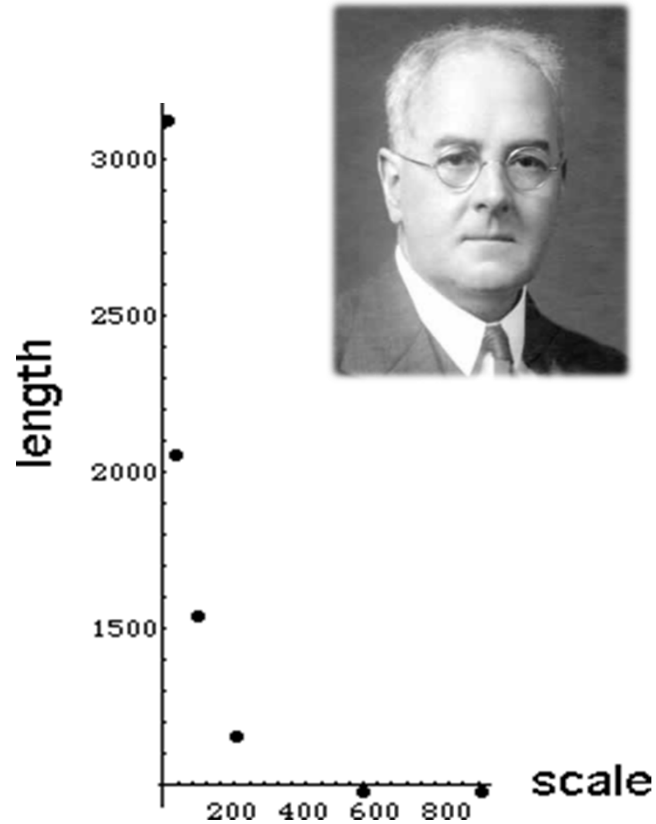
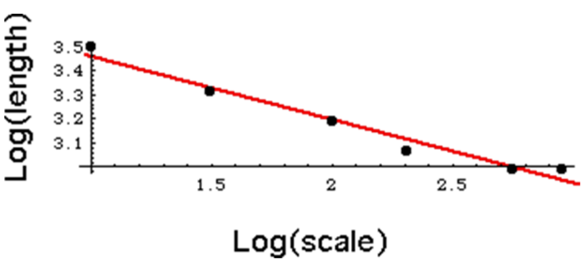
lecture 6

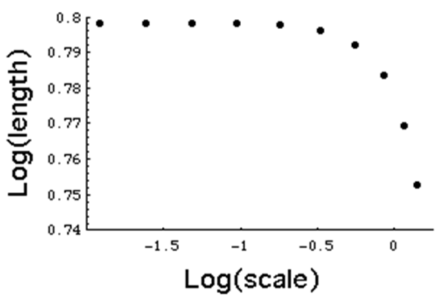
self-similarity and L-Systems

Lewis Richardson's observations (1961)

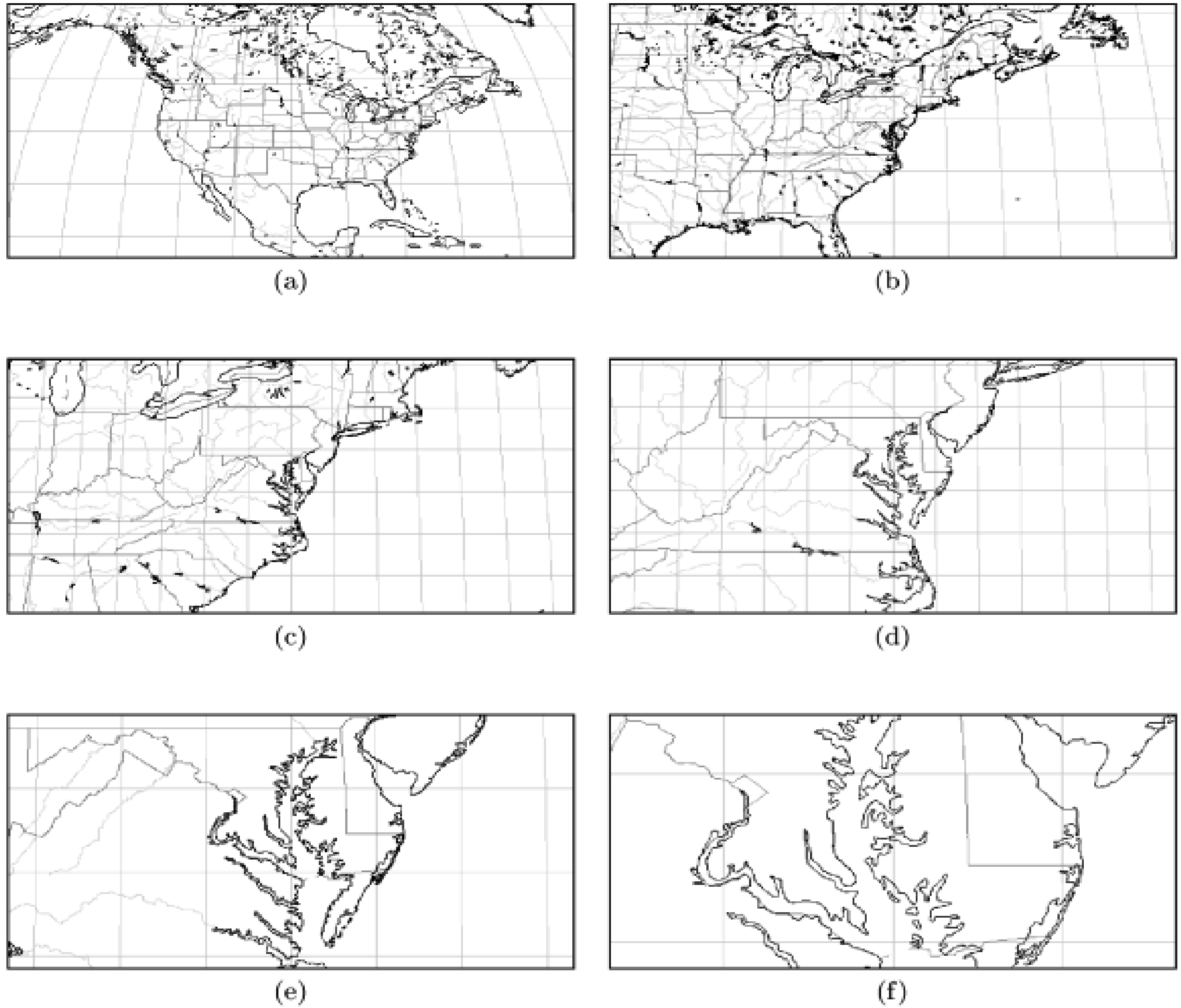
- Measured maps with different scales
  - Coasts of Australia, South Africa, and Britain
  - Land frontiers of Germany and Portugal
  - Measured lengths *L*(*d*) at different scales *d*.
    - As the scale is reduced, the length increases rapidly.
  - Well-fit by a straight line with slopes (s) on log/log plots
    - s = -0.25 for the west coast of Britain, one of the roughest in the atlas,
    - s = -0.15 for the land frontier of Germany,
    - s = -0.14 for the land frontier of Portugal,
    - s = -0.02 for the South African coast, one of the smoothest in the atlas.
    - circles and other smooth curves have line of slope 0.



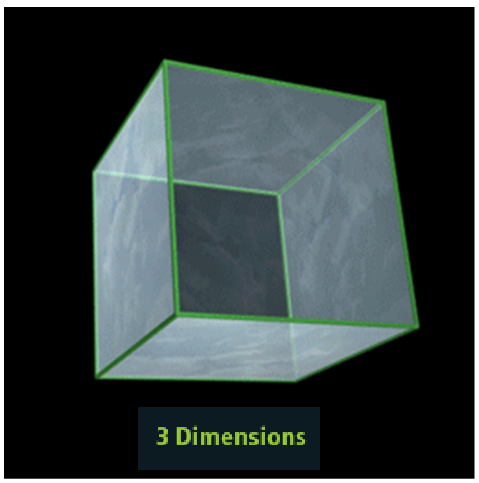


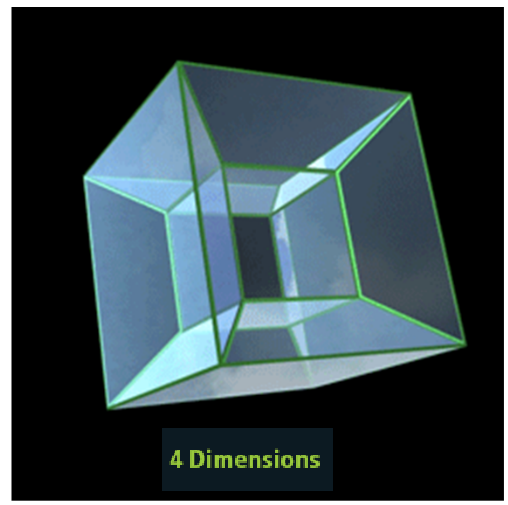


BINGHAMTON UNIVERSITY
STATE UNIVERSITY OF NEW YORK
rocha@indiana.edu
casci.binghamton.edu/academics/i-bic


(a)
(b)
(c)
(d)
(e)
(f)

Integer dimensions

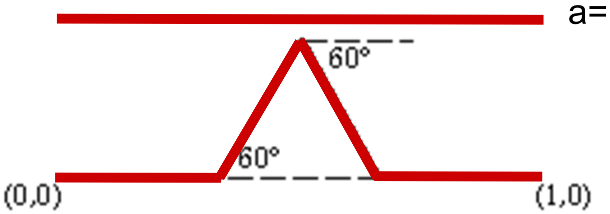




Scientific American, July 2008

the Koch curve example: fractional dimensions

- Koch curve
  - slightly more than line but less than a plane
  - Packing efficiency!



N=4^n

a=$1/3^n$

Measuring scale

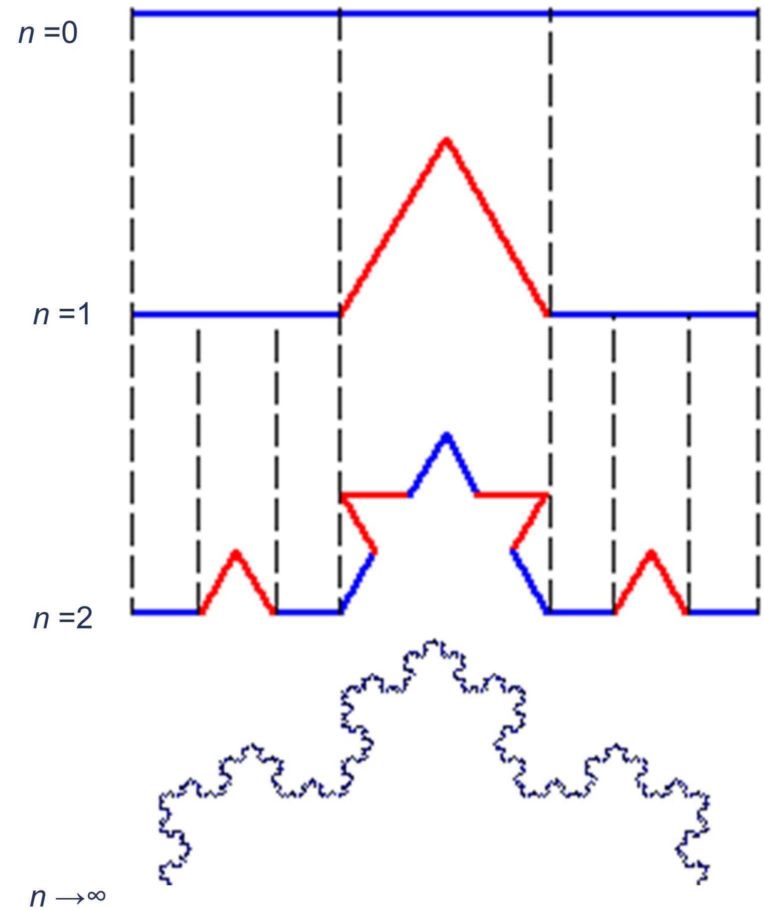
$$D = \frac{\log N}{\log\left(\frac{1}{a}\right)} = \frac{\log 4}{\log 3} = 1.26186...$$

Hausdorff Dimension

Number of units

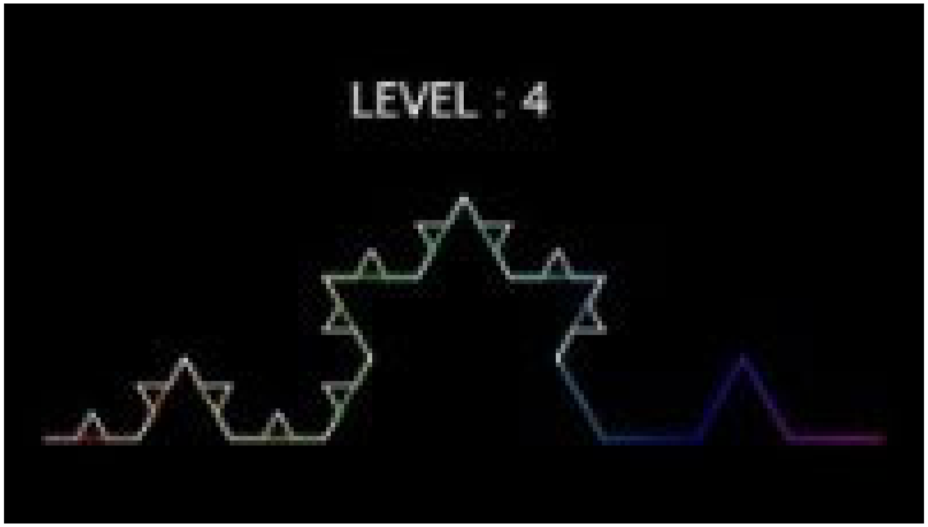
$$N = \left(\frac{1}{a}\right)^D \Rightarrow D = \frac{\log N}{\log\left(\frac{1}{a}\right)}$$

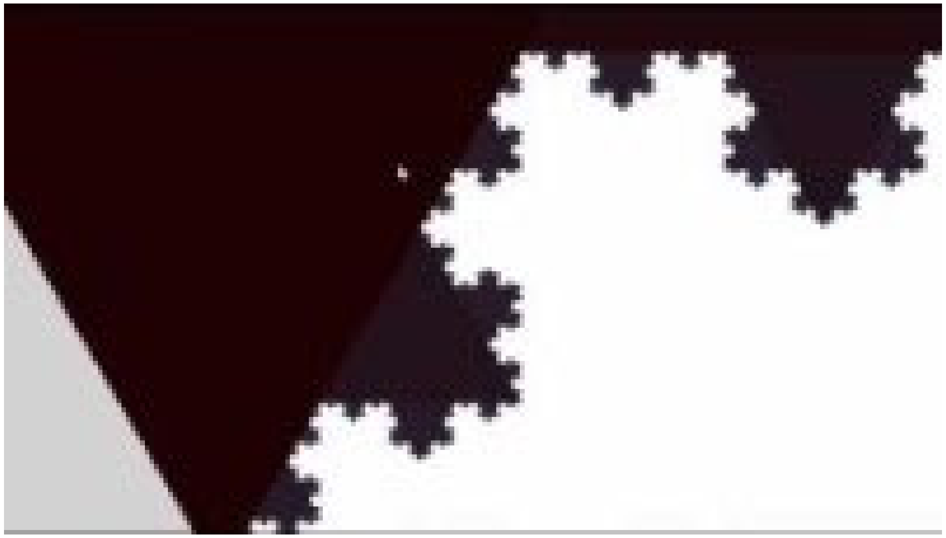
Unit measure



BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

videos

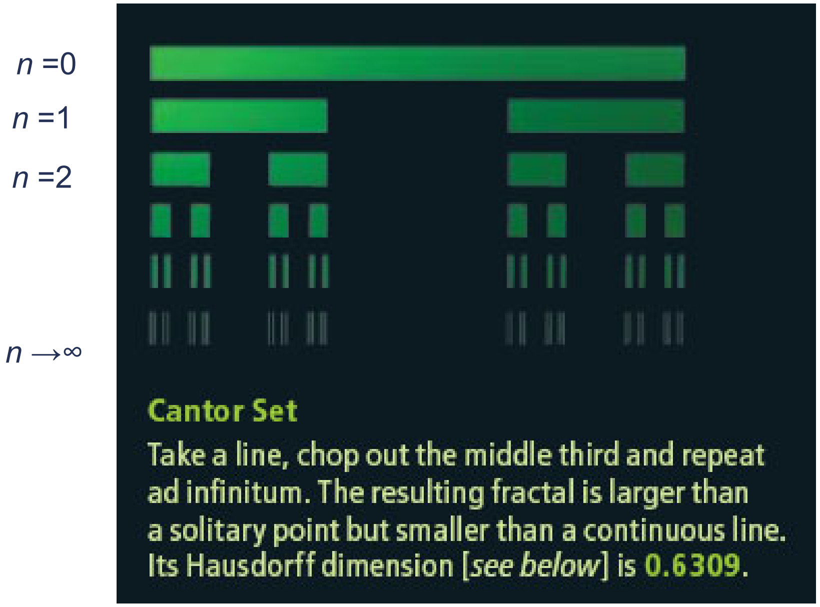




BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

mathematical monsters

- Complex objects are defined by systematically and recursively replacing parts of a simple start object with another object according to a simple rule
  - Cantor Set



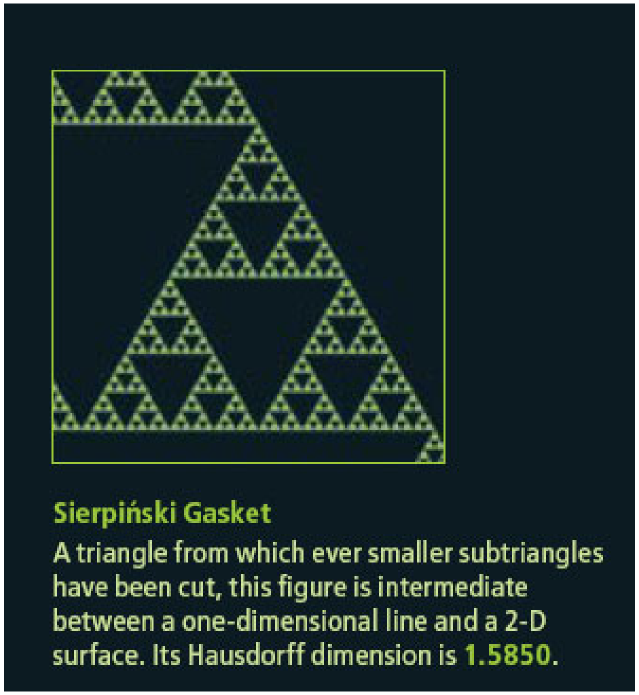
Scientific American, July 2008

$$D = \frac{\log N}{\log\left(\frac{1}{a}\right)} = 0.6309$$

Hausdorff Dimension

mathematical monsters

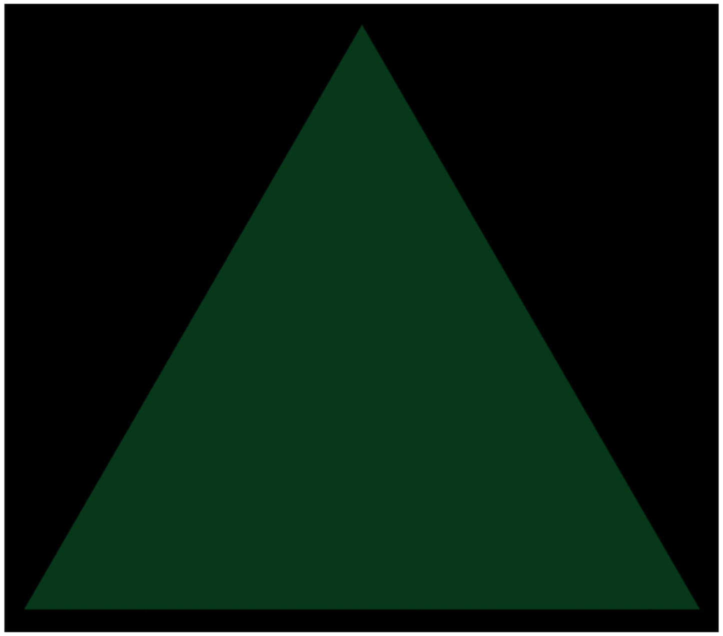
- Complex objects are defined by systematically and recursively replacing parts of a simple start object with another object according to a simple rule
  - Sierpinski Gasket



Scientific American, July 2008

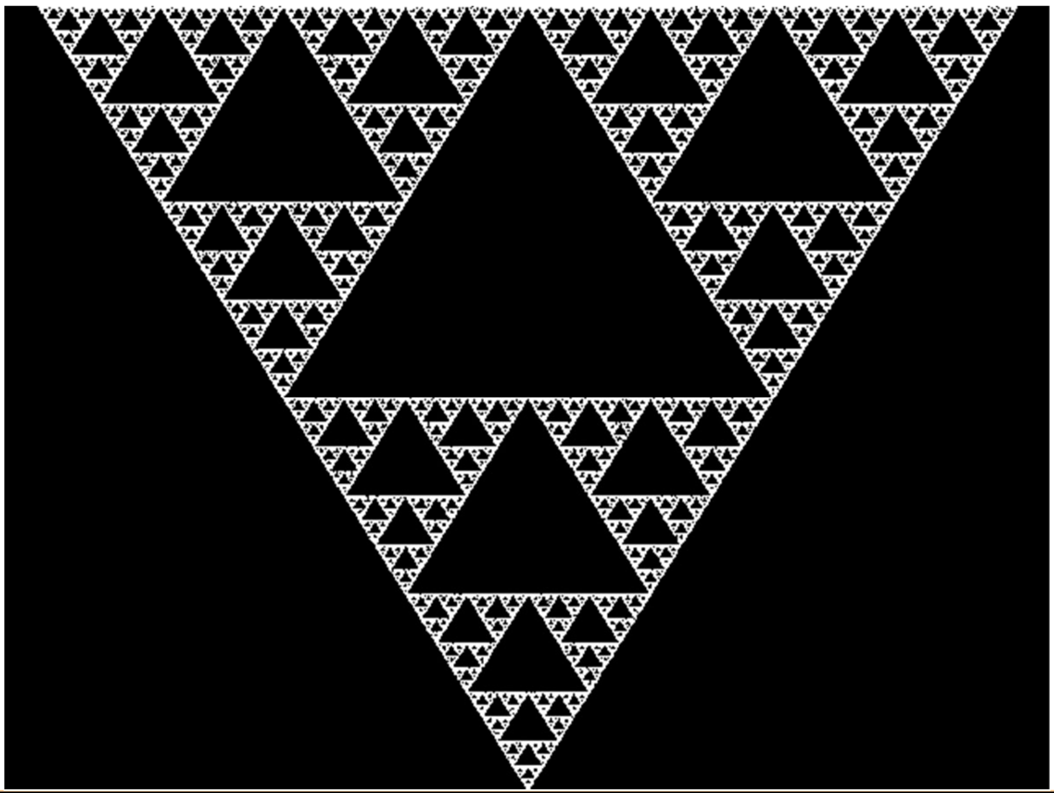
$$D = \frac{\log N}{\log\left(\frac{1}{a}\right)} = 1.585$$

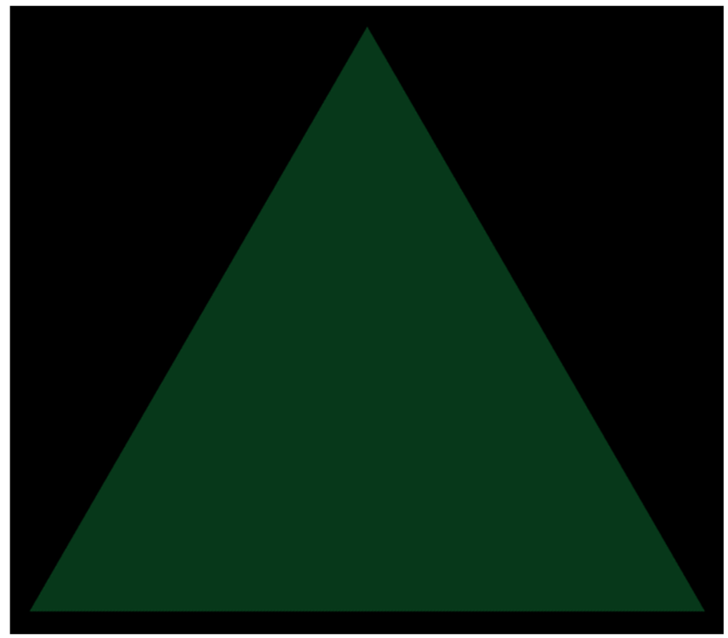
Hausdorff Dimension



mathematical monsters

- Complex objects are defined by systematically and recursively replacing parts of a simple start object with another object according to a simple rule
  - Sierpinski Gasket





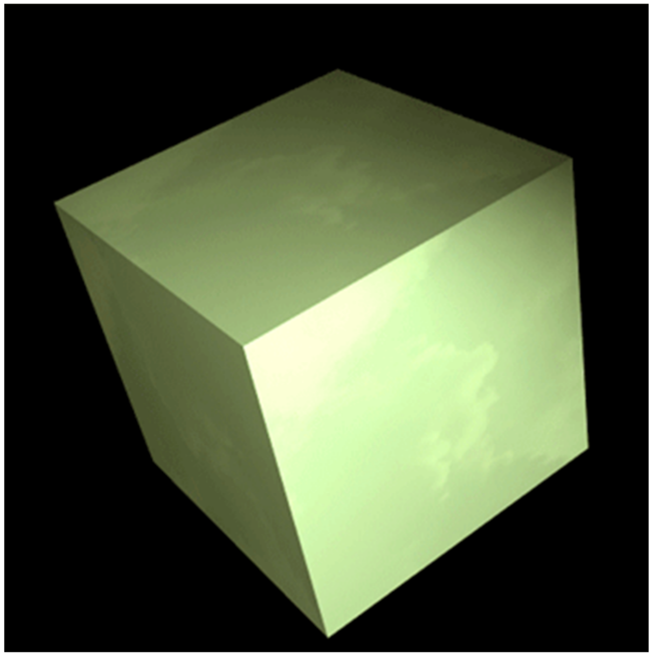
BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

mathematical monsters

- Complex objects are defined by systematically and recursively replacing parts of a simple start object with another object according to a simple rule
  - Menger sponge

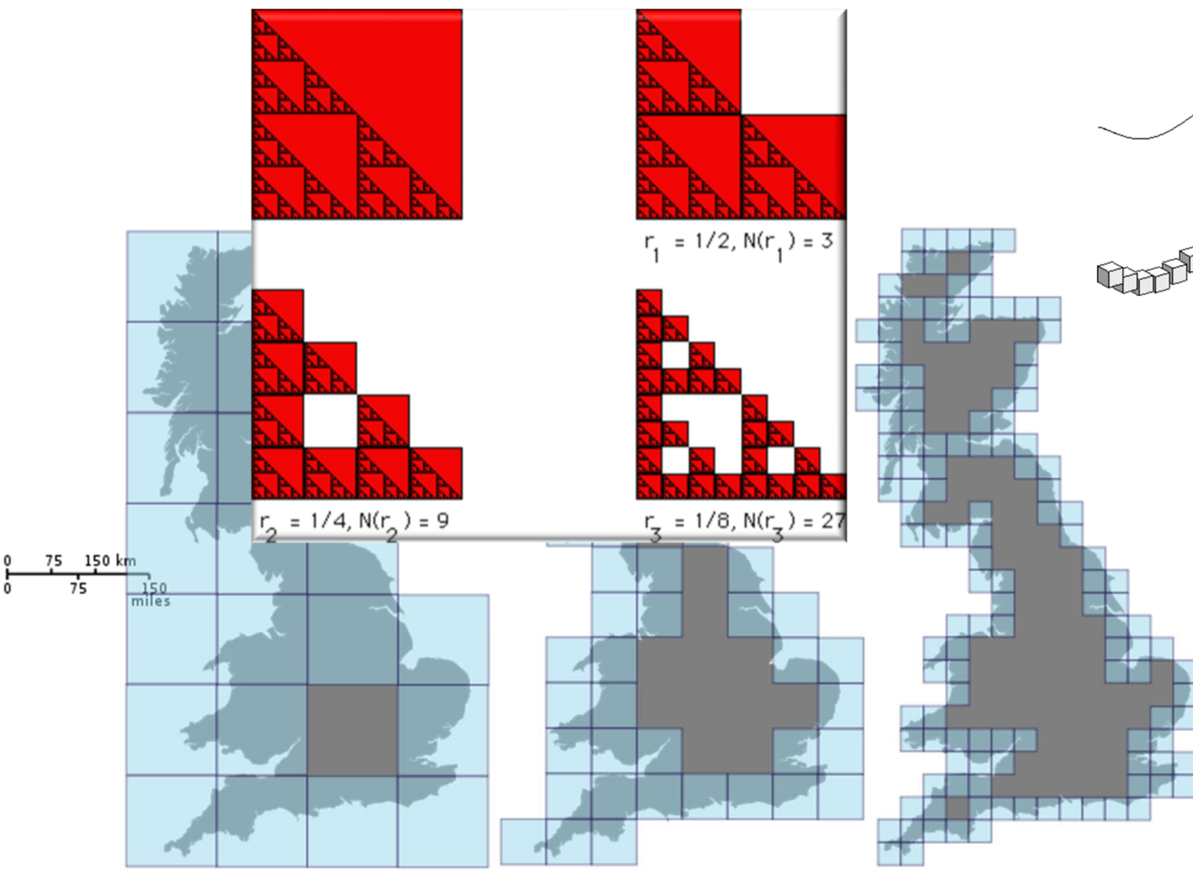
$$D = \frac{\log N}{\log\left(\frac{1}{a}\right)} = 2.7268$$

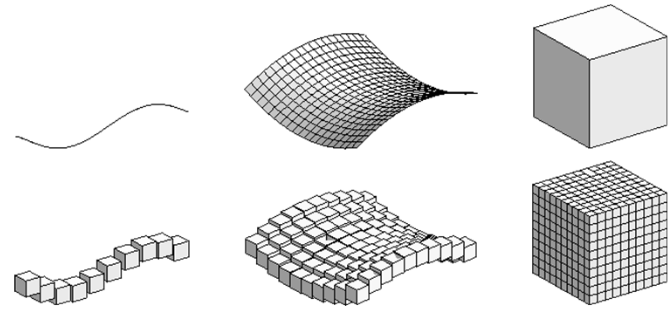
Hausdorff Dimension



Scientific American, July 2008

Box-counting dimension

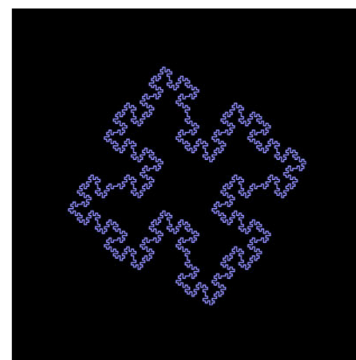




Number of boxes

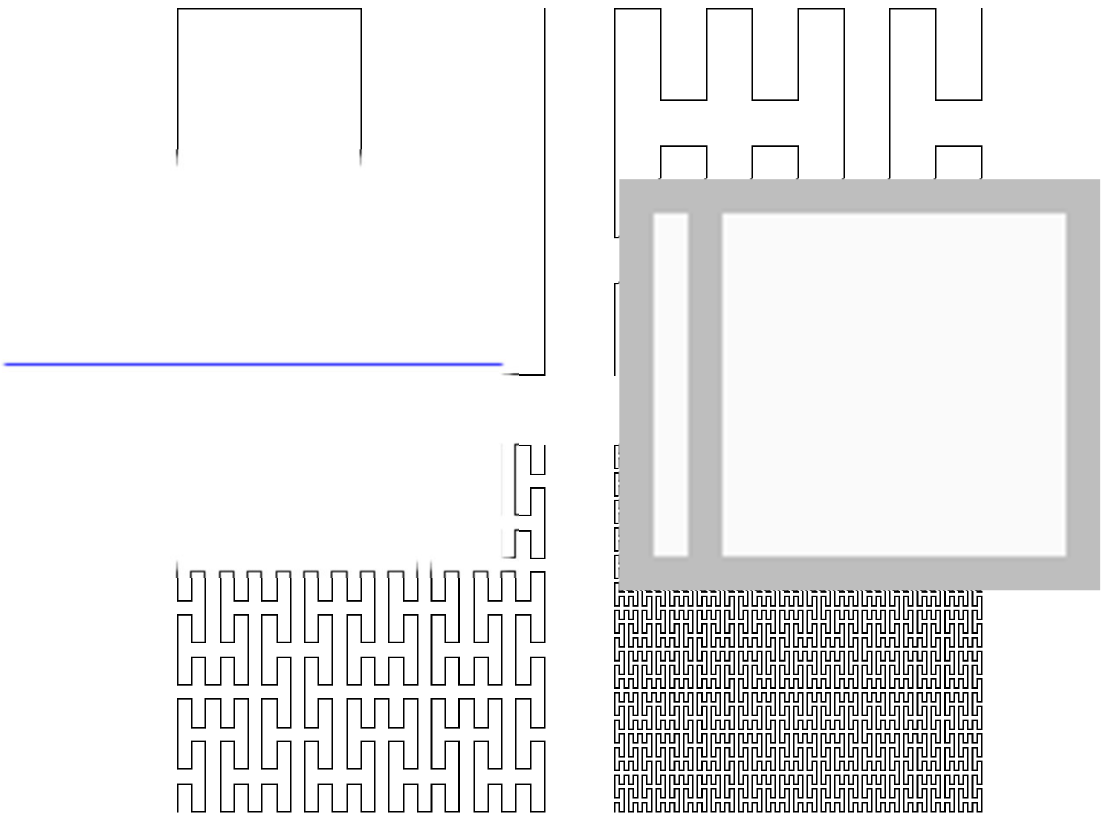
$$D = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log\left(\frac{1}{\varepsilon}\right)}$$

Length of box side

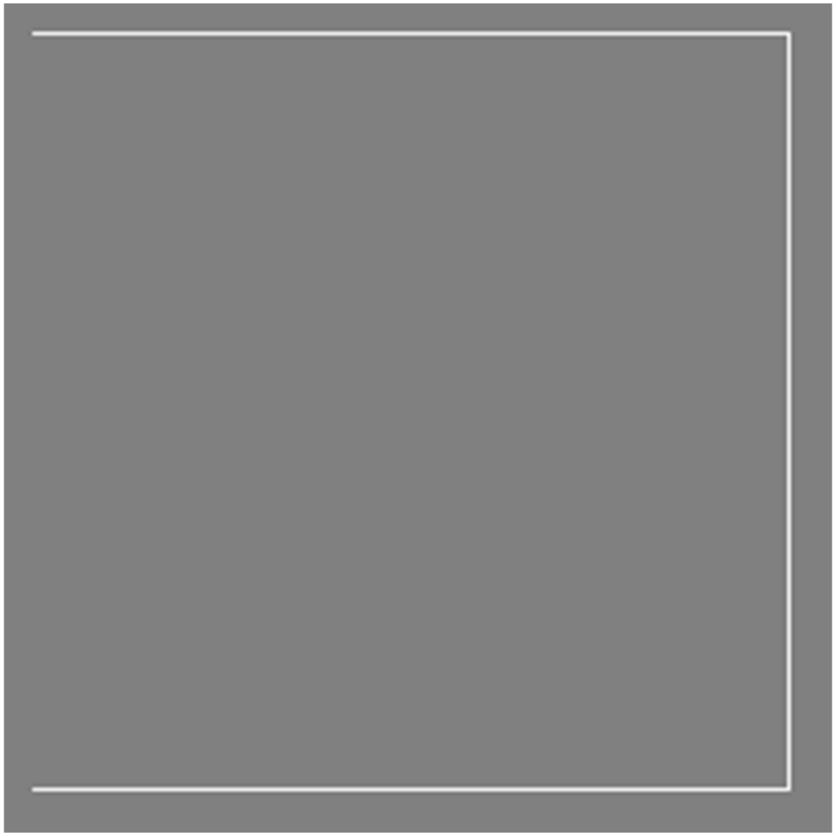


BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

Filling planes and volumes



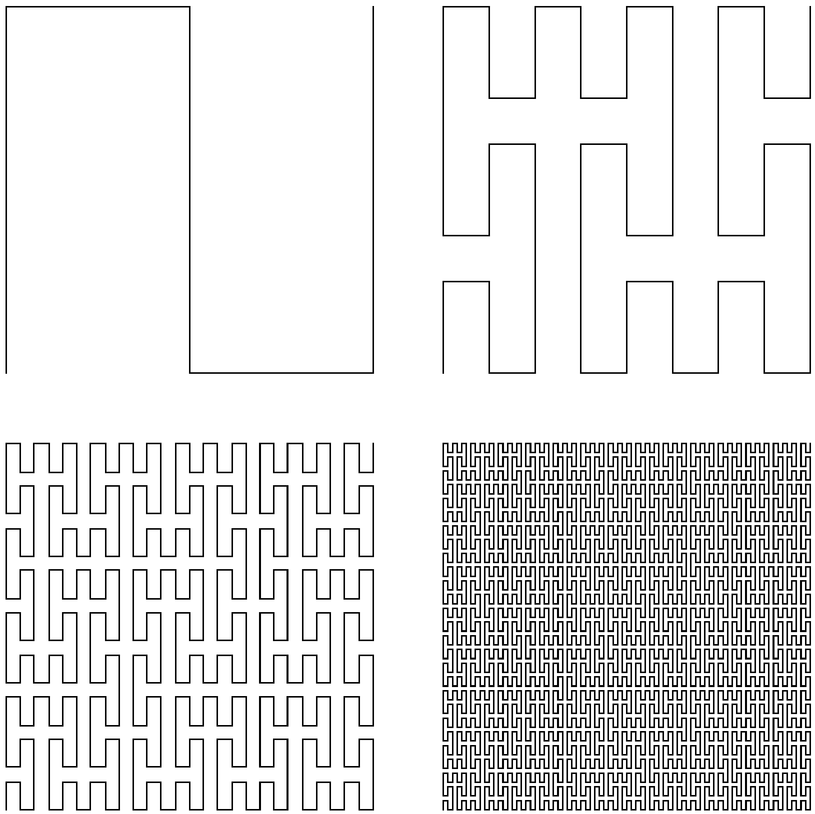
Peano



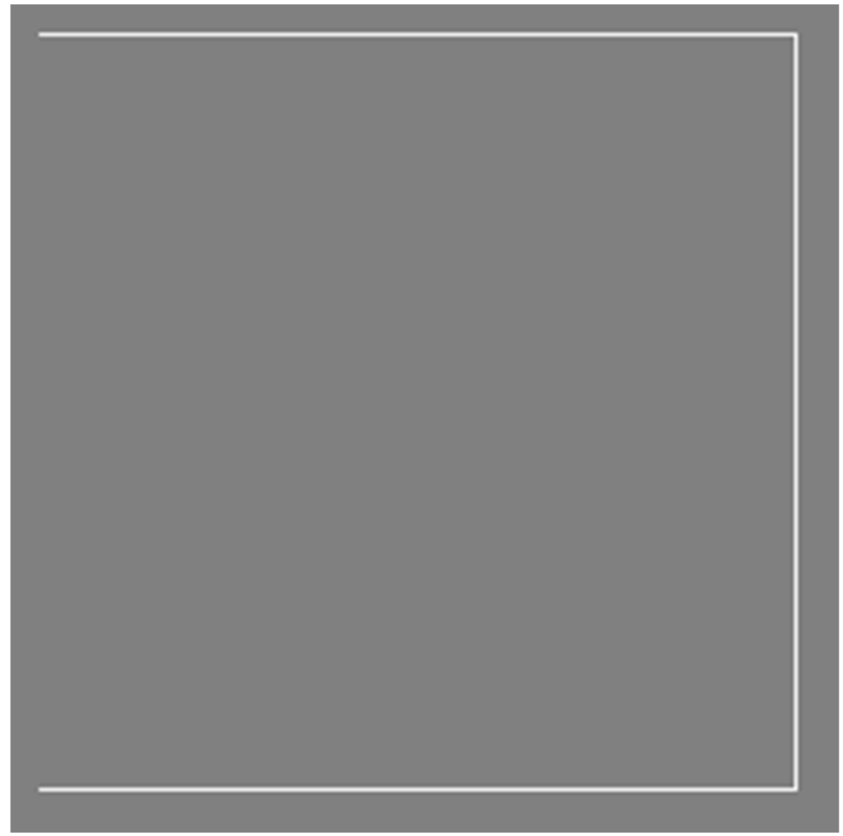
Hilbert

Binghamton University State University of New York
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

Filling planes and volumes



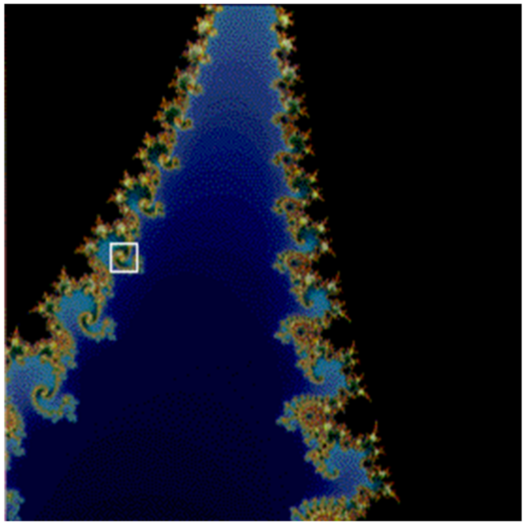
Peano



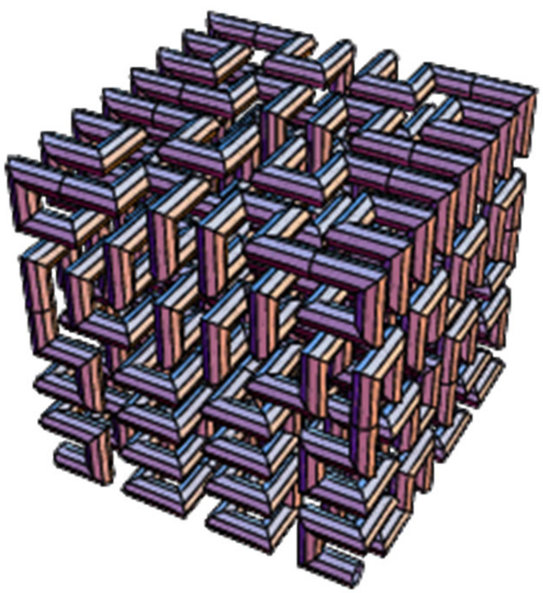
Hilbert

BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

- Self-similarity on multiple scales
  - Due to recursion
- Fractal dimension
  - Enclosed in a given space, but with infinite number of points or measurement



reducing volume

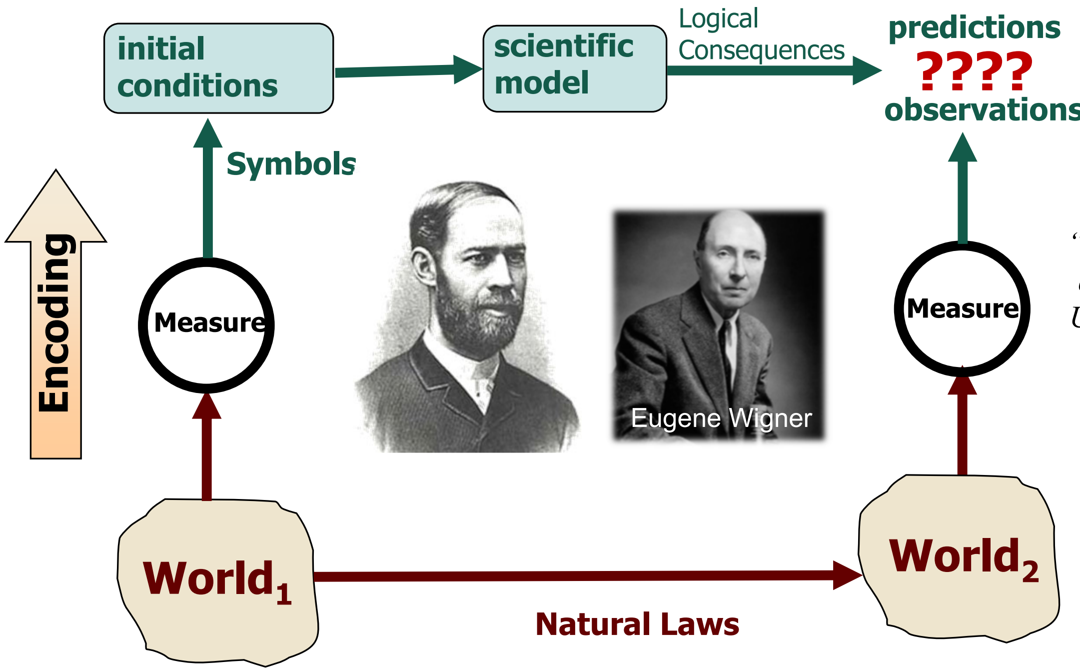


(a) (b) (c)

How do these packed volumes and recursive morphologies grow?

BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

Hertzian scientific modeling paradigm



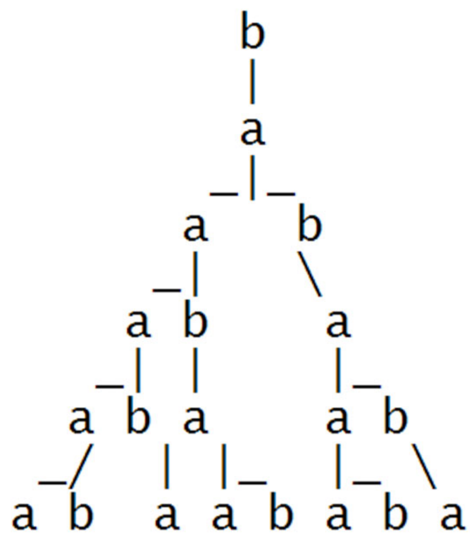
"Every empirical law has the disquieting quality that one does not know its limitations." E. Wigner [1957] in "The Unreasonable Effectiveness of Mathematics in the Natural Sciences"

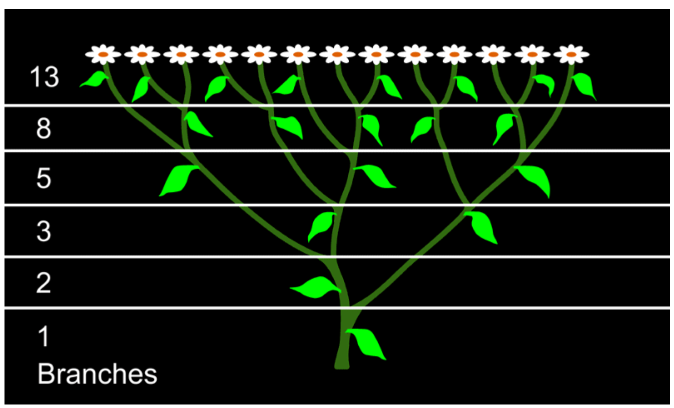
"The most direct and in a sense the most important problem which our conscious knowledge of nature should enable us to solve is the ***anticipation of future events***, so that we may arrange our present affairs in accordance with such anticipation". (Hertz, 1894)

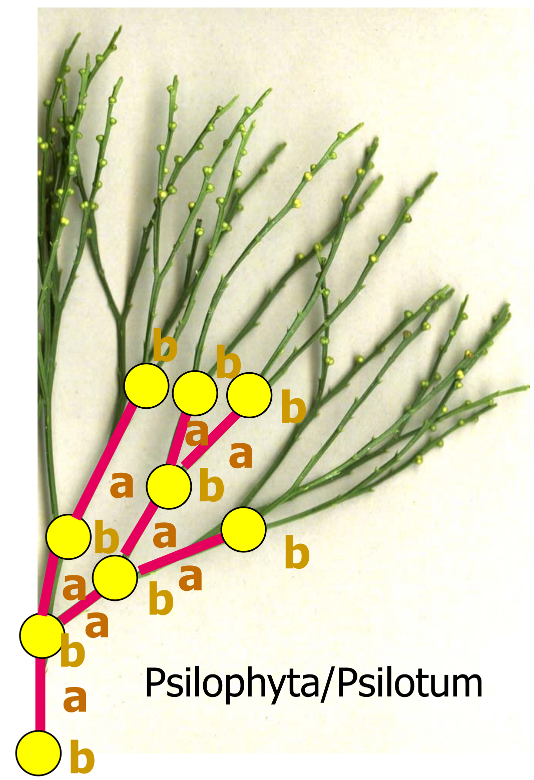
BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

branching as a general system

- An Accurate Model
  - Requires
    - Varying angles
    - Varying stem lengths
    - randomness
  - The Fibonacci Model is similar
    - Initial State: b
    - b -> a
    - a -> ab
  - *sneezewort*



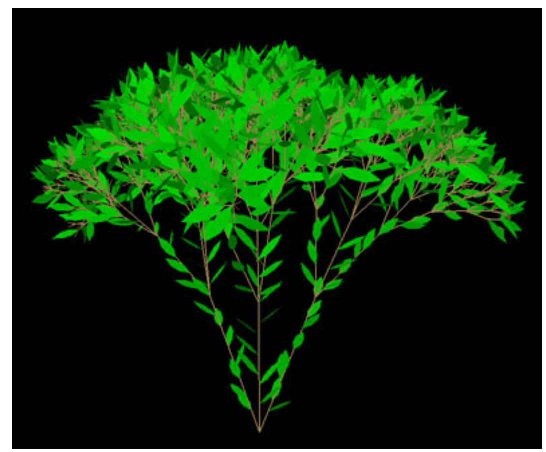




BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

Aristid Lindenmeyer

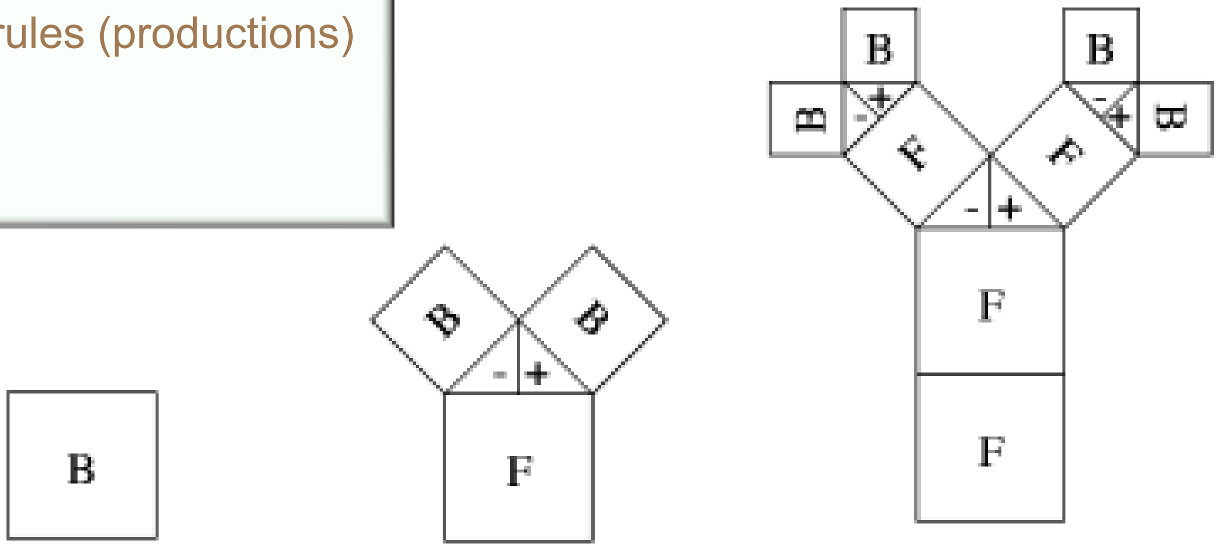
- Mathematical formalism proposed by the biologist Aristid Lindenmayer in 1968 as a foundation for an axiomatic theory of biological development.
  - applications in computer graphics
    - Generation of fractals and realistic modeling of plants
  - Grammar for rewriting Symbols
    - Production Grammar
    - Defines complex objects by successively replacing parts of a simple object using a set of recursive, rewriting rules or productions.
      - Beyond one-dimensional production (Chomsky) grammars
      - Parallel ***recursion***
      - Access to computers



rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

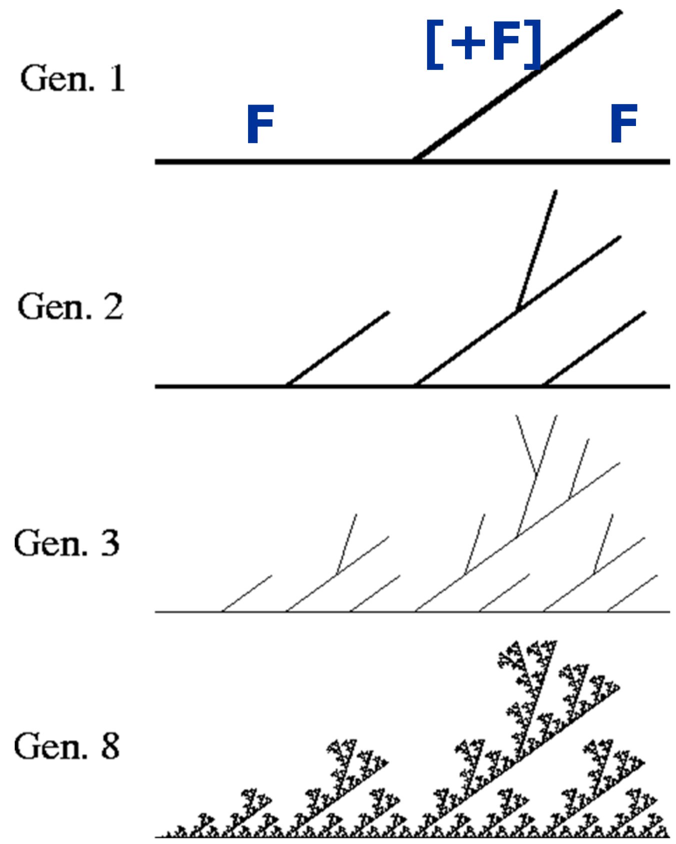
formal notation of the production system

- An L-system is an ordered triplet
  - $G = <V, w, P>$
    - V = alphabet of the symbols in the system
      - V = {F, B}
    - w = nonempty word
      - the axiom: B
    - P = finite set of production rules (productions)
      - B $\rightarrow$ F[-B][+B]
      - F $\rightarrow$ FF

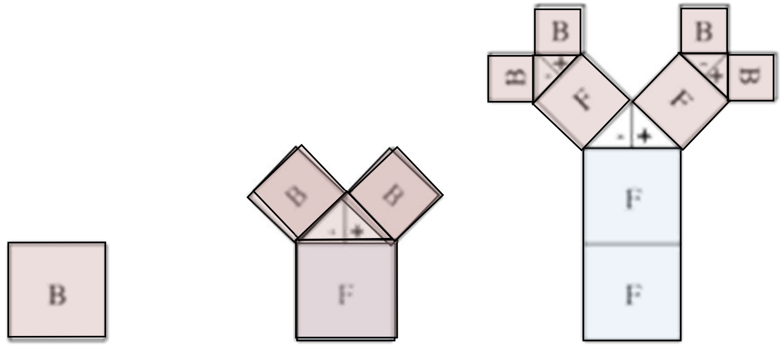


production rules for artificial plants

- Add branching symbols [ ]
  - Main trunk shoots off one side branch
  - simple example
    - Angle 45
    - Axiom: F
      - Seed Cell
    - Rule: F=F[+F]F
- Deterministic, context-free L-systems
  - Simplest class of L-systems
  - Simple re-writing
  - D0L



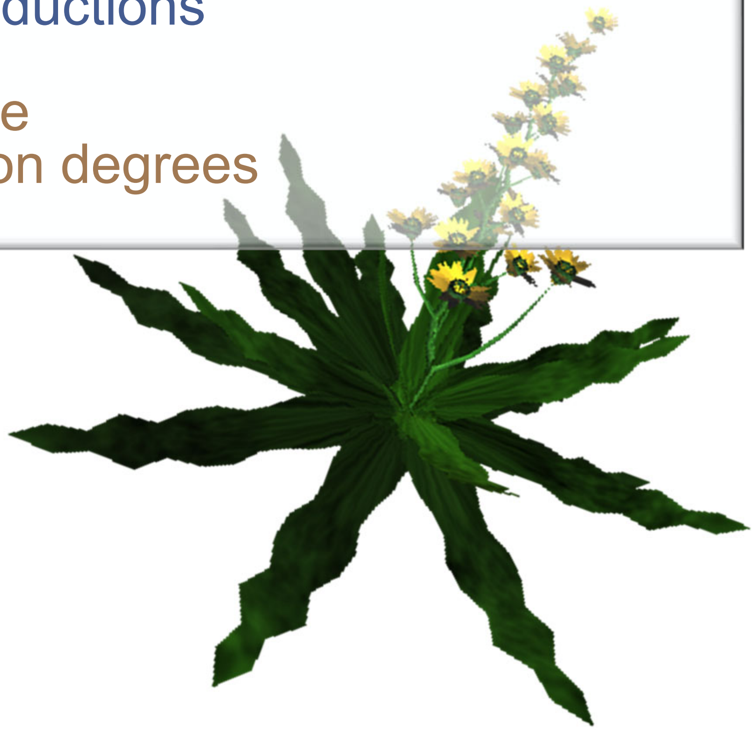
- Axiom
  - B
- Cell Types
  - B,F
- Rules
  - B → F[-B][+B]
  - F → FF



Depth	Resulting String
0	B
1	F[-B][+B]
2	FF[-F[-B][+B]]+[+F[-B][+B]]
3	FFFF[-FF[-F[-B][+B]][+F[-B][+B]]]+[FF[-F[-B][+B]][+F[-B][+B]]]

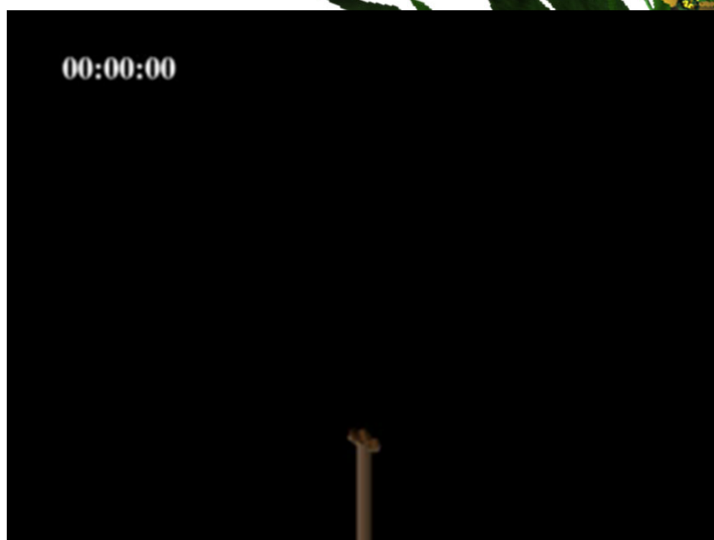
rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

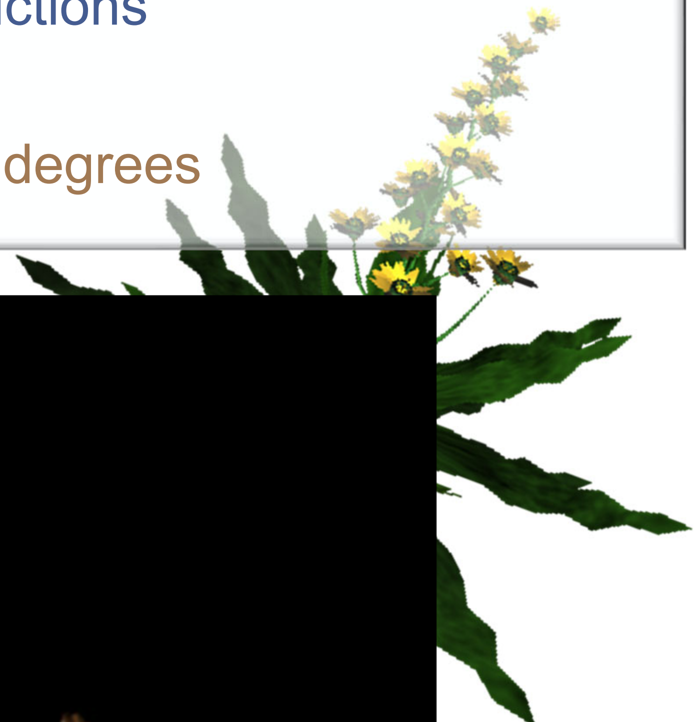
- Discrete nature of L-systems makes it difficult to model continuous phenomena
  - Numerical parameters can be associated with L-system symbols
  - Parameters control the effect of productions
  - A (t) → B (t x 3)
    - Growth can be modulated by time
    - Varying length of braches, rotation degrees



rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

- Discrete nature of L-systems makes it difficult to model continuous phenomena
  - Numerical parameters can be associated with L-system symbols
  - Parameters control the effect of productions
  - A (t) → B (t x 3)
    - Growth can be modulated by time
    - Varying length of braches, rotation degrees





rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

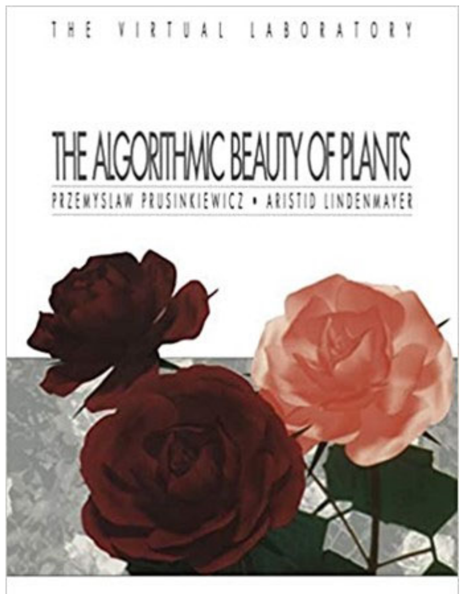
parametric L-system

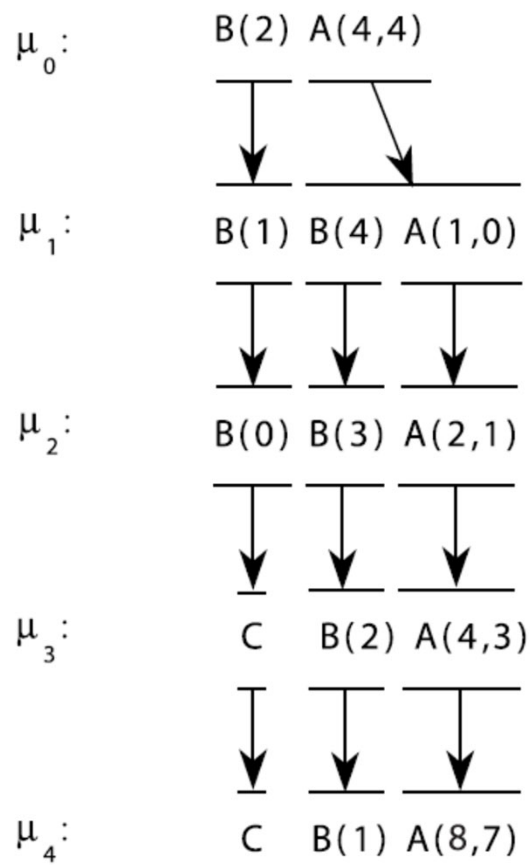
example

$$
\begin{aligned}
\omega &: B(2)A(4,4) \\
p_1 &: A(x,y) \quad : y <= 3 \quad \rightarrow \quad A(x*2, x+y) \\
p_2 &: A(x,y) \quad : y > 3 \quad \rightarrow \quad B(x)A(x/y, 0) \\
p_3 &: B(x) \quad : x < 1 \quad \rightarrow \quad C \\
p_4 &: B(x) \quad : x >= 1 \quad \rightarrow \quad B(x-1)
\end{aligned}
$$

operate on ***parametric words***, which are strings of modules consisting of symbols with associated parameters and their rules

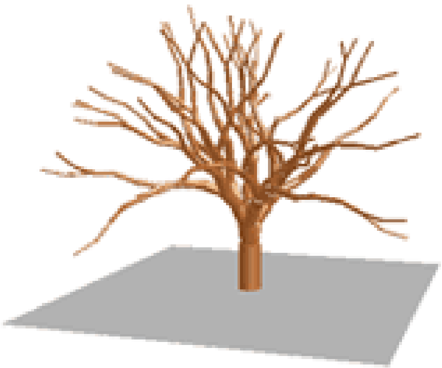
From: P. Prusinkiewicz and A. Lindenmayer [1991]. *The Algorithmic Beauty of Plants*.

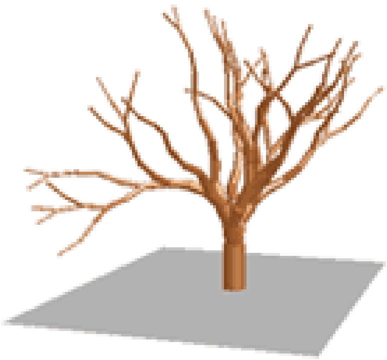




BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

- Probabilistic production rules
  - A → B C (P = 0.3)
  - A → F A (P = 0.5)
  - A → A B (P = 0.2)





http://coco.ccu.uniovi.es/malva/sketchbook/

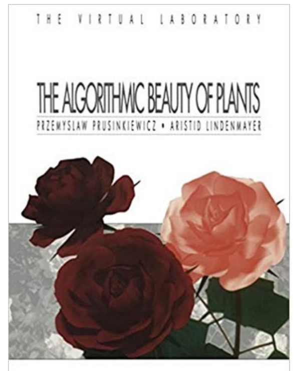
2L-Systems

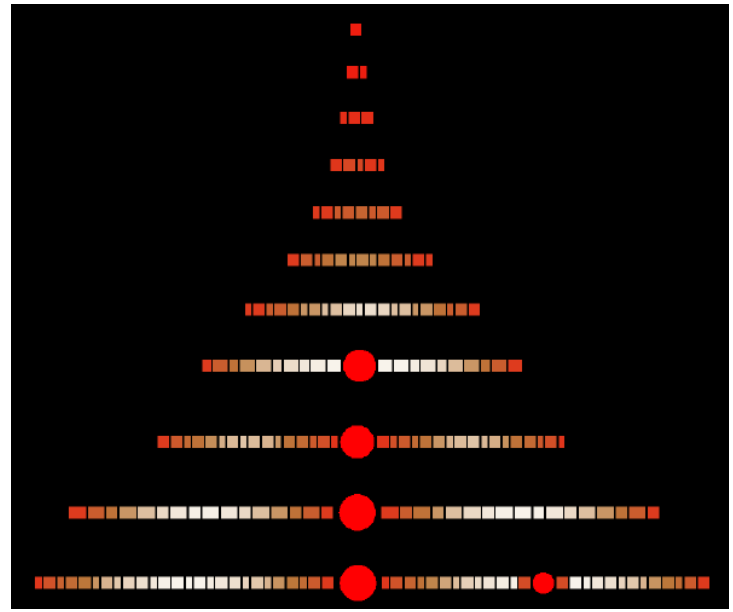
- Production rules depend on neighbor symbols in input string
  - simulates interaction between different parts
    - necessary to model information exchange between neighboring components
- 2L-Systems
  - $P: a_l < a > a_r \rightarrow X$
    - P1: A<F>A → A
    - P2: A<F>F → F
- 1L-Systems
  - $P: a_l < a \rightarrow X$ or $P: a > a_r \rightarrow X$
- Generalized to IL-Systems
  - (k,l)-system
    - left (right) context is a word of length k(l)

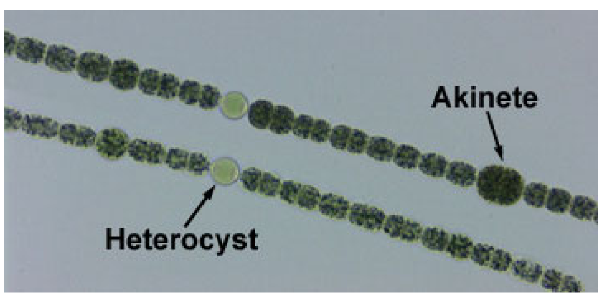
example

```
#define CH 900   /* high concentration */
#define CT 0.4   /* concentration threshold */
#define ST 3.9   /* segment size threshold */
#include H       /* heterocyst shape specification */
#ignore f ~ H

ω  :  -(90)F(0,0,CH)F(4,1,CH)F(0,0,CH)
p1 :  F(s,t,c)  :  t=1 & s>=6 →
                      F(s/3*2,2,c)f(1)F(s/3,1,c)
p2 :  F(s,t,c)  :  t=2 & s>=6 →
                      F(s/3,2,c)f(1)F(s/3*2,1,c)
p3 :  F(h,i,k) < F(s,t,c) > F(o,p,r)  :  s>ST|c>CT →
                      F(s+.1,t,c+0.25*(k+r-3*c))
p4 :  F(h,i,k) < F(s,t,c) > F(o,p,r)  :  !(s>ST|c>CT) →
                      F(0,0,CH) ~ H(1)
p5 :  H(s)  :  s<3 → H(s*1.1)
```







convenient tool for expressing developmental models with ***diffusion of substances***. pattern of cells in *Anabaena catenula* and other blue-green bacteria

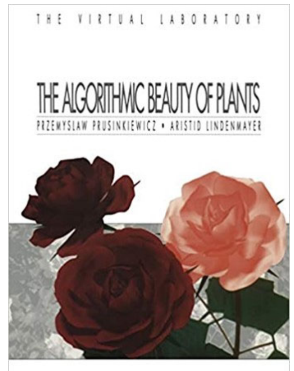
From: P. Prusinkiewicz and A. Lindenmayer [1991]. *The Algorithmic Beauty of Plants*.

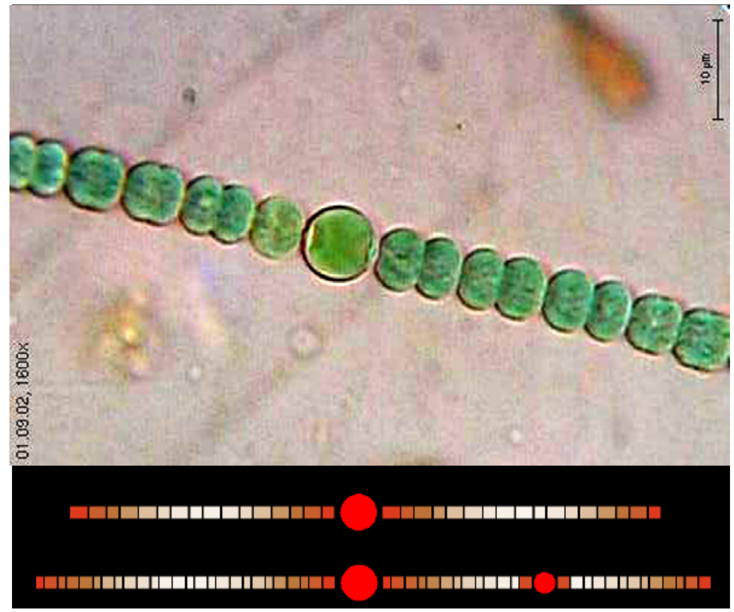
BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

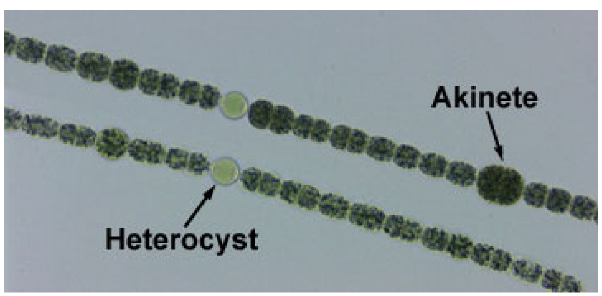
example

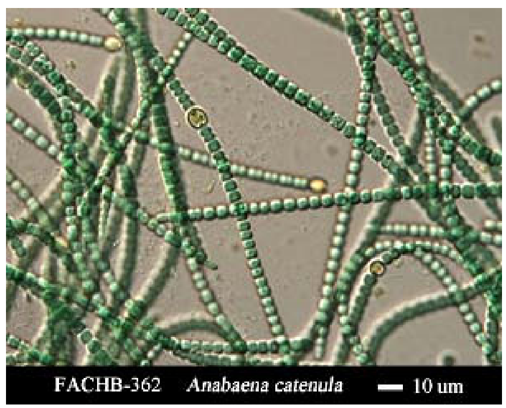
```
#define CH 900   /* high concentration */
#define CT 0.4   /* concentration threshold */
#define ST 3.9   /* segment size threshold */
#include H       /* heterocyst shape specification */
#ignore f ~ H

ω  :  -(90)F(0,0,CH)F(4,1,CH)F(0,0,CH)
p1 :  F(s,t,c) :  t=1 & s>=6 →
                 F(s/3*2,2,c)f(1)F(s/3,1,c)
p2 :  F(s,t,c) :  t=2 & s>=6 →
                 F(s/3,2,c)f(1)F(s/3*2,1,c)
p3 :  F(h,i,k) < F(s,t,c) > F(o,p,r) :  s>ST|c>CT →
                 F(s+.1,t,c+0.25*(k+r-3*c))
p4 :  F(h,i,k) < F(s,t,c) > F(o,p,r) :  !(s>ST|c>CT) →
                 F(0,0,CH) ~ H(1)
p5 :  H(s) :  s<3 → H(s*1.1)
```





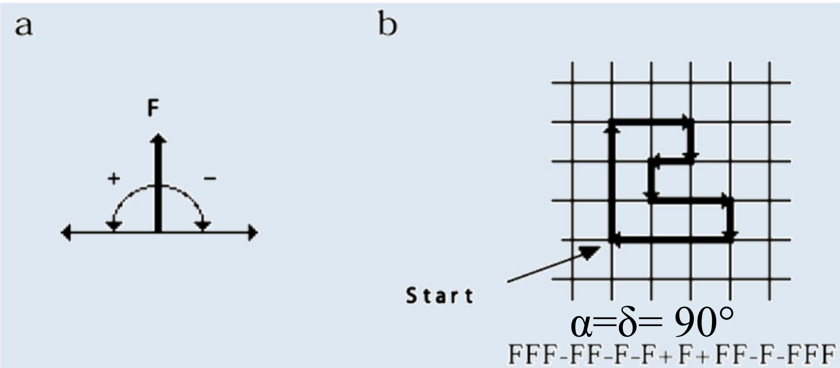




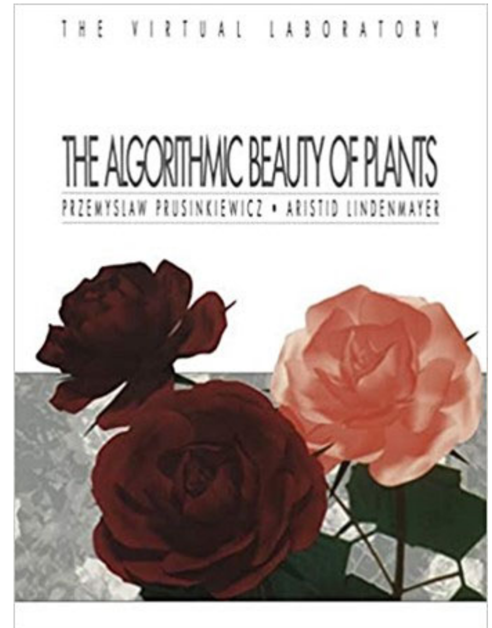
convenient tool for expressing developmental models with ***diffusion of substances***.
pattern of cells in *Anabaena catenula* and other blue-green bacteria

From: P. Prusinkiewicz and A. Lindenmayer [1991]. *The Algorithmic Beauty of Plants*.

Drawing words



state of turtle defined as (x, y, α), coordinates (*position*) and angle (*heading*). Moves according to *step size* d and *angle increment* δ

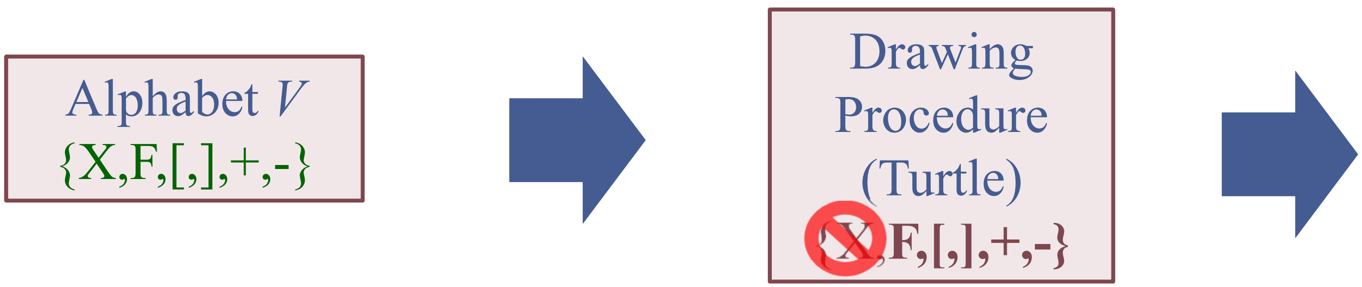


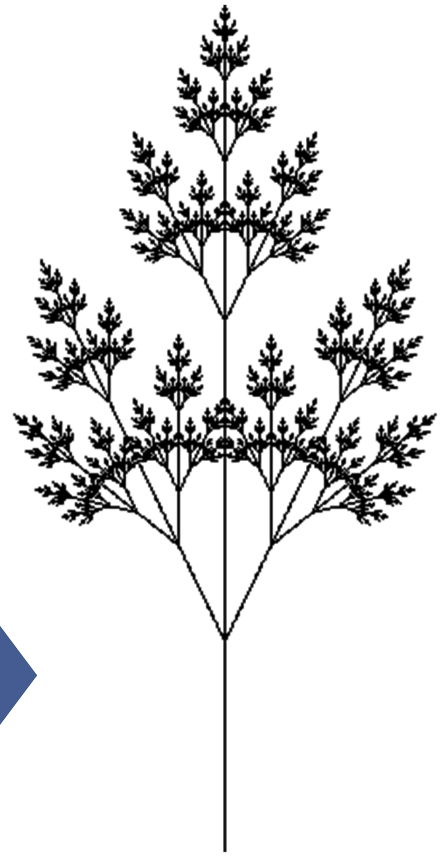
F	Move forward a step of length d. The state of the turtle changes to (x', y', α), where $x' = x + d\cos\alpha$ and $y' = y + d\sin\alpha$. A line segment between points (x, y) and (x', y') is drawn.
f	Move forward a step of length d without drawing a line.
+	Turn left by angle δ. The next state of the turtle is $(x, y, \alpha+\delta)$. The positive orientation of angles is counter-clockwise.
−	Turn right by angle δ. The next state of the turtle is $(x, y, \alpha-\delta)$.

From: P. Prusinkiewicz and A. Lindenmayer [1991]. *The Algorithmic Beauty of Plants*.

alphabet handling by Turtle

- An L-system is an ordered triplet
  - $G = <V, w, P>$
    - V = alphabet of the symbols in the system
      - V = {F, X}
    - w = nonempty word
      - the axiom: X
    - P = finite set of production rules (productions)
      - X → F[+X][-X]FX
      - F → FF





Angle: 14

rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

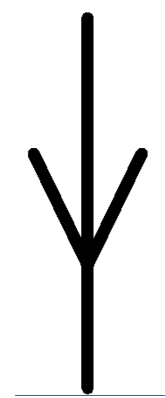
alphabet handling by Turtle

- **Example L-System**
  - V = {F, X}
  - axiom: X
  - Productions
    - X → F[+X][-X]FX
    - F → FF

Alphabet *V*
{X,F,[,],+,-}

Turtle
{~~X~~,F,[,],+,-}

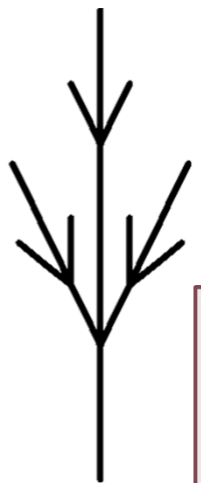
n=1
F[+X][-X]FX
F[+][-]F
FF



n=2
FF[+F[+X][-X]FX][-F[+X][-X]FX]FF[+X][-X]FX
FF[+F[+][-]F][-F[+][-]F]FF[+][-]F
FF[+FF][-FF]FFF

rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

alphabet handling by Turtle



n=2

FF[+F[+X][-X]FX][-F[+X][-X]FX]FF[+X][-X]FX

FF[+F[+][-]F][-F[+][-]F]FF[+][-]F

FF[+FF][-FF]FFF

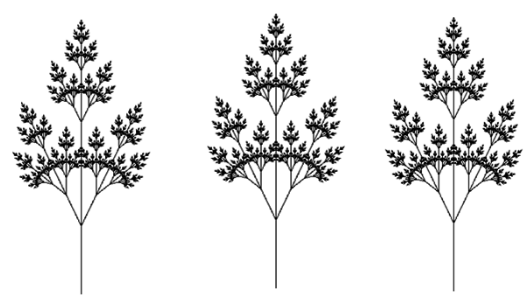
n=3

FFFF[+FF[+F[+X][-X]FX][-F[+X][-X]FX]FFF[+X][-X]FX][-FF[+ F[+X][-X]FX][-
F[+X][-X]FX]FFF[+X][-X]FX]FFFF[+F[+X][-X]FX][- F[+X][-X]FX]FFF[+X][-X]FX

FFFF[+FF[+F[+][-]F][- F[+][-]F]FF F[+][-]F][-FF[+ F[+][-]F][- F[+][-]F]FF F[+][-
]F]FFFF[+ F[+][-]F][- F[+][-]F]FFF[+][-]F

FFFF[+FF[+FF][-FF]FFFF][-FF[+FF][-FF]FFFF]FFFF[+FF][-FF]FFFF

Angle: 14



BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

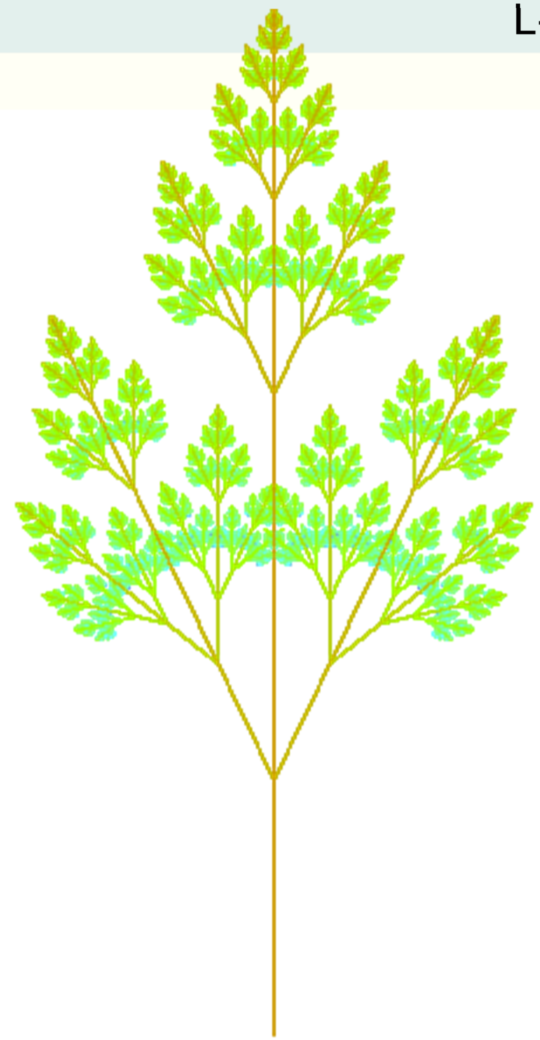
alphabet handling by Turtle (adding color)

- Example L-System
  - V = {F, X}
  - axiom: X
  - Productions
    - X → F[>8+X][>8-X]FX
    - F → FF

Alphabet *V*
{X,F,[,],+,-,>n}

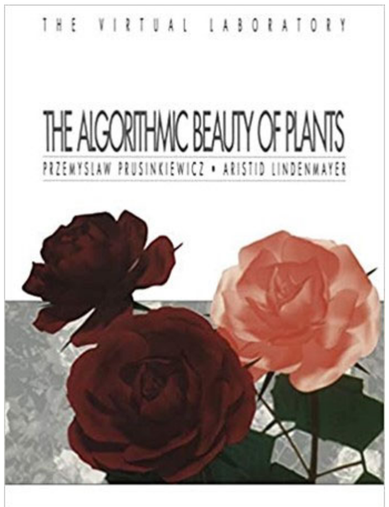
→

Turtle
{X,F,[,],+,-,>n}



Binghamton University State University of New York
rocha@indiana.edu
casci.binghamton.edu/academics/i-bic

example



$$\omega: \; F-F-F-F$$
$$p: \; F \rightarrow F-F+F+FF-F-F+F$$

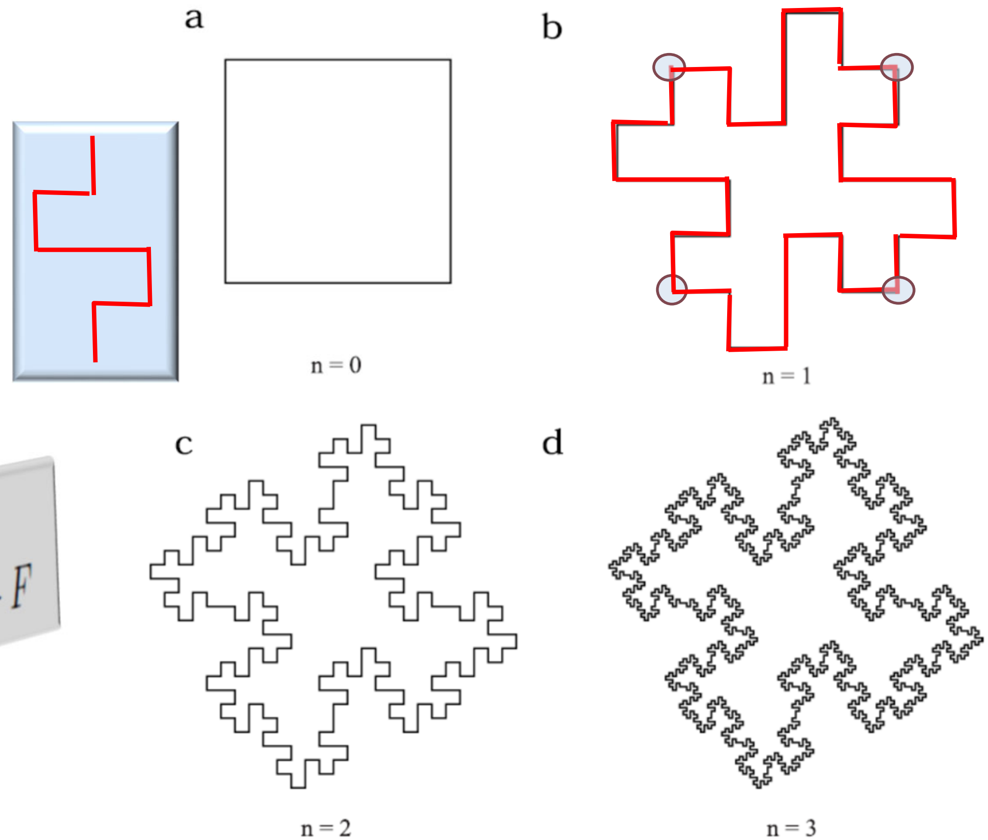


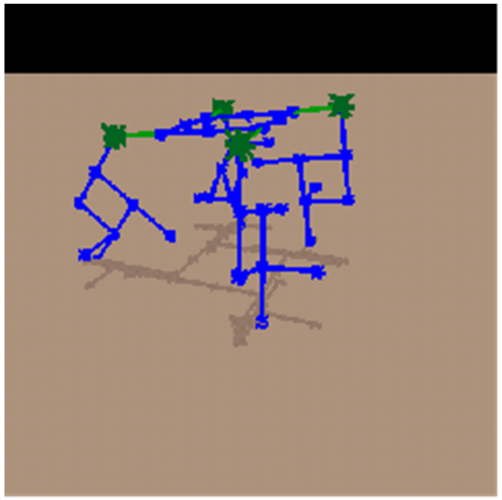
Figure 1.6: Generating a quadratic Koch island

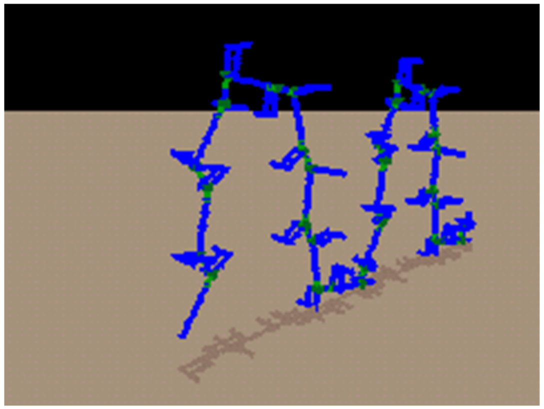
From: P. Prusinkiewicz and A. Lindenmayer [1991]. *The Algorithmic Beauty of Plants*.

BINGHAMTON UNIVERSITY STATE UNIVERSITY OF NEW YORK
rocha@binghamton.edu
casci.binghamton.edu/academics/i-bic

robots

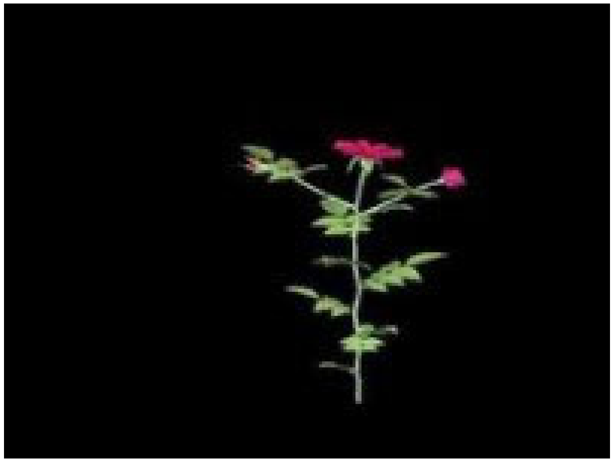
- Evolutionary design of robots
  - Difficult to reach high complexities necessary for practical engineering
  - Karl Sims and Jordan Pollack, Hod Lipson, Gregory Hornby, and Pablo Funes claim that for automatic design to scale in complexity it must employ re-used modules
    - Sims,K. [1994]. “Evolving Virtual Creatures”. *Proceedings of the 21st annual conference on Computer graphics and interactive techniques*, pp. 15 – 22.
    - H. Lipson and J. B. Pollack (2000), "Automatic design and Manufacture of Robotic Lifeforms", *Nature* **406**: 974-978.
  - *generative representation* to encode individuals in the population.
  - Indirect representation: an algorithm for creating a design.
    - using Lindenmayer systems (L-systems)
    - evolved locomotiong robots (called *genobots*).

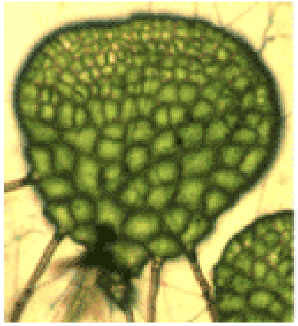


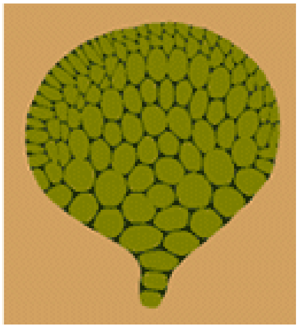


models or realistic imitations?

- Common features (design principle) between artificial and real plants
  - Development of (macro-level) morphology from local (micro-level) logic
  - Parallel application of simple rules
    - Genetic vs. algorithmic
  - Recursion
- But are the algorithms the same as the biological *mechanism*?
  - Real organisms need to economize information for coding complex phenotypes
    - The genome cannot encode every ripple of the brain or lungs
    - Organisms need to encode **compact procedures** for producing the same pattern (with randomness) again and again
- But recursion alone does not explain form and morphogenesis
  - One of the design principles involved
  - There are others
    - Selection, genetic variation, self-organization, epigenetics







fern gametophyte Microsorium linguaeforme (left) and a simulated model using map L systems (right).

readings

- Class Book
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press.
    - **Chapter 2**.
- Lecture notes
  - Chapter 1: What is Life?
  - Chapter 2: The logical Mechanisms of Life
  - Chapter 3: Formalizing and Modeling the World
    - posted online @ http://informatics.indiana.edu/rocha/i-bic

- Papers and other materials
  - Optional
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall.
      - Chapter 2, all sections
      - Chapter 7, sections 7.3 – Cellular Automata
      - Chapter 8, sections 8.1, 8.2, 8.3.10
    - Flake's [1998], *The Computational Beauty of Life*. MIT Press.
      - Chapters 10, 11, 14 – Dynamics, Attractors and chaos
    - Prusinkiewicz and Lindenmeyer [1996] *The algorithmic beauty of plants*.
      - Chapter 1

bit.ly/atBIC

rocha@indiana.edu
casci.binghamton.edu/academics/i-bic