lecture 7

Dynamical Systems And Attractor Behavior

bit.ly/atBIC

key events coming up

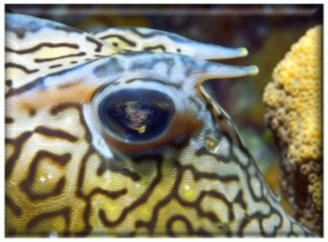
- Labs: 35% (ISE-483)
  - Complete 5 (best 4 graded) assignments based on algorithms presented in class
    - Lab 2 : February 19th
      - *L-Systems* (Assignment 2)
        - Delivered by SSIE583 Group 1
        - Due: February 26th
    - Lab 3: March 11th
      - Cellular Automata and Boolean Networks (Assignment 3)
        - Delivered by SSIE583 Group 3
        - Due: March 18th
- SSIE – 583 -Presentation and Discussion: 25%
  - Present and lead the discussion of an article related to the class materials
    - Enginet students post/send video or join by Zoom
  - February 26th
    - Kauffman, S.A. [1969]. "Metabolic stability and epigenesis in randomly constructed genetic nets". *Journal of Theoretical Biology* **22**(3):437-467.
      - Yoshiaki Fujita
  - Dates TBA
    - Conrad, M. [1990]. "The geometry of evolution.“ *Biosystems* **24**: 61-81.
      - Mario Franco
    - Stanley, Kenneth O., Jeff Clune, Joel Lehman, and Risto Miikkulainen. “Designing Neural Networks through Neuroevolution.” *Nature Machine Intelligence* **1**, no. 1 (January 2019): 24–35.
      - Jessica Lasebikan
    - Discussion by all

bit.ly/atBIC

readings

until now

- Class Book
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press. Preface**, Sections 4.1, 4.2, Chapter 2.**
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall. **Chapter 1**, pp. 1-23. Chapter 7, sections **7.1-7.4**, **Appendix B.3.1**, **Chapter 2**, Chapter 8, sections **8.1, 8.2, 8.3.10**
- Lecture notes
  - Chapter 1: What is Life?
  - Chapter 2: The logical Mechanisms of Life
  - Chapter 3: Formalizing and Modeling the World
    - posted online @ http://informatics.indiana.edu/rocha/i-bic



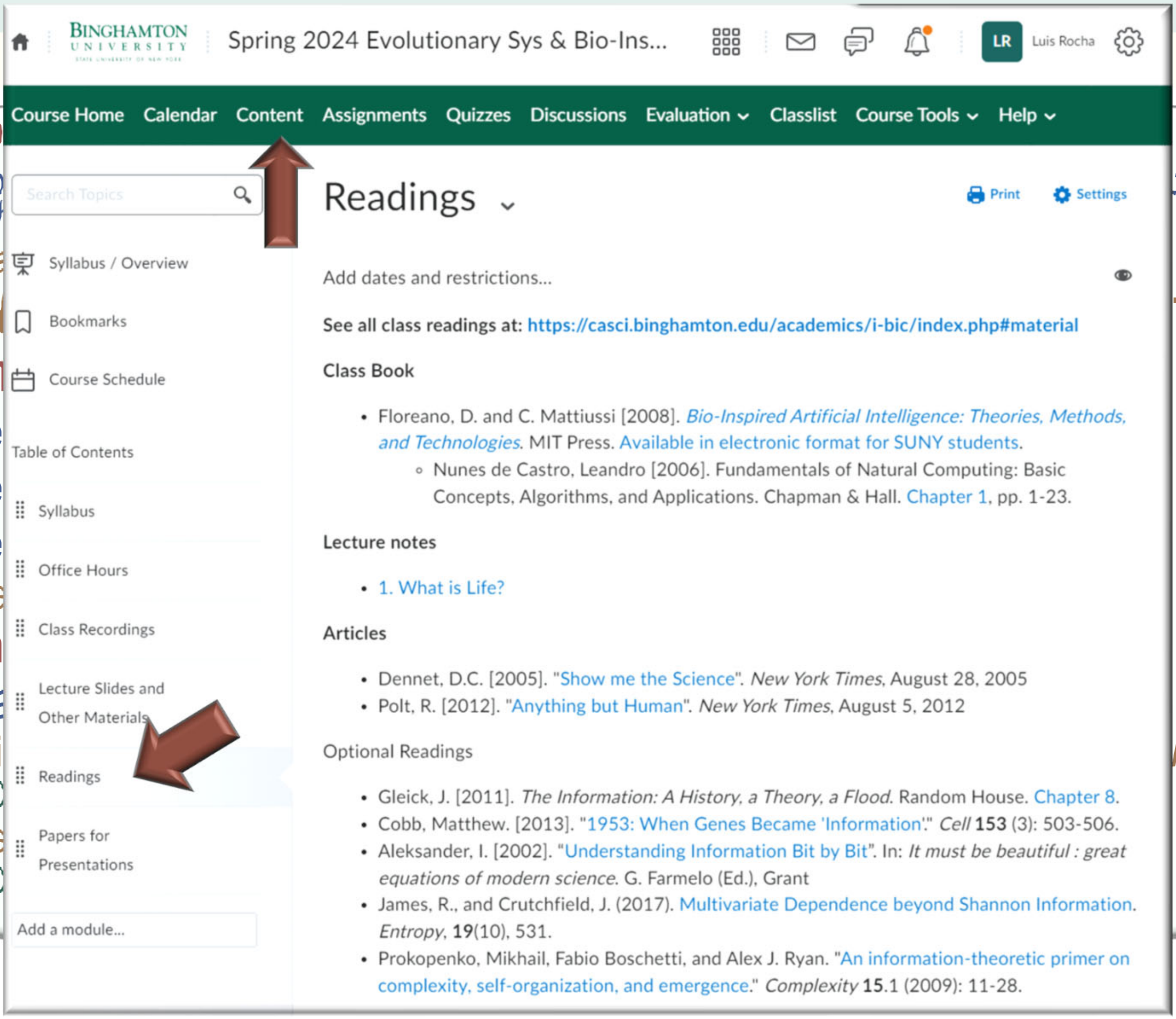
- Papers and other materials
  - Optional
    - Prusinkiewicz and Lindenmeyer [1996] *The algorithmic beauty of plants*.
      - Chapter 1
    - Flake's [1998], *The Computational Beauty of Life*. MIT Press.
      - Chapters 10, 11, 14 – Dynamics, Attractors and chaos

bit.ly/atBIC

readings

until now

- Class Boo
  - Floreano
    and Tech
    - Nunes
      and A
      Chapt
- Lecture n
  - Chapte
  - Chapte
  - Chapte
    - poste
- Papers an
  - Optiona
    - Prusi
      - C
    - Flake
      - C



Readings

Add dates and restrictions...

See all class readings at: https://casci.binghamton.edu/academics/i-bic/index.php#material

Class Book

- Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press. Available in electronic format for SUNY students.
  - Nunes de Castro, Leandro [2006]. Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications. Chapman & Hall. Chapter 1, pp. 1-23.

Lecture notes

- 1. What is Life?

Articles

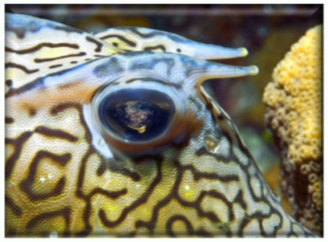
- Dennet, D.C. [2005]. "Show me the Science". *New York Times*, August 28, 2005
- Polt, R. [2012]. "Anything but Human". *New York Times*, August 5, 2012

Optional Readings

- Gleick, J. [2011]. *The Information: A History, a Theory, a Flood*. Random House. Chapter 8.
- Cobb, Matthew. [2013]. "1953: When Genes Became 'Information'." *Cell* **153** (3): 503-506.
- Aleksander, I. [2002]. "Understanding Information Bit by Bit". In: *It must be beautiful : great equations of modern science*. G. Farmelo (Ed.), Grant
- James, R., and Crutchfield, J. (2017). Multivariate Dependence beyond Shannon Information. *Entropy*, **19**(10), 531.
- Prokopenko, Mikhail, Fabio Boschetti, and Alex J. Ryan. "An information-theoretic primer on complexity, self-organization, and emergence." *Complexity* **15**.1 (2009): 11-28.

: Theories, Methods,

Concepts, Algorithms,
.1-7.4, Appendix B.3.1,



lants.

final project schedule

- Projects
  - Due by May 6th in Brightspace, “Final Project Paper” assignment
    - ALIFE 2023
      - Not to submit to actual conference due date (April 3rd , 2024)
      - https://2024.alife.org/
      - 8 pages, author guidelines:
      - https://2024.alife.org/call_paper.html
      - MS Word and Latex/Overleaf templates
    - Preliminary ideas **by March 15**
      - Submit to “Project Idea” assignment in Brightspace.
  - Individual or group
    - With very definite tasks assigned per member of group

ALIFE 2024

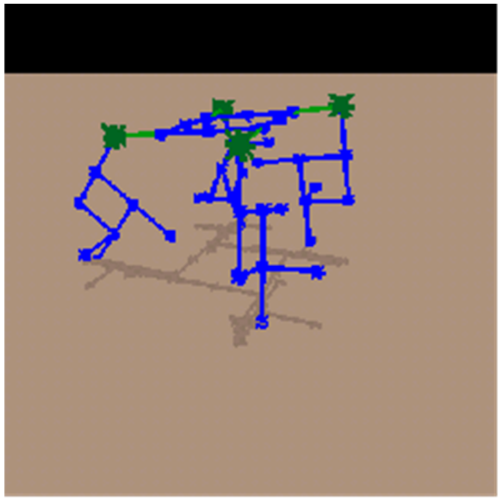
Tackle a real problem using bio-inspired algorithms, such as those used in the labs.

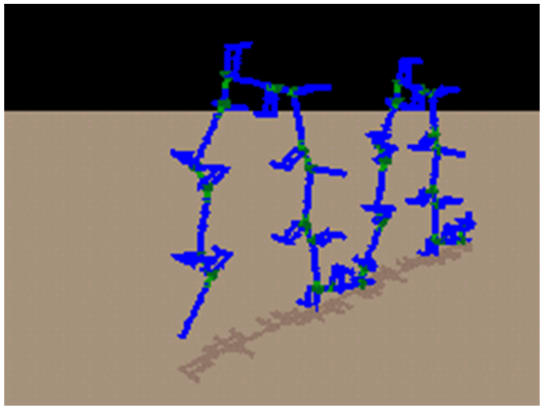
The 2024 Conference on Artificial Life

Copenhagen, Denmark | July 22-26, 2024

robots

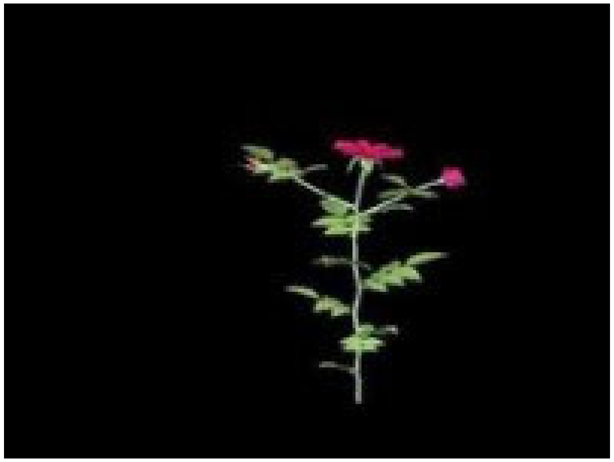
- generative design of robots
  - Karl Sims and Jordan Pollack, Hod Lipson, Gregory Hornby, and Pablo Funes claim that for automatic design to scale in complexity it must employ re-used modules
    - Sims,K. [1994]. “Evolving Virtual Creatures”. *Proceedings of the 21st annual conference on Computer graphics and interactive techniques*, pp. 15 – 22.
    - H. Lipson and J. B. Pollack (2000), "Automatic design and Manufacture of Robotic Lifeforms", *Nature* **406**: 974-978.
  - *Generative, iterative growth/development*
    - an algorithm for creating a design
  - Indirect representation of solutions (for evolutionary algorithms)
    - using Lindenmayer systems (L-systems)
    - evolved locomotiong robots (called *genobots*).

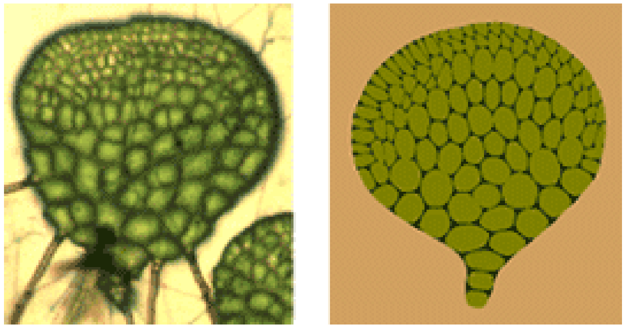




models or realistic imitations?

- Common features (design principle) between artificial and real plants
  - Development of (macro-level) morphology from local (micro-level) logic
  - Parallel application of simple rules
    - Genetic vs. algorithmic
  - Recursion
- But are the algorithms the same as the biological *mechanism*?
  - Real organisms need to economize information for coding complex phenotypes
    - The genome cannot encode every ripple of the brain or lungs
    - Organisms need to encode **compact procedures** for producing the same pattern (with randomness) again and again
- But recursion alone does not explain form and morphogenesis
  - One of the design principles involved
  - There are others
    - Selection, genetic variation, self-organization, epigenetics





fern gametophyte Microsorium linguaeforme (left) and a simulated model using map L systems (right).

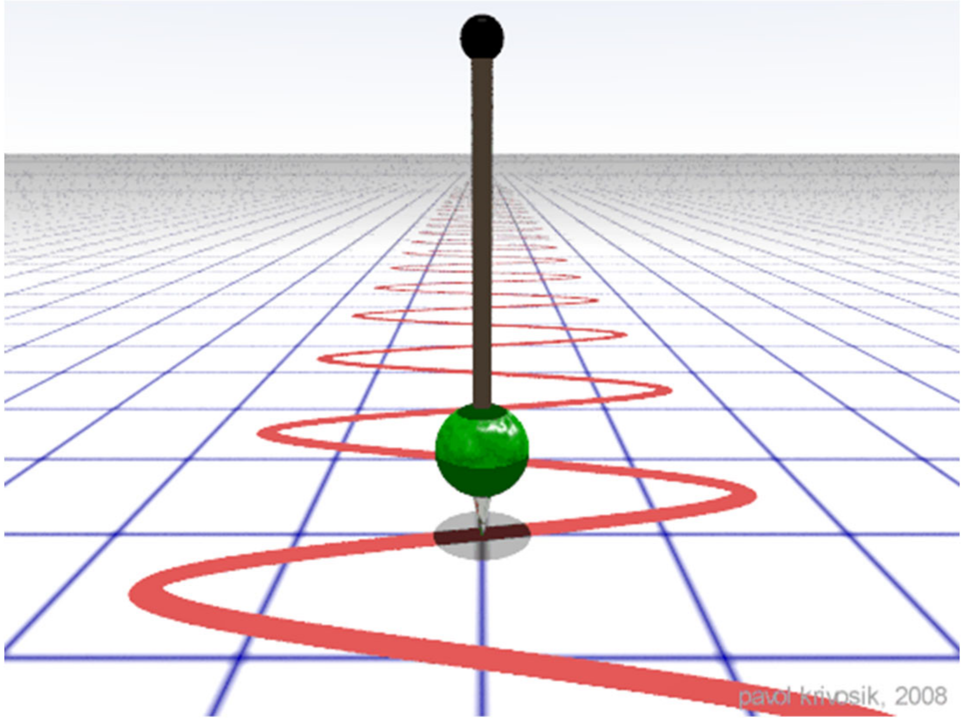
Natural design principles

exploring similarities across nature

- self-similar structures
  - Trees, plants, clouds, mountains
    - morphogenesis
  - Mechanism
    - Iteration, recursion, feedback
- dynamical systems and unpredictability
  - From limited knowledge or inherent in nature?
  - Mechanism
    - Chaos, measurement
- self-organization, collective behavior, emergence
  - Complex behavior from collectives of many simple units or agents
    - cellular automata, dynamical networks, morphogenesis, swarms, brains, social systems
  - Mechanism
    - Parallelism, multiplicity, multi-solutions, redundancy
- evolution
  - Adaptation, learning, social evolution
  - Mechanism
    - Reproduction, transmission, variation, selection, Turing’s tape
- Network causality (heterogenous complexity)
  - Behavior derived from many inseparable sources
    - Immune system, anticipatory systems, brain-body-environment-culture, embodiment, epigenetics, culture
  - Mechanism
    - Modularity, control, hierarchy, connectivity, stigmergy, redundancy

bodies in motion

- Mathematical models of systems containing the rules describing the way some quantity undergoes a change in time
    - What changes in time
        - a variable
            - Position, quantity, concentration
    - How does something change in time
        - Deterministic rules that define change
            - Set of differential equations defining rates of change


pavol krivosik, 2008

gravitational pendulum example

- What changes in time
  - a variable
    - Angle
  - Rules that define change
    - Set of differential equations defining rates of change

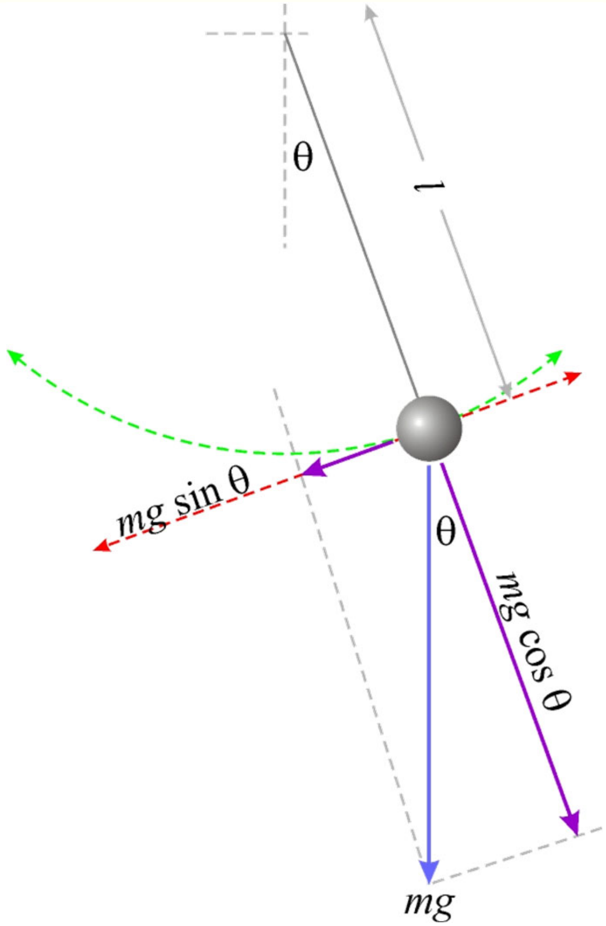
$$F = mg\sin\theta = ma$$

$$a = g\sin\theta$$

$$a = \frac{d^2s}{dt^2} = l\frac{d^2\theta}{dt^2}$$

$$l\frac{d^2\theta}{dt^2} = g\sin\theta$$

$$\boxed{l\frac{d^2\theta}{dt^2} - g\sin\theta = 0}$$



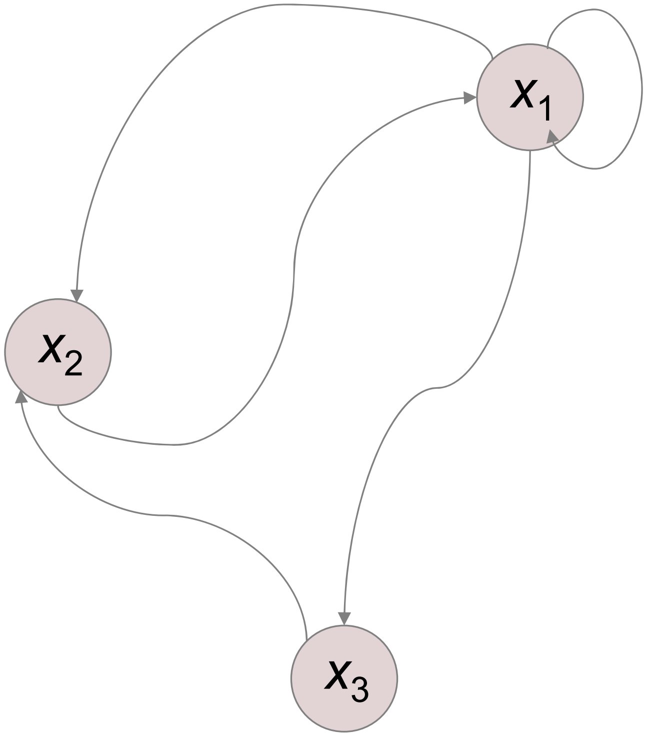
chemical reaction example

- What changes in time
  - a variable
    - concentrations
  - Rules that define change
    - Set of differential equations defining rates of change

$$\frac{dx_1}{dt} = f_1(x_1, x_2) = x_1 - K_1 x_2$$

$$\frac{dx_2}{dt} = f_2(x_1, x_3) = x_1^2 + K_2 x_3$$

$$\frac{dx_3}{dt} = f_3(x_1) = K_3 x_1$$

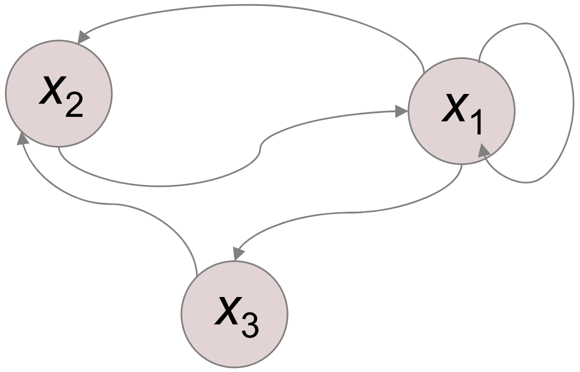


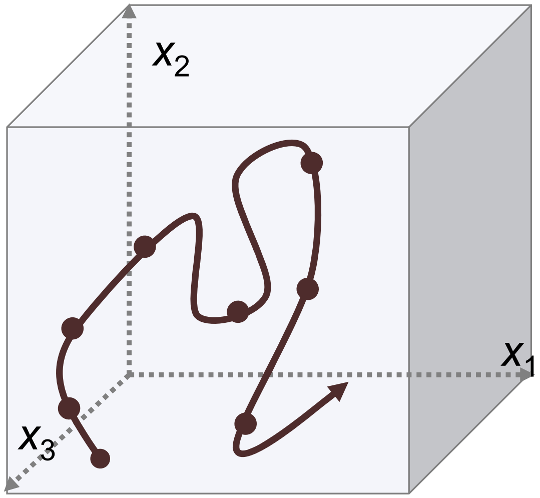
phase or state-space

- Map of variables in time
  - Time is parameter
    - Trajectory (orbit) in state space

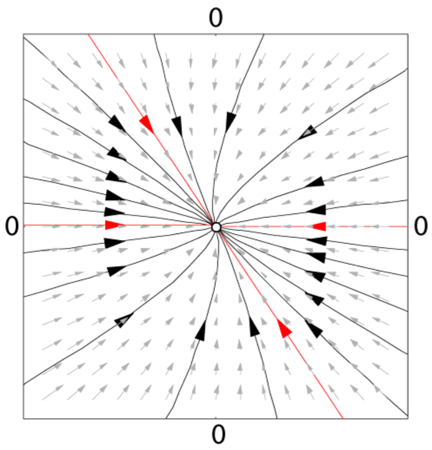
$$X(t) = \left(x_1(t), x_2(t), x_3(t)\right)$$

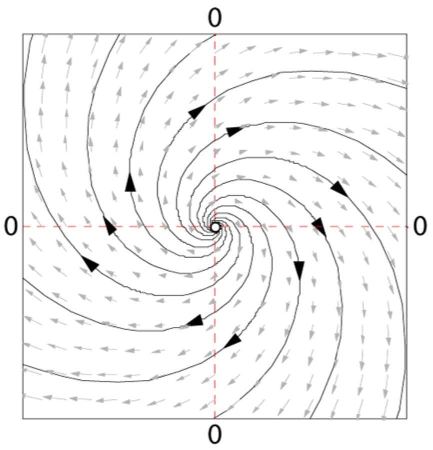
- Continuous (reversible) systems
  - Only one trajectory passes through each point of a state-space
    - State-determined system
    - 2 points on different trajectories will always be on different trajectories
      - Albeit arbitrarily close
    - Not true in discrete systems
  - Determinism, strict causality
    - Laplace

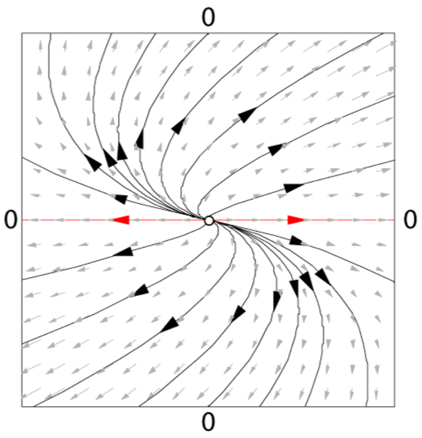


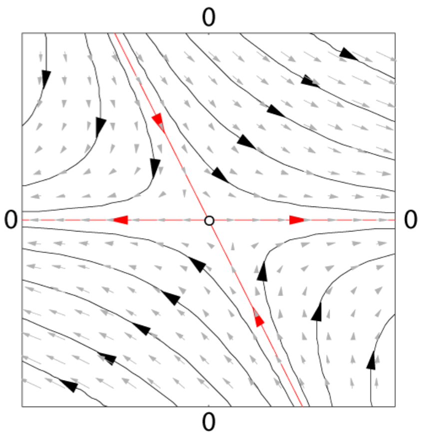


vector fields represent basins of attraction in phase-space



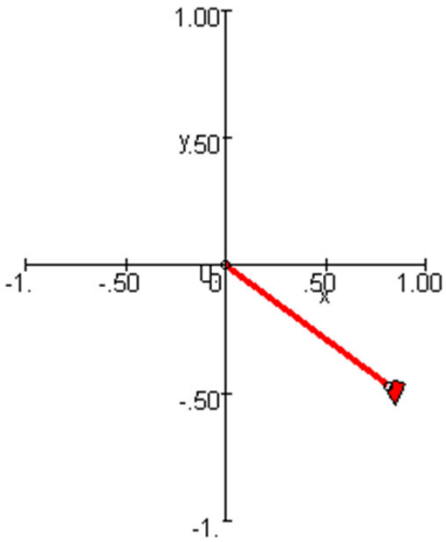


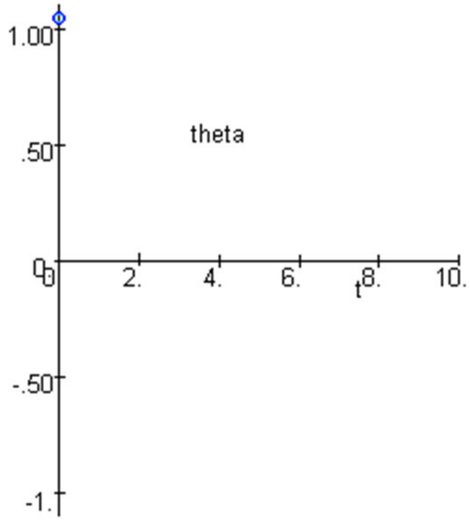


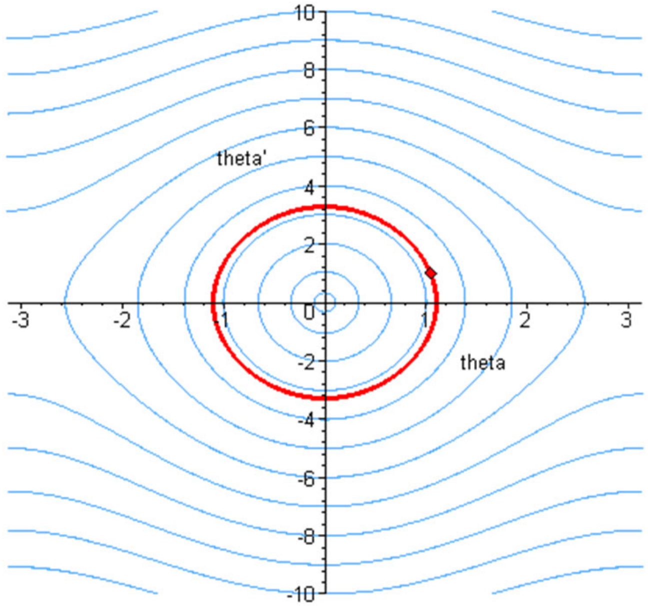


frictionless gravitational pendulum

phase space





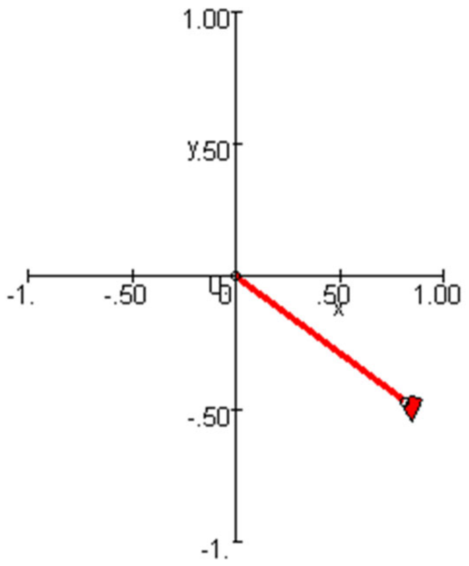


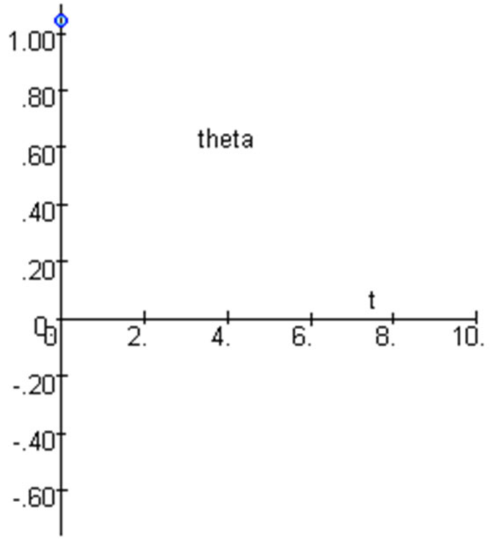
$$X(t) = \big(\theta(t), \dot{\theta}(t)\,\big)$$

displacement and velocity

Gravitational pendulum with friction

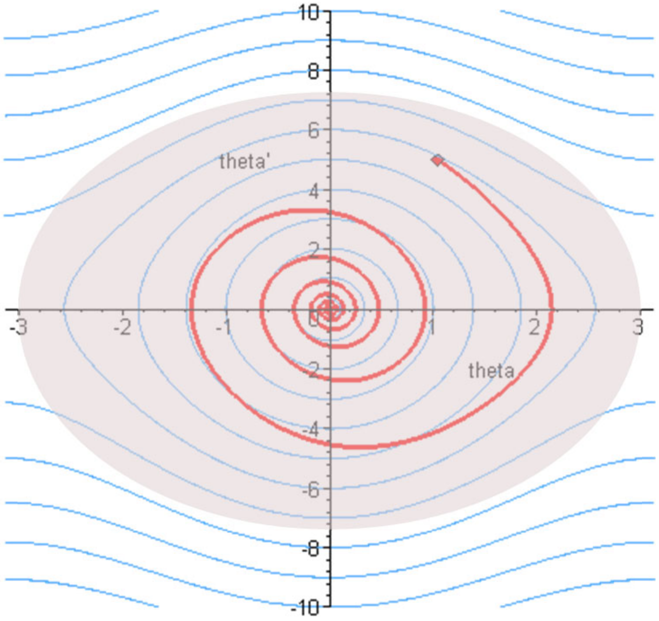
attractor behavior: where motion leads to





Fixed-point behavior

(0-dimensional attractor)



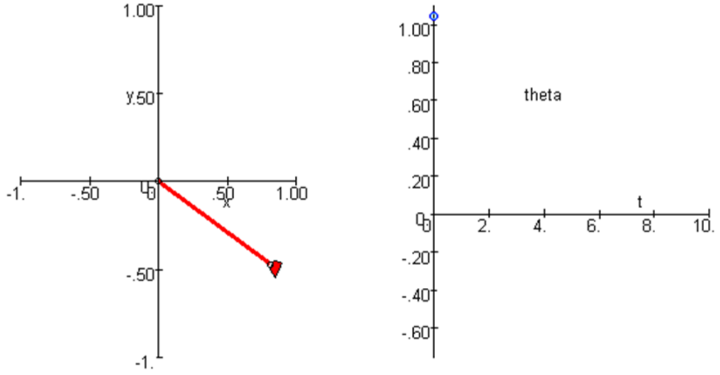
volumes of phase space to which the system converges after a long enough time

Basin of attraction

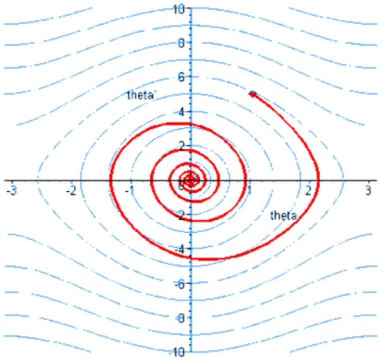
Volume of the phase-space defined by all trajectories leading into the attractor

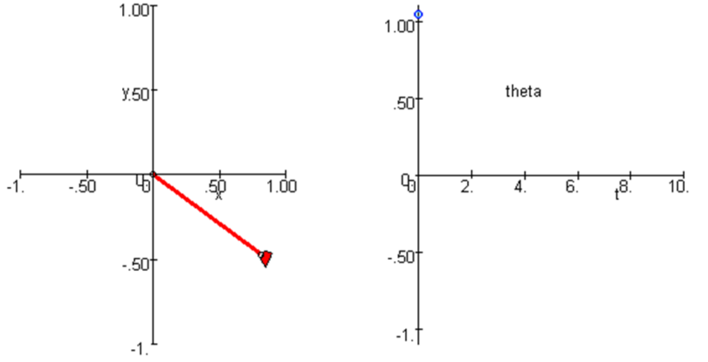
BINGHAMTON UNIVERSITY
STATE UNIVERSITY OF NEW YORK
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

why the attractor behavior?

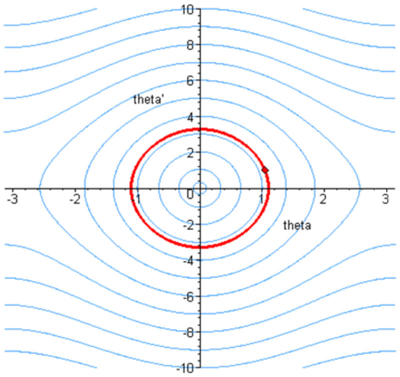


- Energy dissipation (thermodynamic systems)
  - Friction, thermodynamic losses, loss of material, etc.
  - Volume contraction in phase-space
    - System tends to restrict itself to small basins of attraction
  - Self-organization
    - Dissipative systems (Prigogine)



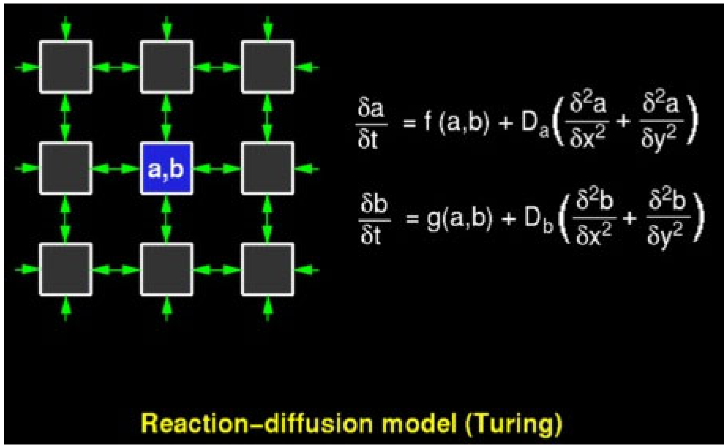


- Hamiltonian systems
  - Frictionless, no attractors
  - Conservation of energy
  - ergodicity



simple dynamics yield complex patterns

- Morphogenesis
  - development of the structure of an organism or part
    - phenotype develops in time under the direction of the genotype + dynamic constraints
  - The process in complex system-environment exchanges that tends to elaborate a system's given form or structure.
- Fischer (1924)
  - Reaction-diffusion equation
    - Propagation of a gene a population
- Nicolas Rashevsky
  - Embryogenesis
- Alan Turing
  - spent the last few years of his life developing his morphogenetic theory and using the new computer to generate solutions to reaction-diffusion systems.



$$\frac{\delta a}{\delta t} = f(a,b) + D_a\left(\frac{\delta^2 a}{\delta x^2} + \frac{\delta^2 a}{\delta y^2}\right)$$

$$\frac{\delta b}{\delta t} = g(a,b) + D_b\left(\frac{\delta^2 b}{\delta x^2} + \frac{\delta^2 b}{\delta y^2}\right)$$

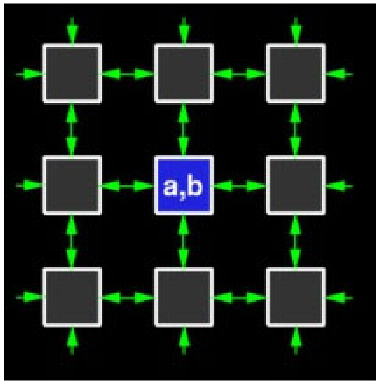
Reaction–diffusion model (Turing)

Turing, A. M. [1952] “The chemical basis of morphogenesis”. *Phil. Trans. R. Soc. Lond. B* **237**, 37–72

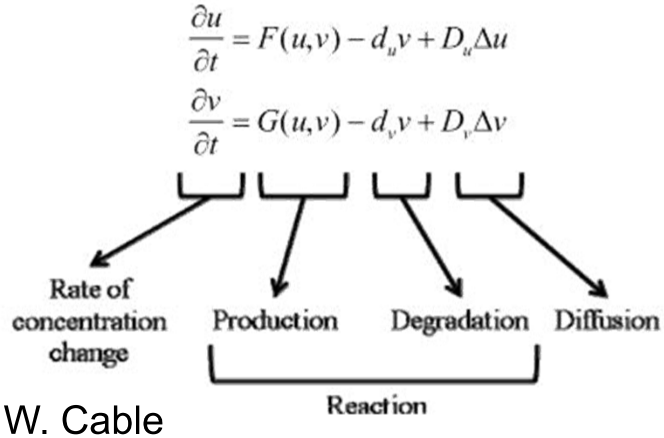
two homogeneously distributed substances within a certain space, one “locally activated” and the other capable of “long-range inhibition,” can produce novel shapes and gradients.

simple dynamics yield complex patterns

- Reaction-diffusion model
  - Stable tension between production and transformation
    - When balance is disturbed, tension restores balance:
  - Metaphor
    - Island populated by cannibals and (celibate) missionaries.
    - Missionaries do not reproduce, but can recruit and die (transform)
    - Cannibals reproduce and die (produce)
    - Two missionaires convert a cannibal leading to tension between production and transformation



By Kele W. Cable



$$\underbrace{\frac{\partial u}{\partial t}}_{\text{Rate of concentration change}} = \underbrace{F(u,v)}_{\text{Production}} \underbrace{- d_u v}_{\text{Degradation}} + \underbrace{D_u \Delta u}_{\text{Diffusion}}$$

$$\frac{\partial v}{\partial t} = G(u,v) - d_v v + D_v \Delta v$$

two homogeneously distributed substances within a certain space, one “locally activated” and the other capable of “long-range inhibition,” can produce novel shapes and gradients.

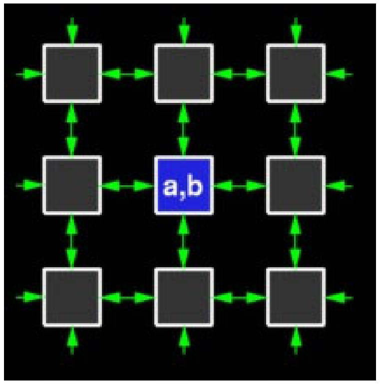
substance interactions depend on just four variables per *morphogen* – the rate of production, the rate of degradation, the rate of diffusion and the strength of activating/inhibiting interactions.

Turing, A. M. [1952] “The chemical basis of morphogenesis”.
Phil. Trans. R. Soc. Lond. B **237**, 37–72

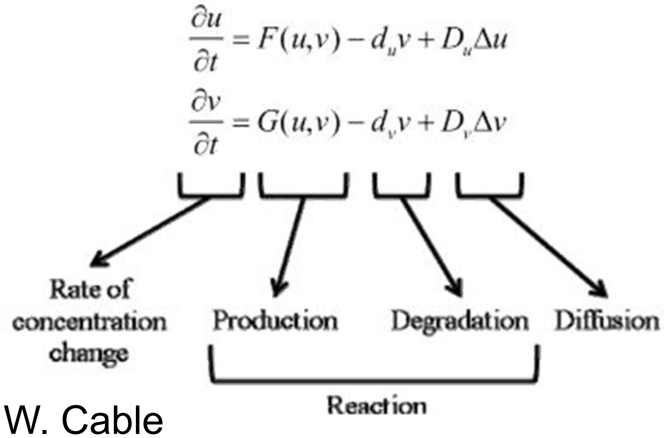
morphogenesis

simple dynamics yield complex patterns

- Reaction-diffusion model
  - Stable tension between production and transformation
    - When balance is disturbed, tension restores balance:
  - Metaphor
    - Island populated by cannibals and (celibate) missionaries.
    - Missionaries do not reproduce, but can recruit and die (transform)
    - Cannibals reproduce and die (produce)
    - Two missionaires convert a cannibal leading to tension between production and transformation

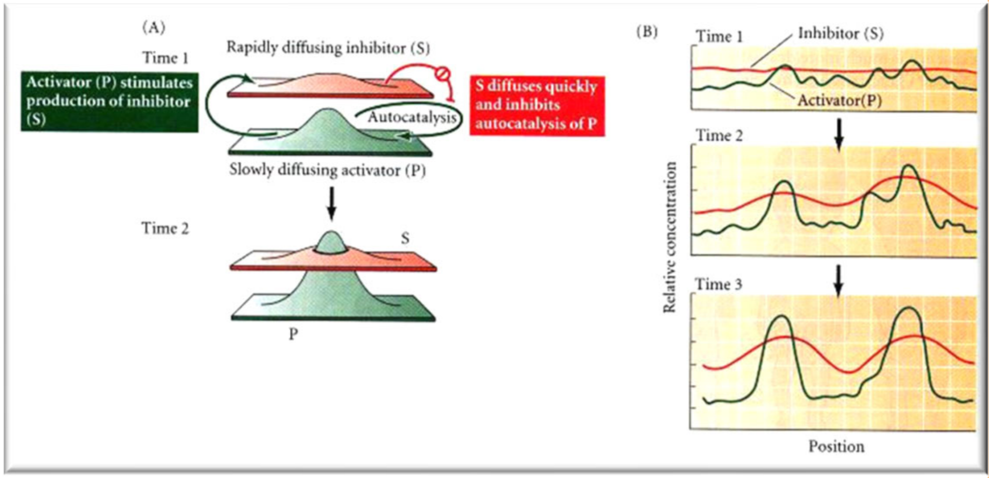


By Kele W. Cable



$$\underbrace{\frac{\partial u}{\partial t}}_{\text{Rate of concentration change}} = \underbrace{\overbrace{F(u,v)}^{}}_{\text{Production}} - \underbrace{d_u v}_{\text{Degradation}} + \underbrace{D_u \Delta u}_{\text{Diffusion}}$$

$$\frac{\partial v}{\partial t} = G(u,v) - d_v v + D_v \Delta v$$

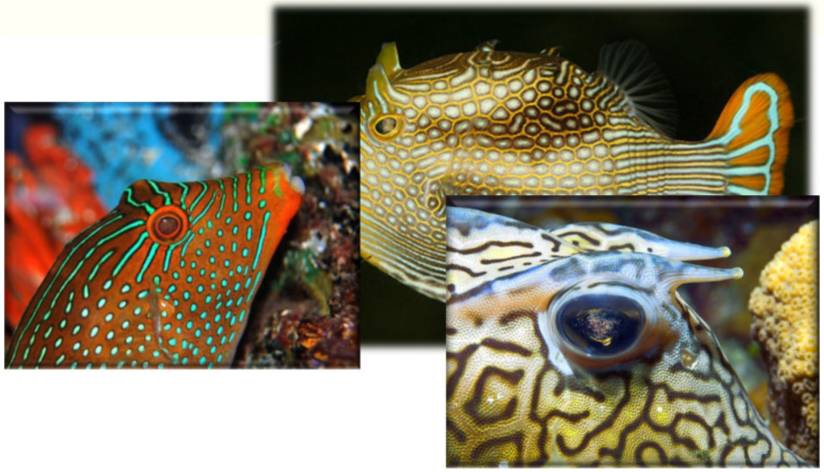


Turing, A. M. [1952] "The chemical basis of morphogenesis".
Phil. Trans. R. Soc. Lond. B **237**, 37–72

BINGHAMTON UNIVERSITY
STATE UNIVERSITY OF NEW YORK
rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

modeling nature

- Reaction-diffusion model
  - Stable tension between production and transformation
    - When balance is disturbed, tension restores balance:
  - Metaphor
    - Island populated by cannibals and (celibate) missionaries.
    - Missionaries do not reproduce, but can recruit and die (transform)
    - Cannibals reproduce and die (produce)
    - Two missionaires convert a cannibal leading to tension between production and transformation



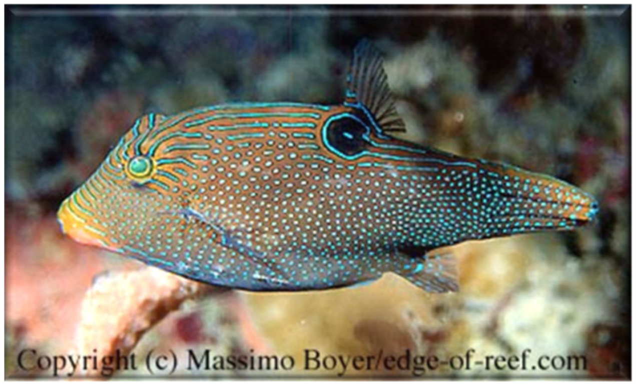
Copyright (c) Ken Body/kenbondy.com

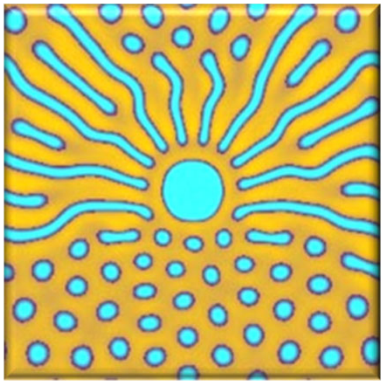
Allen Sanderson

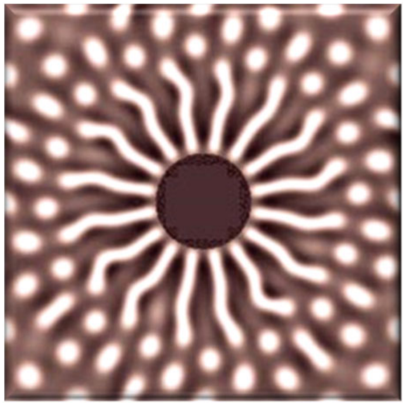
modeling nature

- Reaction-diffusion model
  - Stable tension between production and transformation
    - When balance is disturbed, tension restores balance:
  - Metaphor
    - Island populated by cannibals and (celibate) missionaries.
    - Missionaries do not reproduce, but can recruit and die (transform)
    - Cannibals reproduce and die (produce)
    - Two missionaires convert a cannibal leading to tension between production and transformation


Copyright (c) Massimo Boyer/edge-of-reef.com


Copyright (c) Massimo Boyer/edge-of-reef.com

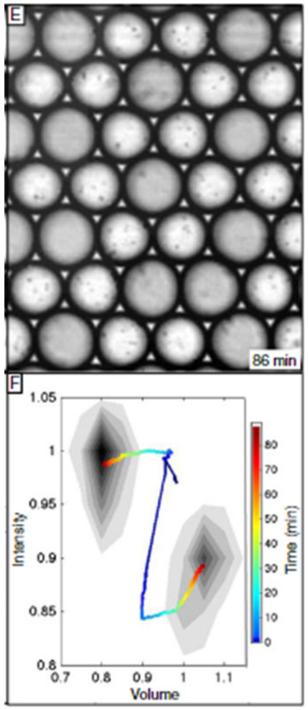


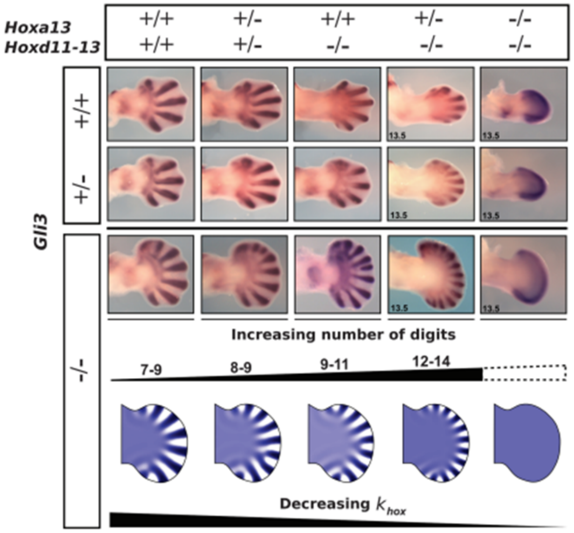


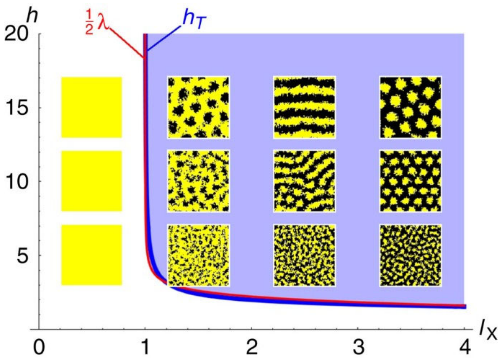
Allen Sanderson

Turing morphogenesis

in biology, chemistry, and complex systems science







Gene expression of digit determination (in mouse)

Sheth et al [2012]. “Hox Genes Regulate Digit Patterning by Controlling the Wavelength of a Turing-Type Mechanism.” *Science.* **338** (6113): 1476–80.

Validation of predicted patterns (in abiological droplets)

Tompkins et al [2014]. “Testing Turing’s Theory of Morphogenesis in Chemical Cells.” *PNAS* **111 (**12) : 4397–4402.

Revising the model with biological evidence (in zebrafish)

Bullara, D, and Y De Decker [2015]. “Pigment Cell Movement Is Not Required for Generation of Turing Patterns in Zebrafish Skin.” *Nature Communications* **6** : 6971.

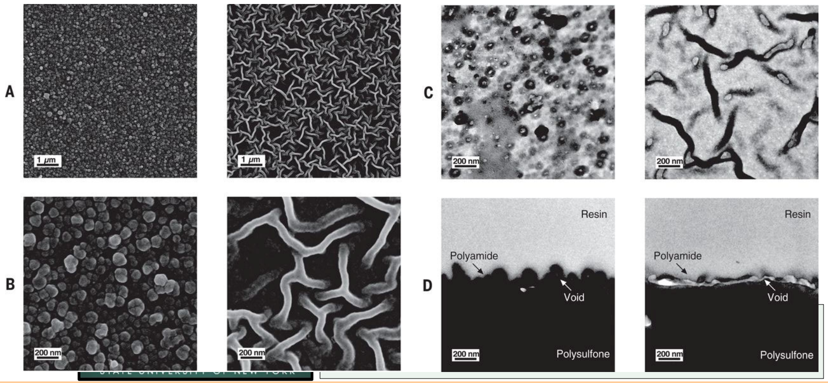
Turing-type polyamide membranes for water purification

Tan, et al [2018]. “Polyamide Membranes with Nanoscale Turing Structures for Water Purification.” *Science* **360 (**6388): 518–21.

Expanding theoretical models (ABM and others)

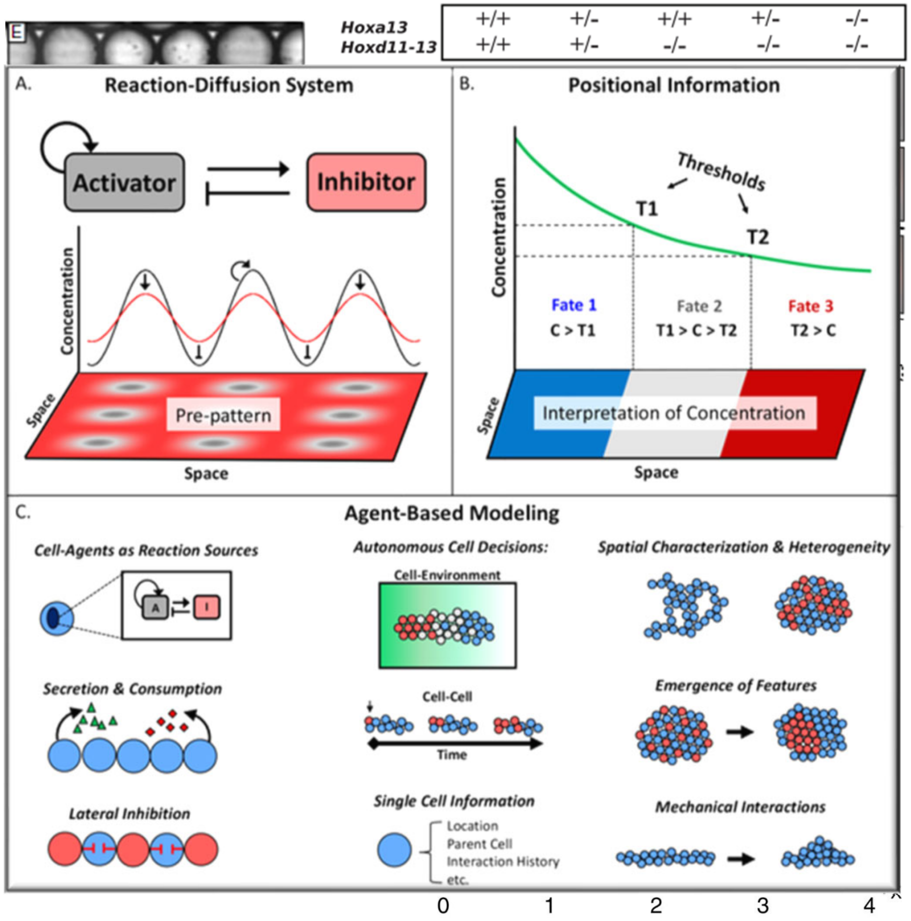
Glen et al [2019] “Agent-Based Modeling of Morphogenetic Systems: Advantages and Challenges.” *PLOS Computational Biology* **15** (3): e1006577.

Steinbock, Wackerbauer, and Horváth [2019]. “Nonlinear Chemical Dynamics and Its Interdisciplinary Impact: Dedicated to Ken Showalter on the Occasion of His 70th Birthday.” *Chaos: An Interdisciplinary Journal of Nonlinear Science* **29** (8): 080401.



Turing morphogenesis

in biology, chemistry, and complex systems science



Gene expression of digit determination (in mouse)

Sheth et al [2012]. "Hox Genes Regulate Digit Patterning by Controlling the Wavelength of a Turing-Type Mechanism." *Science.* **338** (6113): 1476–80.

Validation of predicted patterns (in abiological droplets)

Tompkins et al [2014]. "Testing Turing's Theory of Morphogenesis in Chemical Cells." *PNAS* **111** **(**12) : 4397–4402.

Revising the model with biological evidence (in zebrafish)

Bullara, D, and Y De Decker [2015]. "Pigment Cell Movement Is Not Required for Generation of Turing Patterns in Zebrafish Skin." *Nature Communications* **6** : 6971.

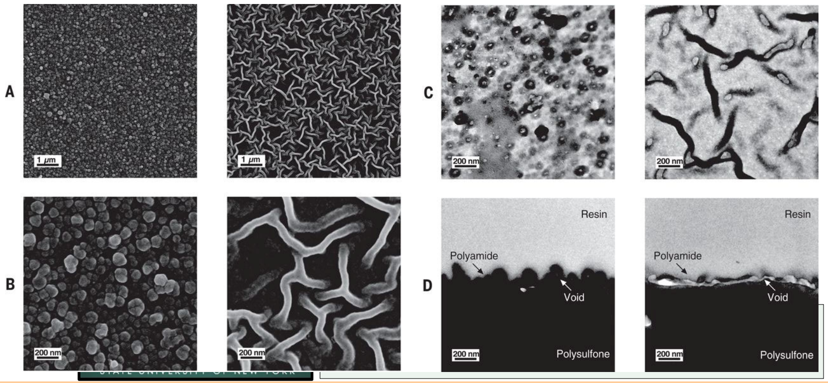
Turing-type polyamide membranes for water purification

Tan, et al [2018]. "Polyamide Membranes with Nanoscale Turing Structures for Water Purification." *Science* **360** **(**6388): 518–21.

Expanding theoretical models (ABM and others)

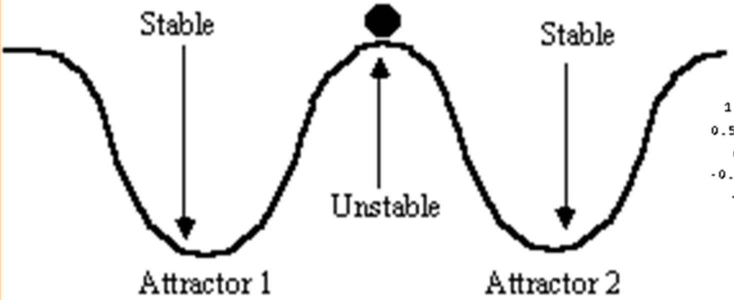
Glen et al [2019] "Agent-Based Modeling of Morphogenetic Systems: Advantages and Challenges." *PLOS Computational Biology* **15** (3): e1006577.

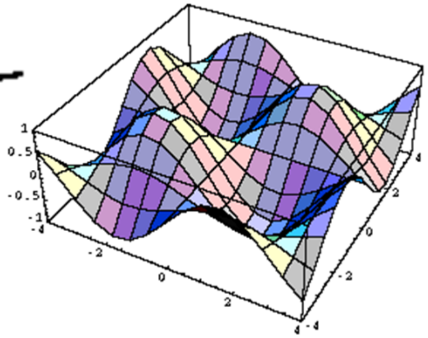
Steinbock, Wackerbauer, and Horváth [2019]. "Nonlinear Chemical Dynamics and Its Interdisciplinary Impact: Dedicated to Ken Showalter on the Occasion of His 70th Birthday." *Chaos: An Interdisciplinary Journal of Nonlinear Science* **29** (8): 080401.

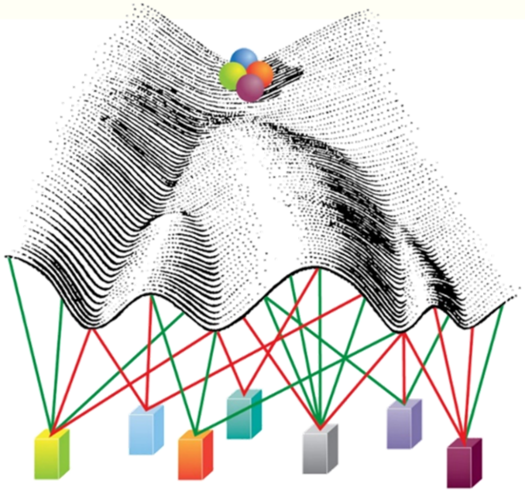


(energy) landscape metaphor

- Phase-space as landscape
  - State of the system as a drop of water released in hills and valleys



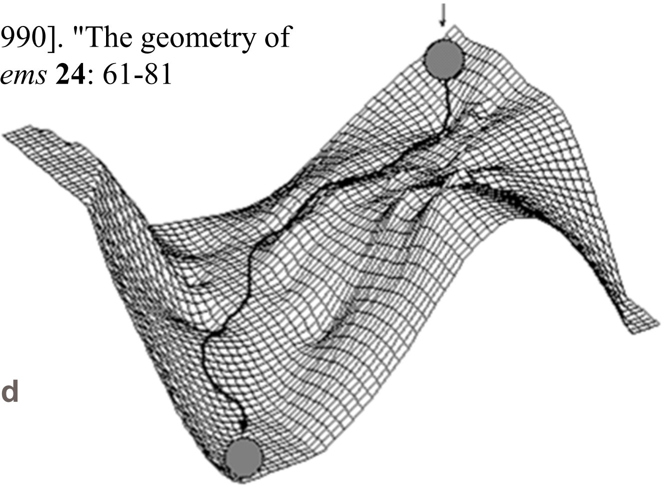


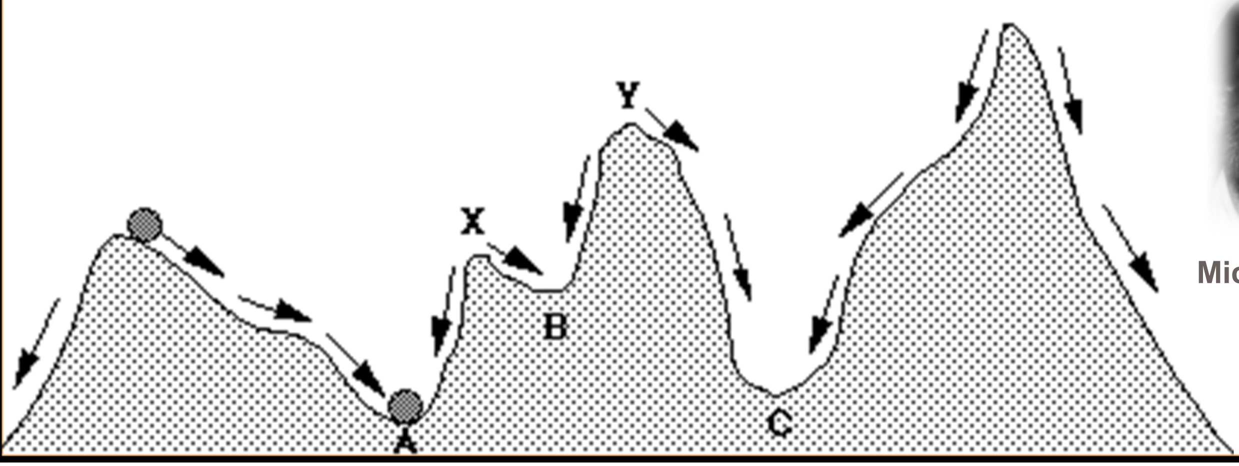


See: Conrad, M. [1990]. "The geometry of evolution.“ *Biosystems* **24**: 61-81

Waddington CH (1942). *Nature*.**150** (3811):563–565

Michael Conrad





readings

- Class Book
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press.
    - **Chapter 2**.
- Lecture notes
  - Chapter 1: What is Life?
  - Chapter 2: The logical Mechanisms of Life
  - Chapter 3: Formalizing and Modeling the World
    - posted online @ http://informatics.indiana.edu/rocha/i-bic
- Papers and other materials
  - Discussions
    - Kauffman, S.A. [1969]. "Metabolic stability and epigenesis in randomly constructed genetic nets". *Journal of Theoretical Biology* **22**(3):437-467.
  - Optional
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall.
      - Chapter 2, all sections
      - Chapter 7, sections 7.3 – Cellular Automata
      - Chapter 8, sections 8.1, 8.2, 8.3.10
    - Flake's [1998], *The Computational Beauty of Life*. MIT Press.
      - Chapters 10, 11, 14 – Dynamics, Attractors and chaos