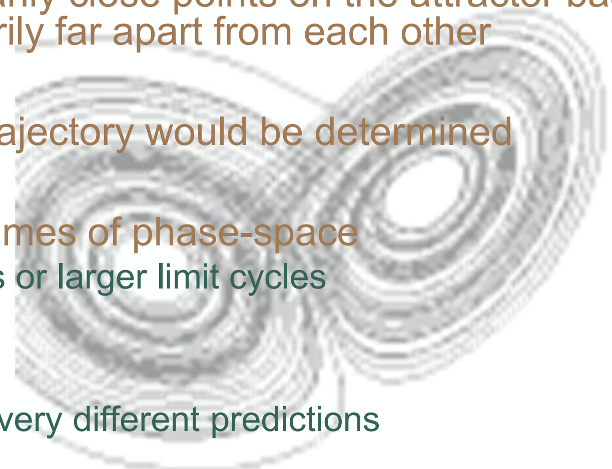
strange attractors

- **Strange or chaotic attractors**
  - Sensitivity to initial conditions
    - If system is released from two distinct, arbitrarily close points on the attractor basin, after sufficient time their trajectories will be arbitrarily far apart from each other
  - Deterministic Chaos
    - If we could know the exact initial condition, trajectory would be determined
  - Low-dimensional chaos
    - Strange attractors are restricted to small volumes of phase-space
      - Can be more ordered than Hamiltonian chaos or larger limit cycles
  - Weak Causality
    - 3-body problem
      - Any slight measurement difference results in very different predictions
    - Butterfly effect
      - Lorenz attractor



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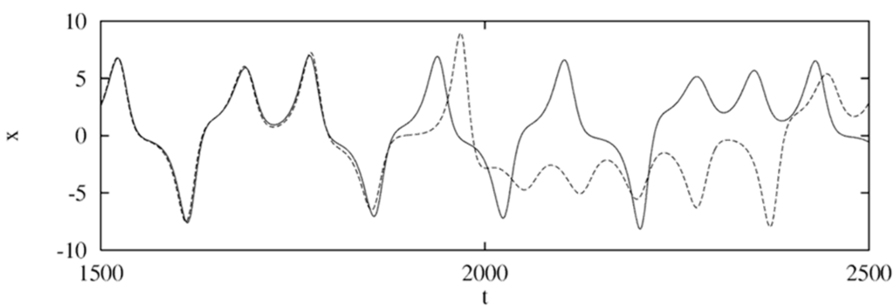
Edward Lorenz

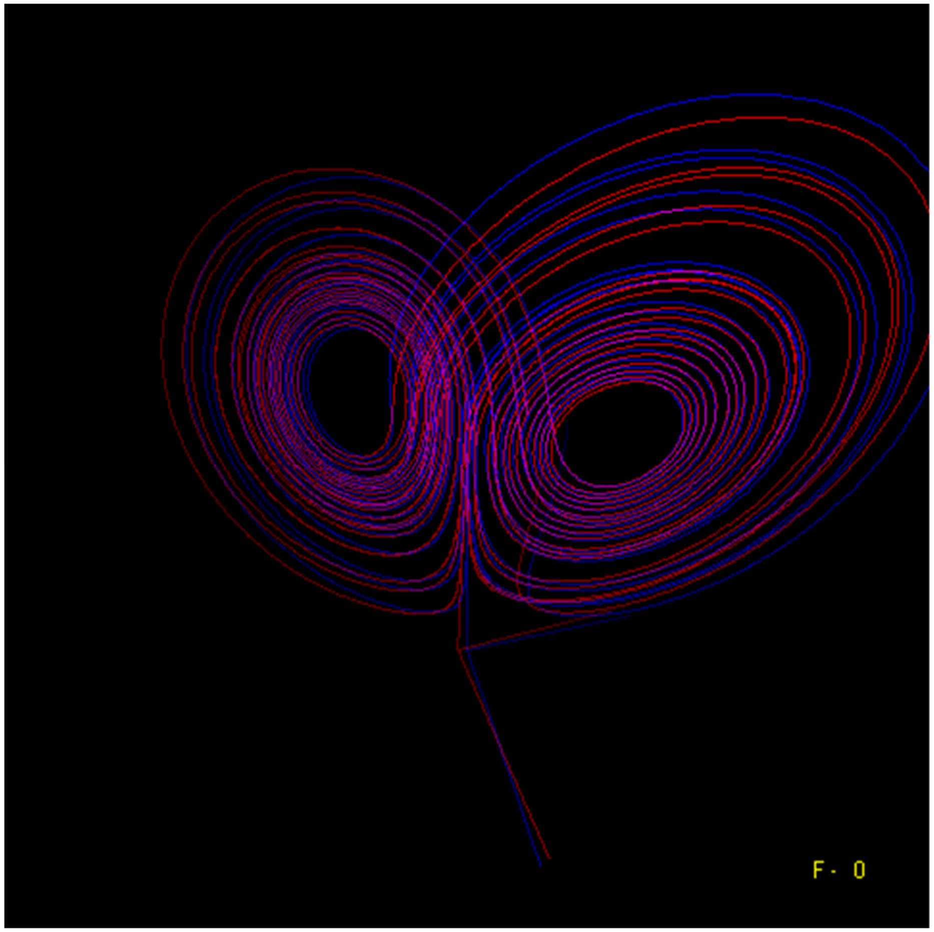
- Discovered sensitivity to initial conditions in a simple 3-variable dynamical system
  - A simplified model of weather
    - Convection flows in the atmosphere

$$\frac{dx}{dt} = \sigma(y-x)$$

$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dz}{dt} = xy - bz$$





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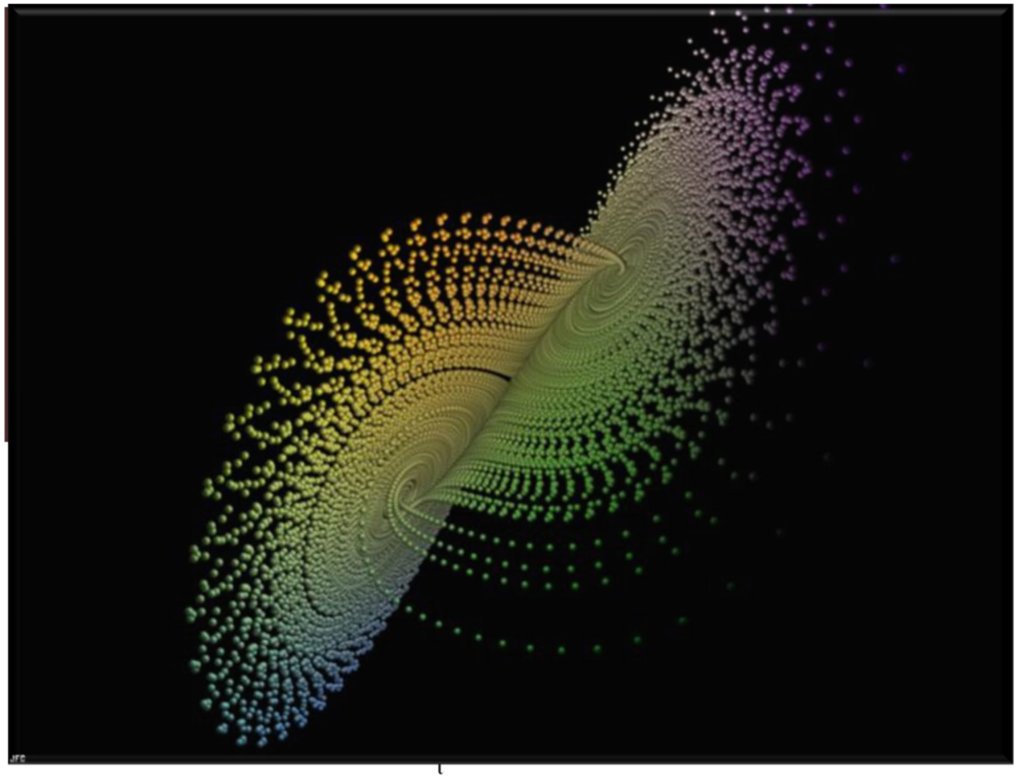

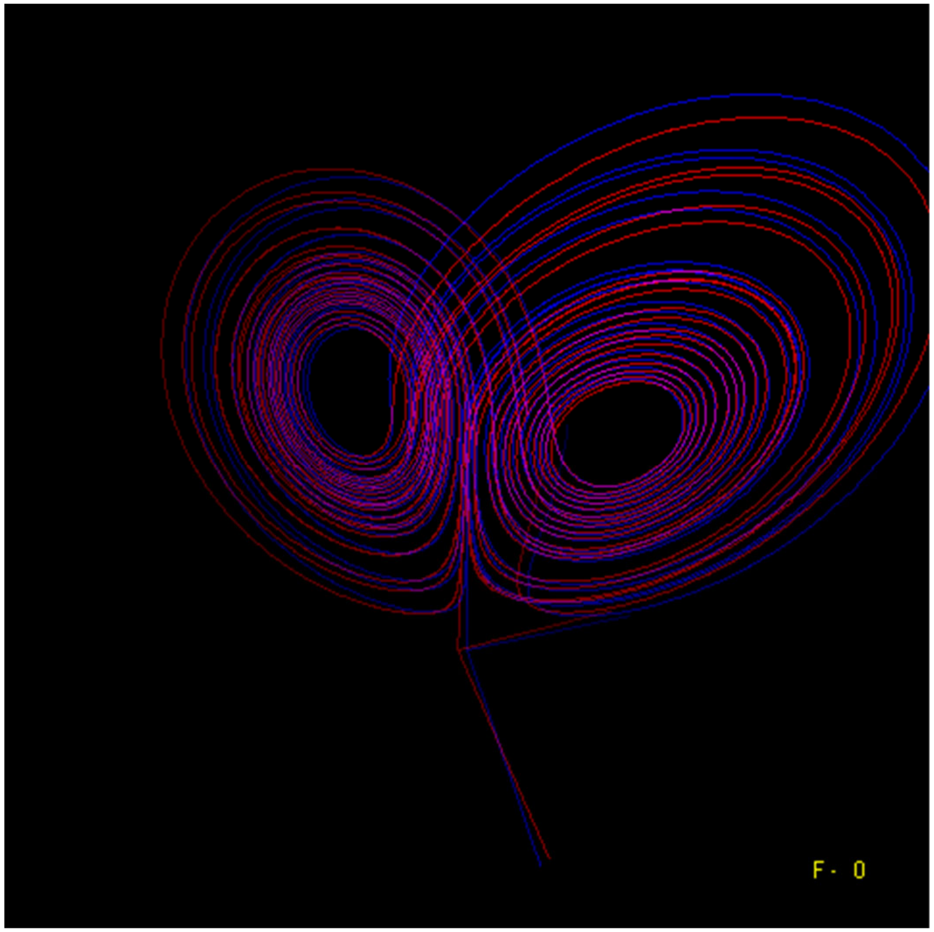

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quadratic equation

- Demographic model
  - introduced by Pierre François Verhulst in 1838
- Continuous state-determined system
  - Memory of the previous state only
- Observations
  - X=0: population extinct
  - X=1: Overpopulation, leads to extinction

Population size

positive feedback

negative feedback

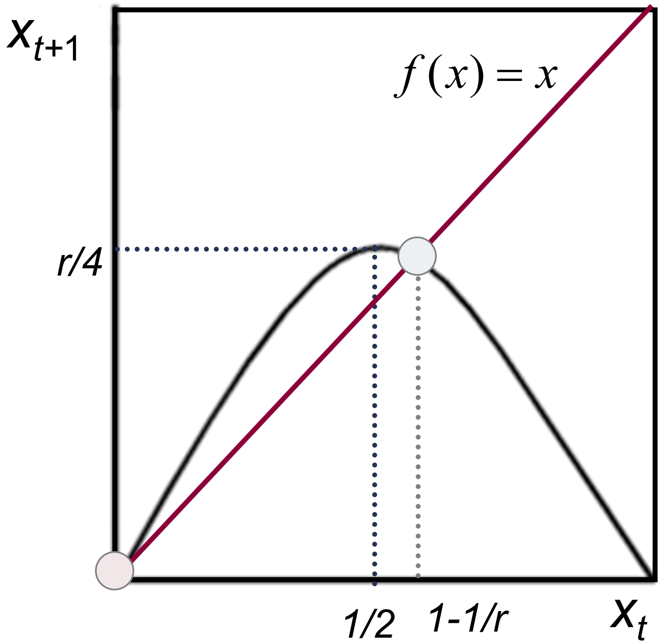
$$x_{t+1} = r x_t (1 - x_t)$$

Reproduction rate

$$x \in [0,1]$$

$$r \in [0,4]$$

plot



$$f(x) = rx(1-x)$$

$$x_{t+1} = rx_t(1-x_t)$$

$$x \in [0,1]$$

$$r \in [0,4]$$

$$f'(x) = r(1-2x), \begin{cases} |f'(x)| < 1 \Rightarrow \text{stable} \\ |f'(x)| > 1 \Rightarrow \text{unstable} \end{cases}$$

$$x = 0 \vee x = 1 - \frac{1}{r}$$

Fixed-point attractors

$$f(x) = x \Leftrightarrow rx(1-x) = x \Leftrightarrow x\big(r(x-1)+1\big) = 0$$

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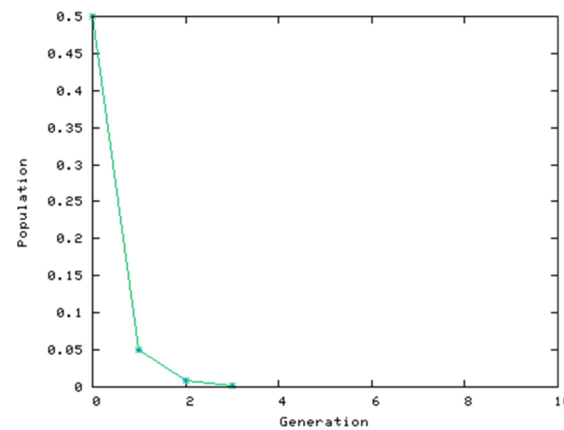
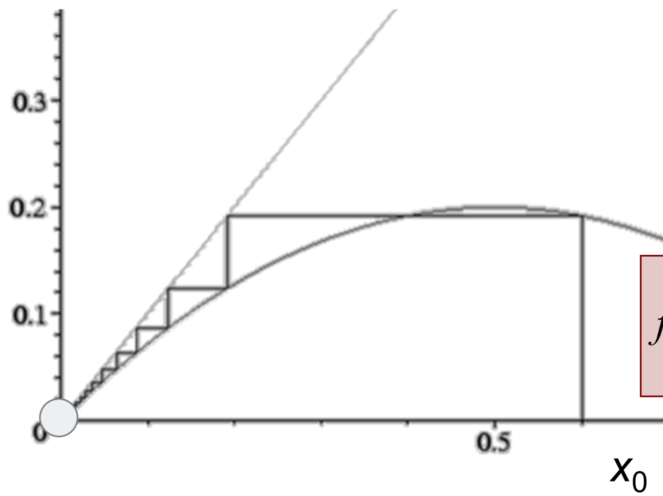
r ≤ 1 (population goes extinct)

$$x = 0 \vee x = 1 - \frac{1}{r}$$





$$f'(x) = r(1-2x), \begin{cases} |f'(x)| < 1 \Rightarrow \text{stable} \\ |f'(x)| > 1 \Rightarrow \text{unstable} \end{cases}$$

Stable attractor = population extinction

$$x = 0 \Rightarrow |f'(x)| = |r(1-2x)| = r, \begin{cases} r < 1 \Rightarrow \text{stable} \\ r > 1 \Rightarrow \text{unstable} \end{cases}$$

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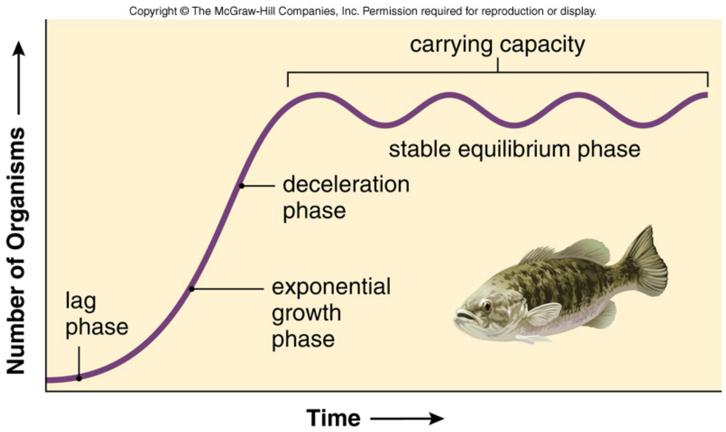
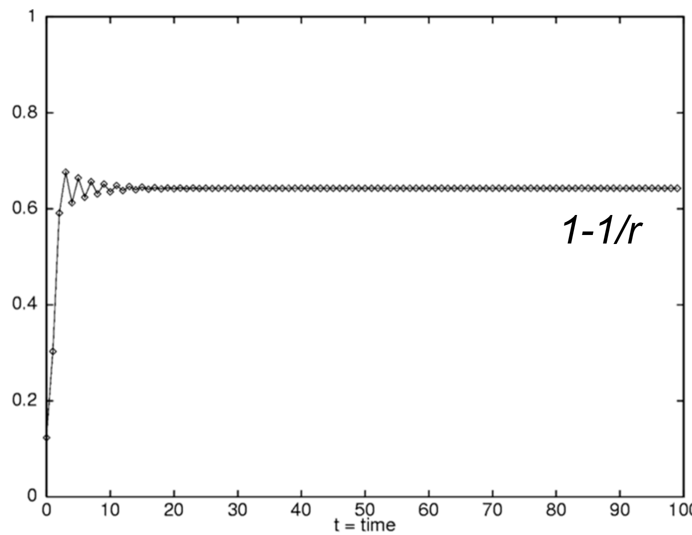
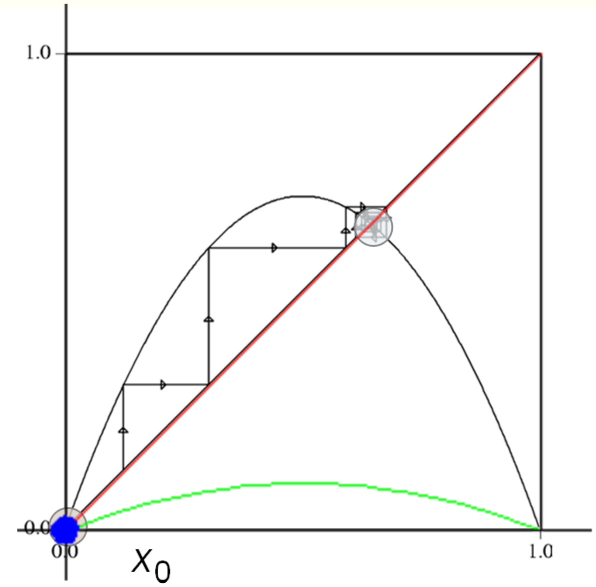
1 ≤ r ≤ 3







$$x = 0 \vee x = 1 - \frac{1}{r}$$

$$f'(x) = r(1-2x), \begin{cases} |f'(x)| < 1 \Rightarrow \text{stable} \\ |f'(x)| > 1 \Rightarrow \text{unstable} \end{cases}$$

Stable attractor = 1-1/*r*

$$x = 0 \Rightarrow |f'(x)| = r$$

$$x = 1 - \frac{1}{r} \Rightarrow |f'(x)| = |2 - r|$$

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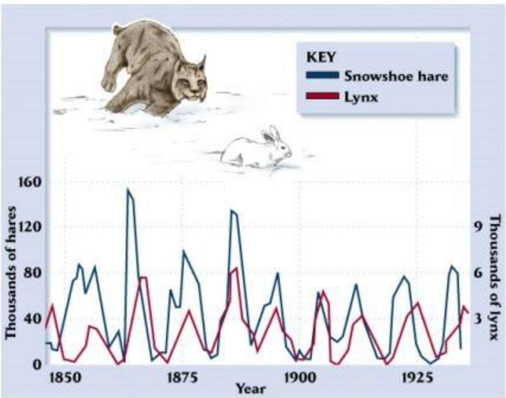
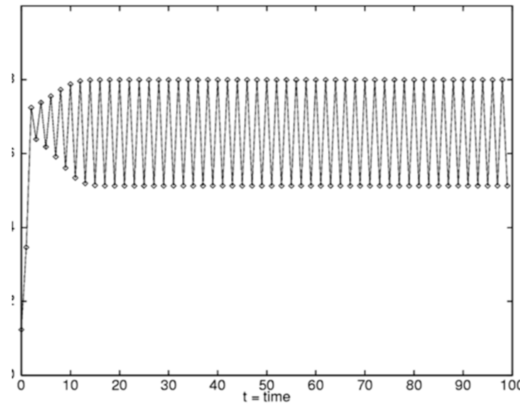
$3 \le r \le 4$ ($r \le 3.44$)



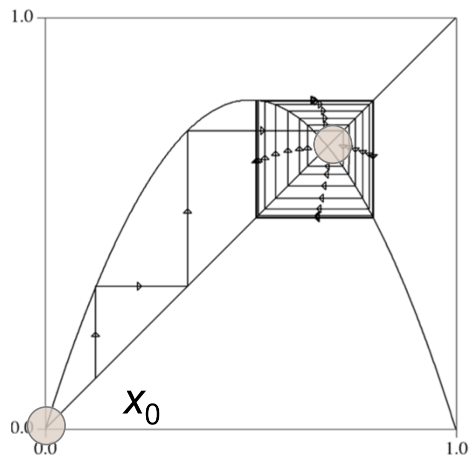


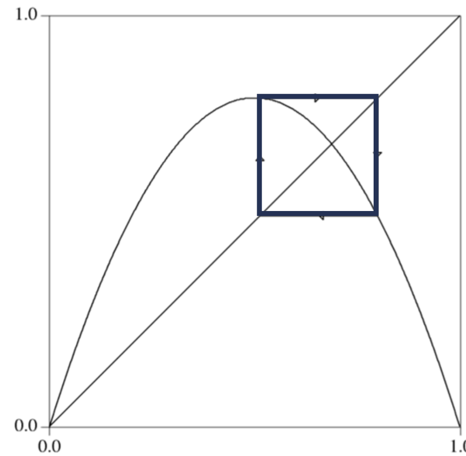
$$x = 0 \vee x = 1 - \frac{1}{r}$$

$$f'(x) = r(1-2x), \begin{cases} |f'(x)| < 1 \Rightarrow \text{stable} \\ |f'(x)| > 1 \Rightarrow \text{unstable} \end{cases}$$

$$x = 1 - \frac{1}{r} \Rightarrow |f'(x)| = |2 - r|$$

$$x = 0 \Rightarrow |f'(x)| = r$$





Limit cycle

$$f(f(x)) = x$$

Stable attractor = 2-point limit-cycle (oscillation)

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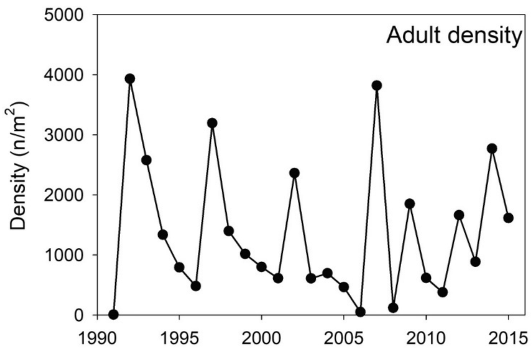
$3 \leq r \leq 4$ ($3.44 \leq r \leq 3.54$)



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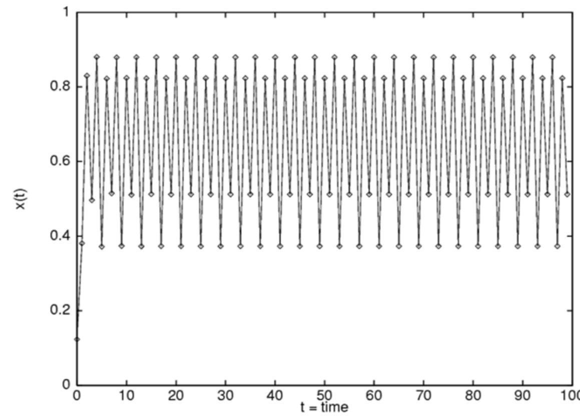
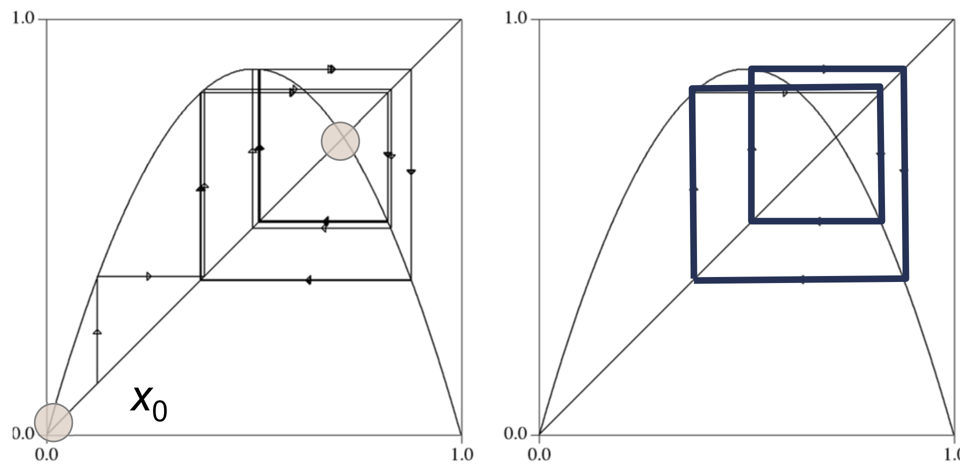


(a)

$$x = 0 \vee x = 1 - \frac{1}{r}$$

$$f(f(f(f(x)))) = x$$



Limit cycle

Stable attractor = 4-point limit-cycle

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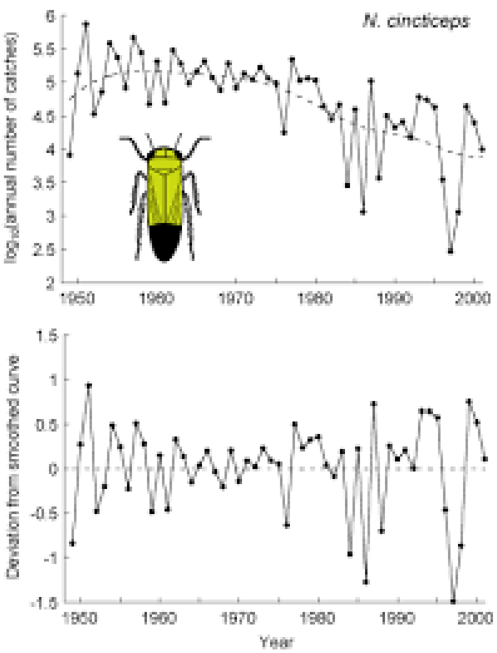
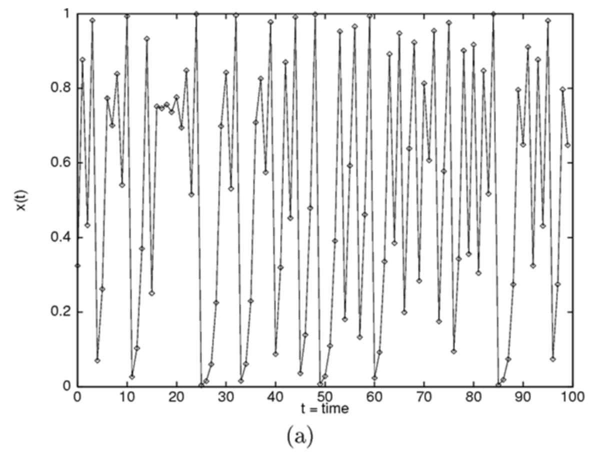
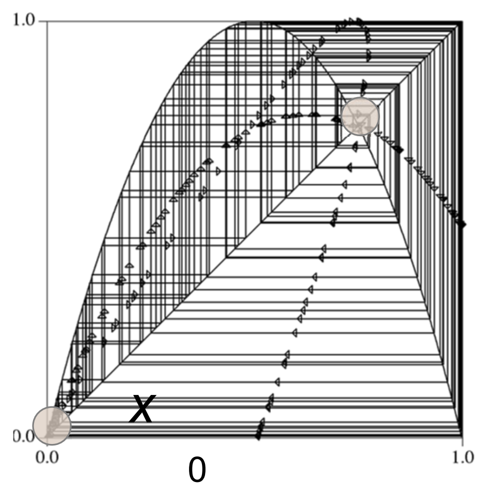
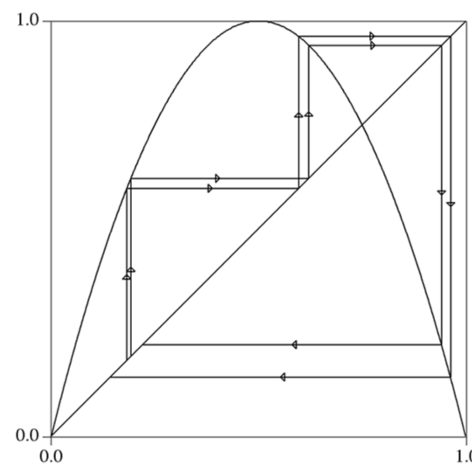
r = 4





$$x = 0 \vee x = 1 - \frac{1}{r}$$





Chaotic

Deterministic

Sensitive

ergodic

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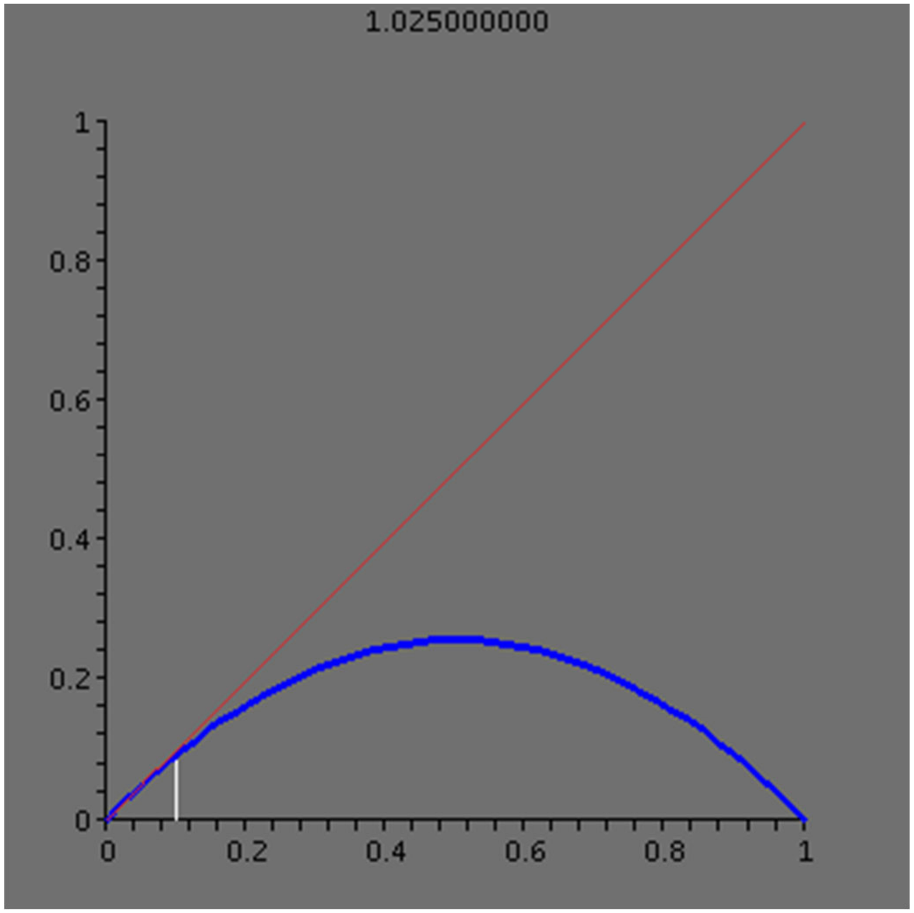
movie



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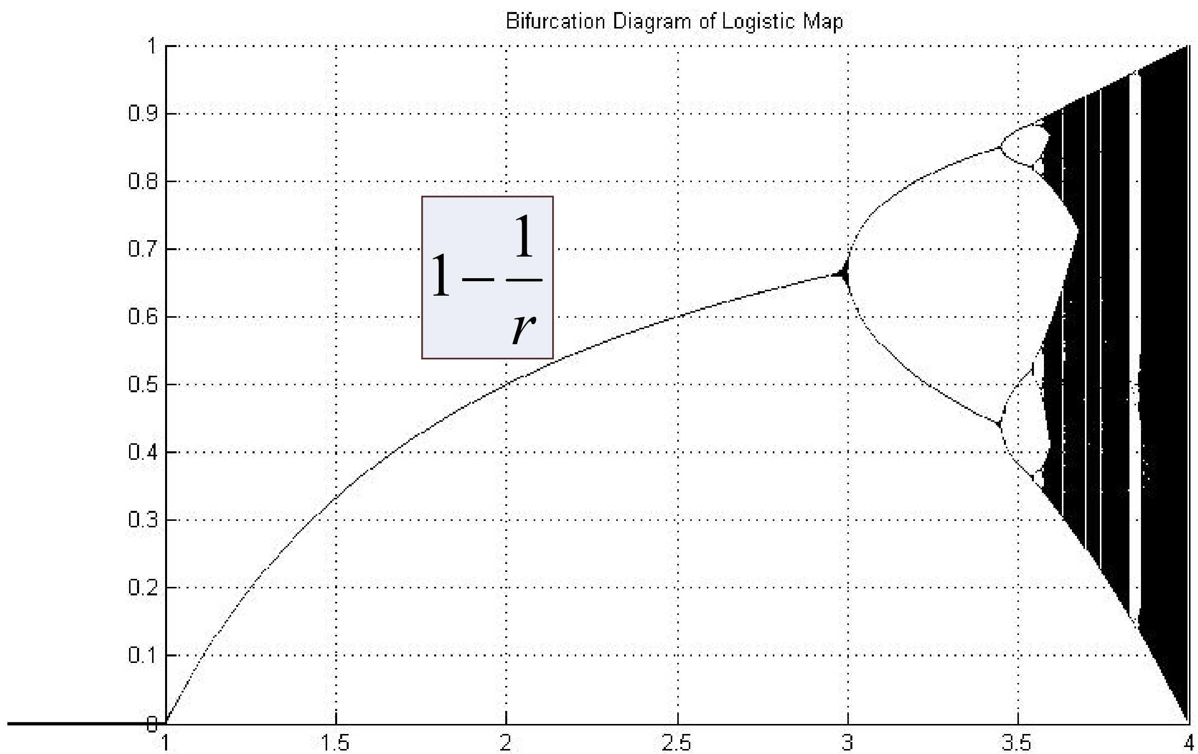
bifurcation map



Period doubling

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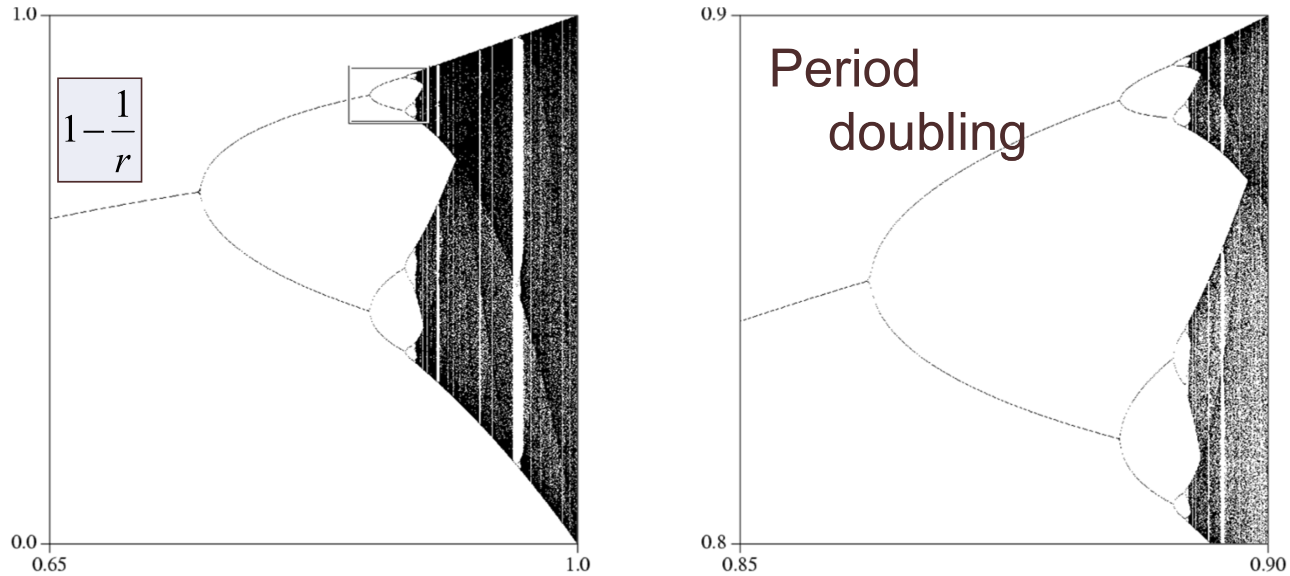
bifurcation map



Chaotic

Deterministic (not random)

Sensitive

ergodic

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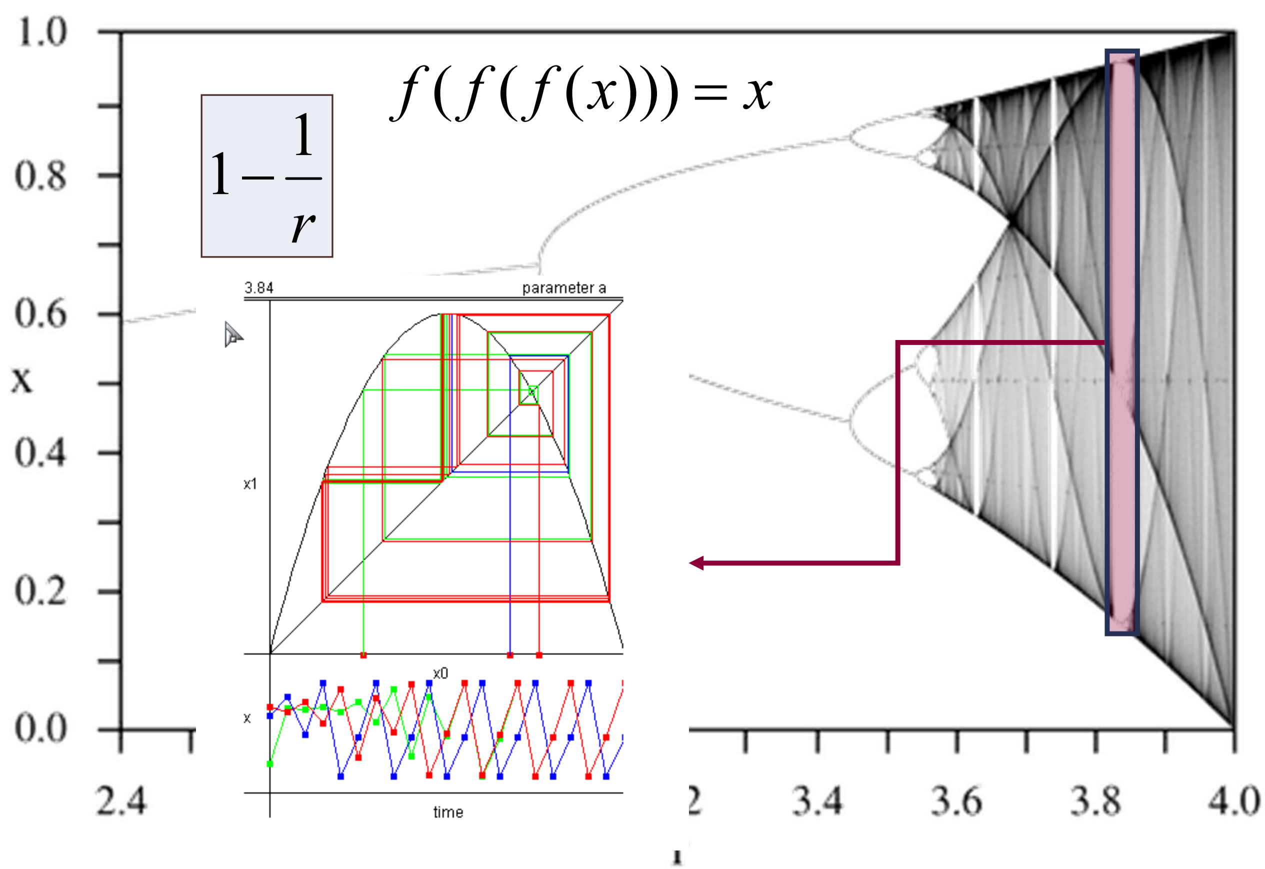
bifurcation map: cycle of 3



Period doubling

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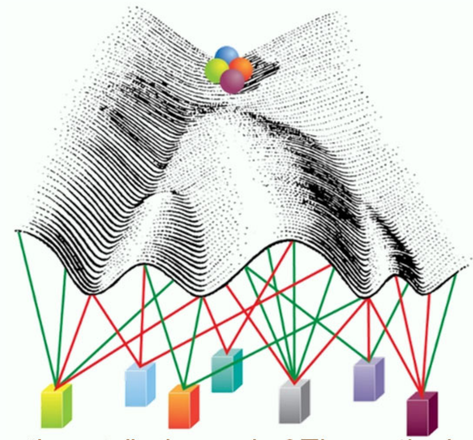
readings

- Class Book
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press.
    - **Chapter 2**.
- Lecture notes
  - Chapter 1: What is Life?
  - Chapter 2: The logical Mechanisms of Life
  - Chapter 3: Formalizing and Modeling the World
  - Chapter 4: Self-Organization and Emergent Complex Behavior
    - posted online @ http://informatics.indiana.edu/rocha/i-bic



- Papers and other materials
  - Discussions
    - Kauffman, S.A. [1969]. "Metabolic stability and epigenesis in randomly constructed genetic nets". *Journal of Theoretical Biology* **22**(3):437-467.
  - Optional
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall.
      - Chapter 2, all sections
      - Chapter 7, sections 7.3 – Cellular Automata
      - Chapter 8, sections 8.1, 8.2, 8.3.10
    - Flake's [1998], *The Computational Beauty of Life*. MIT Press.
      - Chapters 10, 11, 14 – Dynamics, Attractors and chaos

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