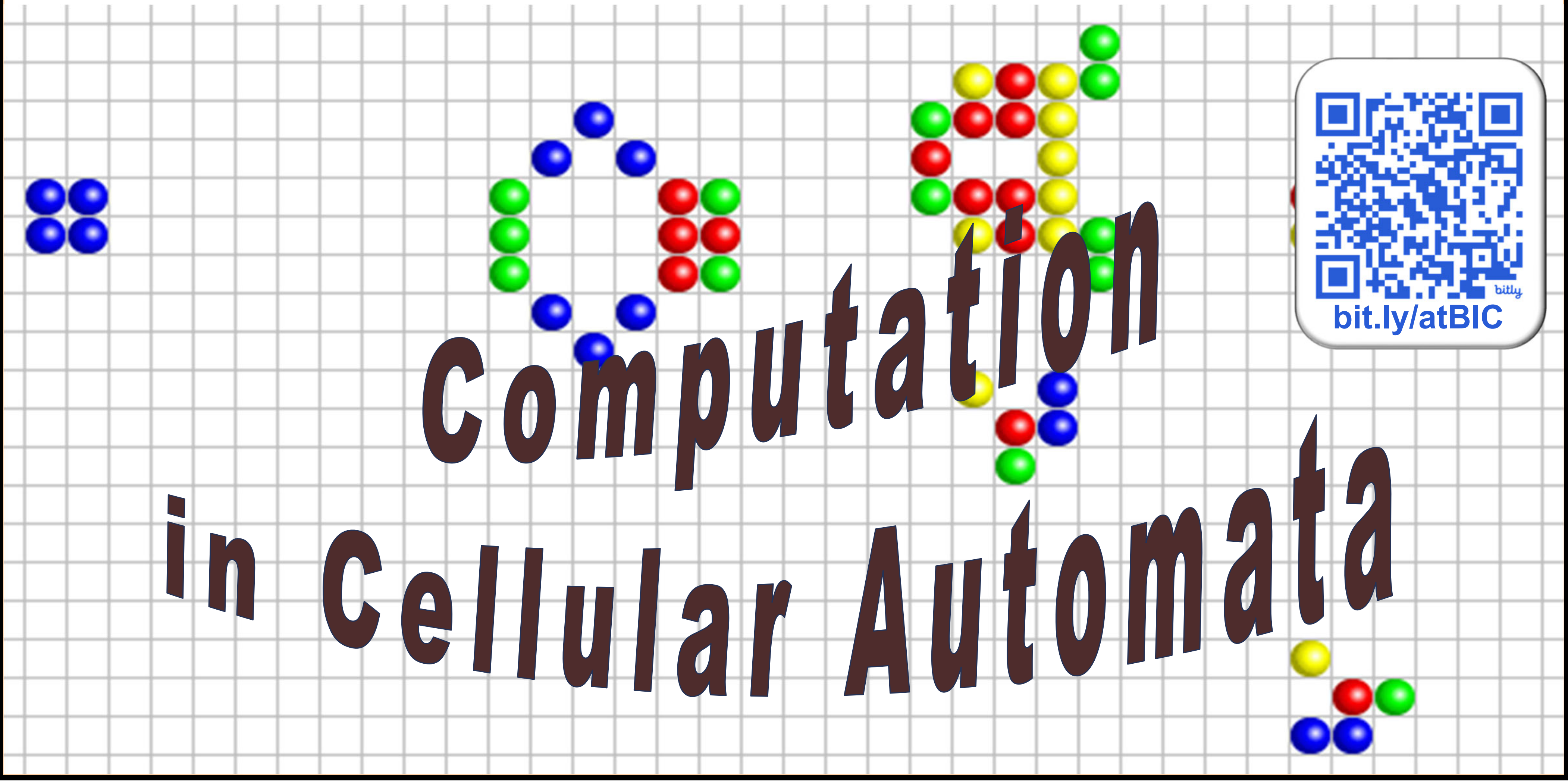
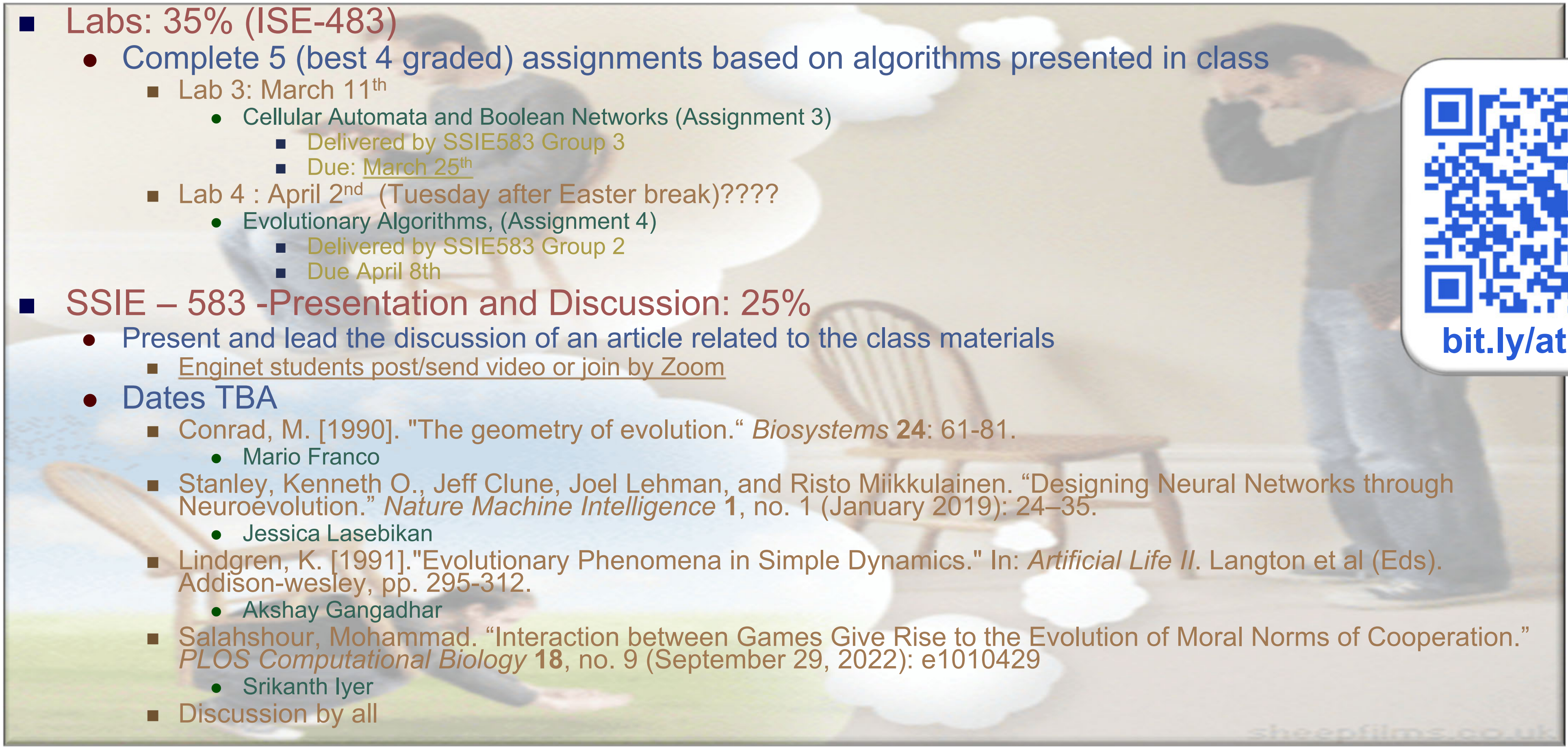


Computation in Cellular Automata

bit.ly/atBIC

key events coming up



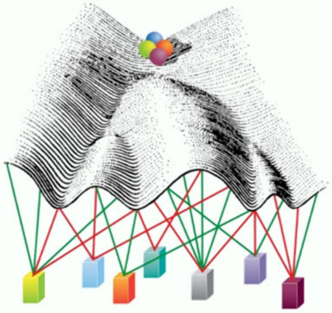
- Labs: 35% (ISE-483)
  - Complete 5 (best 4 graded) assignments based on algorithms presented in class
    - Lab 3: March 11th
      - Cellular Automata and Boolean Networks (Assignment 3)
        - Delivered by SSIE583 Group 3
        - Due: March 25th
    - Lab 4 : April 2nd (Tuesday after Easter break)????
      - Evolutionary Algorithms, (Assignment 4)
        - Delivered by SSIE583 Group 2
        - Due April 8th
- SSIE – 583 -Presentation and Discussion: 25%
  - Present and lead the discussion of an article related to the class materials
    - Enginet students post/send video or join by Zoom
  - Dates TBA
    - Conrad, M. [1990]. "The geometry of evolution.“ *Biosystems* **24**: 61-81.
      - Mario Franco
    - Stanley, Kenneth O., Jeff Clune, Joel Lehman, and Risto Miikkulainen. “Designing Neural Networks through Neuroevolution.” *Nature Machine Intelligence* **1**, no. 1 (January 2019): 24–35.
      - Jessica Lasebikan
    - Lindgren, K. [1991]."Evolutionary Phenomena in Simple Dynamics." In: *Artificial Life II*. Langton et al (Eds). Addison-wesley, pp. 295-312.
      - Akshay Gangadhar
    - Salahshour, Mohammad. “Interaction between Games Give Rise to the Evolution of Moral Norms of Cooperation.” *PLOS Computational Biology* **18**, no. 9 (September 29, 2022): e1010429
      - Srikanth Iyer
    - Discussion by all

bit.ly/atBIC

readings

until now

- Class Book
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press. Preface**, Chapters 1 and 4.**
- Lecture notes
  - Chapter 1: What is Life?
  - Chapter 2: The logical Mechanisms of Life
  - Chapter 3: Formalizing and Modeling the World
  - Chapter 4: Self-Organization and Emergent Complex Behavior
  - Chapter 5: Reality is Stranger than Fiction
    - posted online @ http://informatics.indiana.edu/rocha/i-bic



- Papers and other materials
  - Optional
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall.
      - Chapter 2, 7, 8
      - **Chapter 3, sections 3.1 to 3.5**
    - Flake's [1998], *The Computational Beauty of Life*. MIT Press.
      - Chapters 10, 11, 14 – Dynamics, Attractors and chaos

final project schedule

- Projects
  - Due by May 6th in Brightspace, “Final Project Paper” assignment
    - ALIFE 2023
      - Not to submit to actual conference due date (April 3rd , 2024)
      - https://2024.alife.org/
      - 8 pages, author guidelines:
      - https://2024.alife.org/call_paper.html
      - MS Word and Latex/Overleaf templates
    - Preliminary ideas **by March 15**
      - Submit to “Project Idea” assignment in Brightspace.
  - Individual or group
    - With very definite tasks assigned per member of group

ALIFE 2024

Tackle a real problem using bio-inspired algorithms, such as those used in the labs.

The 2024 Conference on Artificial Life

Copenhagen, Denmark | July 22-26, 2024

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STATE UNIVERSITY OF NEW YORK

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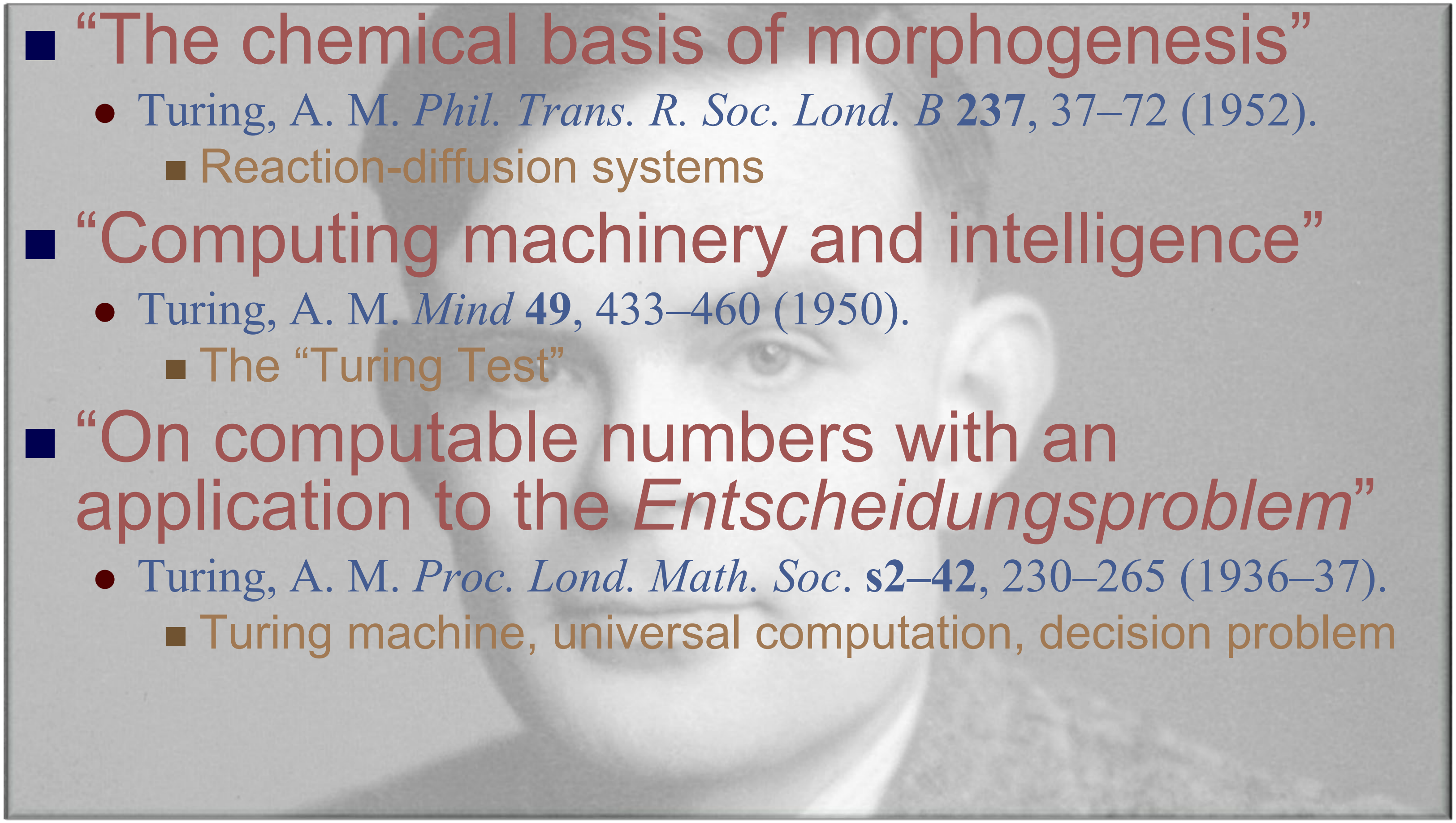
casci.binghamton.edu/academics/i-bic

Alan Turing (1912-1954)

key contributions (most relevant to biocomplexity)



- “The chemical basis of morphogenesis”
  - Turing, A. M. *Phil. Trans. R. Soc. Lond. B* **237**, 37–72 (1952).
    - Reaction-diffusion systems
- “Computing machinery and intelligence”
  - Turing, A. M. *Mind* **49**, 433–460 (1950).
    - The “Turing Test”
- “On computable numbers with an application to the *Entscheidungsproblem*”
  - Turing, A. M. *Proc. Lond. Math. Soc.* **s2–42**, 230–265 (1936–37).
    - Turing machine, universal computation, decision problem

Brenner, Sydney. [2012]. “Life’s code script.” *Nature* **482** (7386): 461-461.

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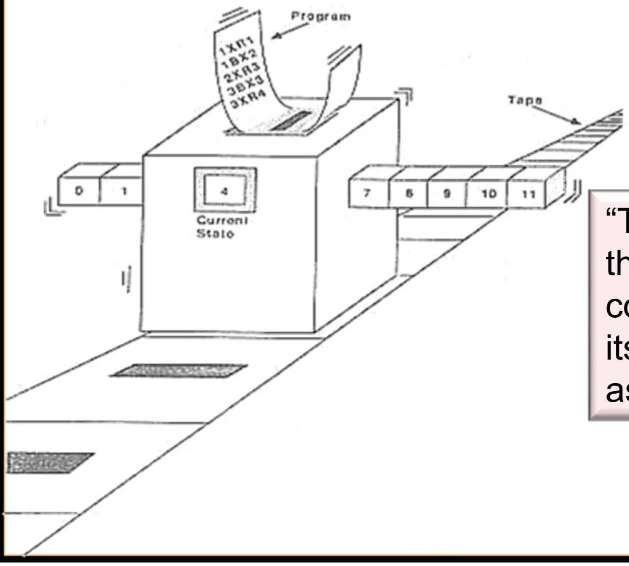
Turing’s tape

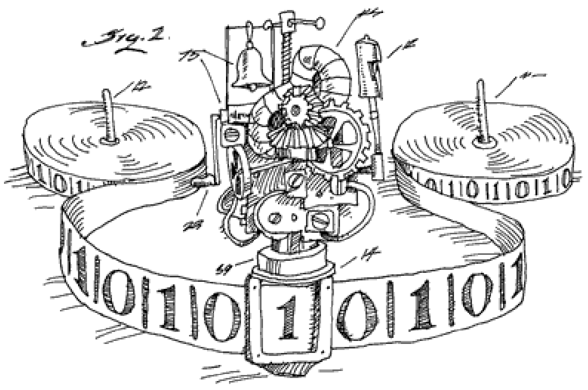
A fundamental principle of computation

- “On computable numbers with an application to the *Entscheidungsproblem*”
  - Turing, A. M. *Proc. Lond. Math. Soc.* **s2–42**, 230–265 (1936–37).
    - **Turing machine**, universal computation, decision problem
  - **Machine**’s *state* is *controlled* by a *program*, while *data* for program is on limitless **external tape**
    - every machine can be described as a **number** that can be stored on the tape (for itself or another machine)
      - Including a Universal machine
    - **distinction** between *numbers that mean things* (data) and *numbers that do things* (program)

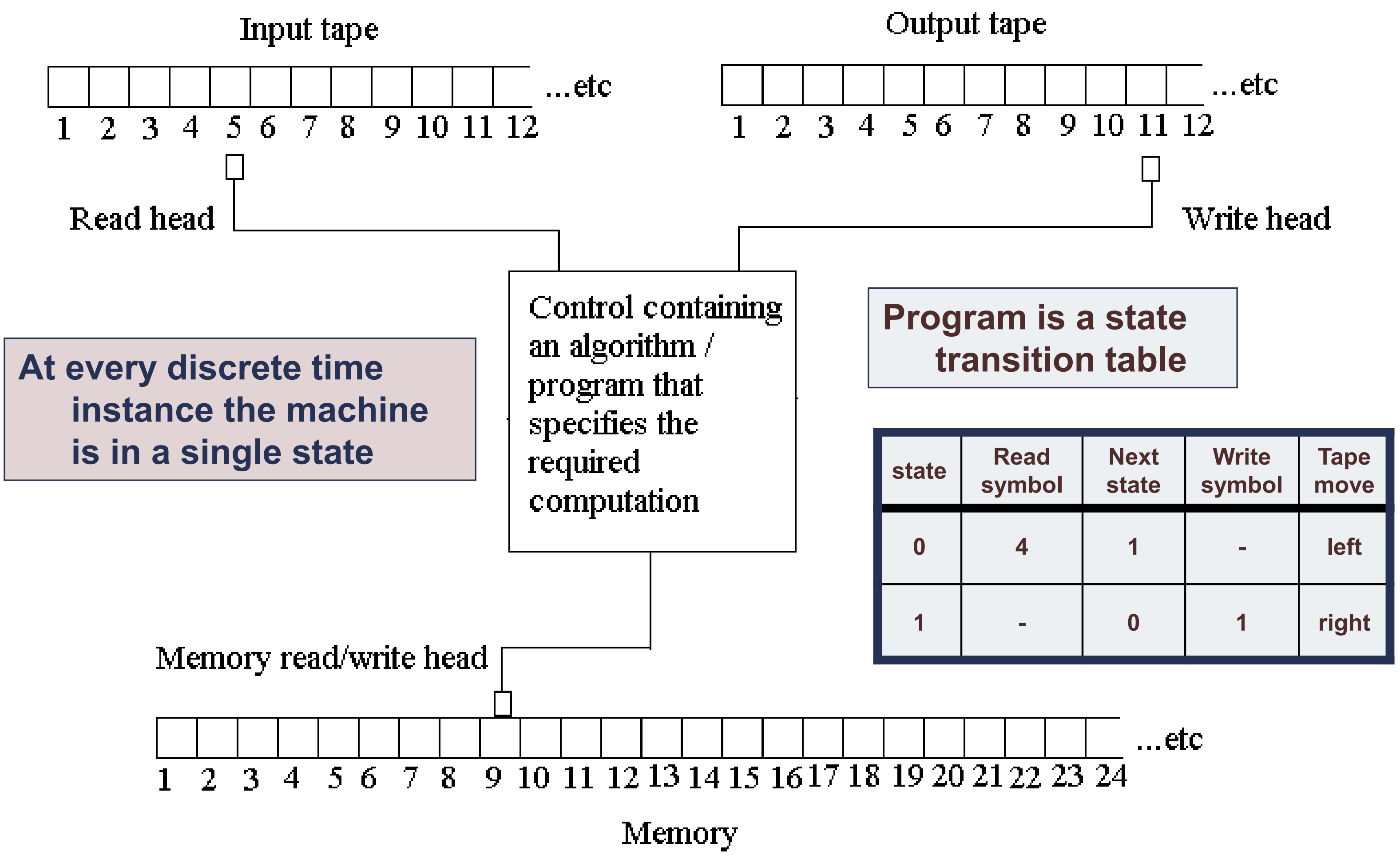


code

> “The fundamental, indivisible unit of information is the bit. The fundamental, indivisible unit of digital computation is the transformation of a bit between its two possible forms of existence: as [memory] or as [code]. George Dyson, 2012.



A Turing Machine



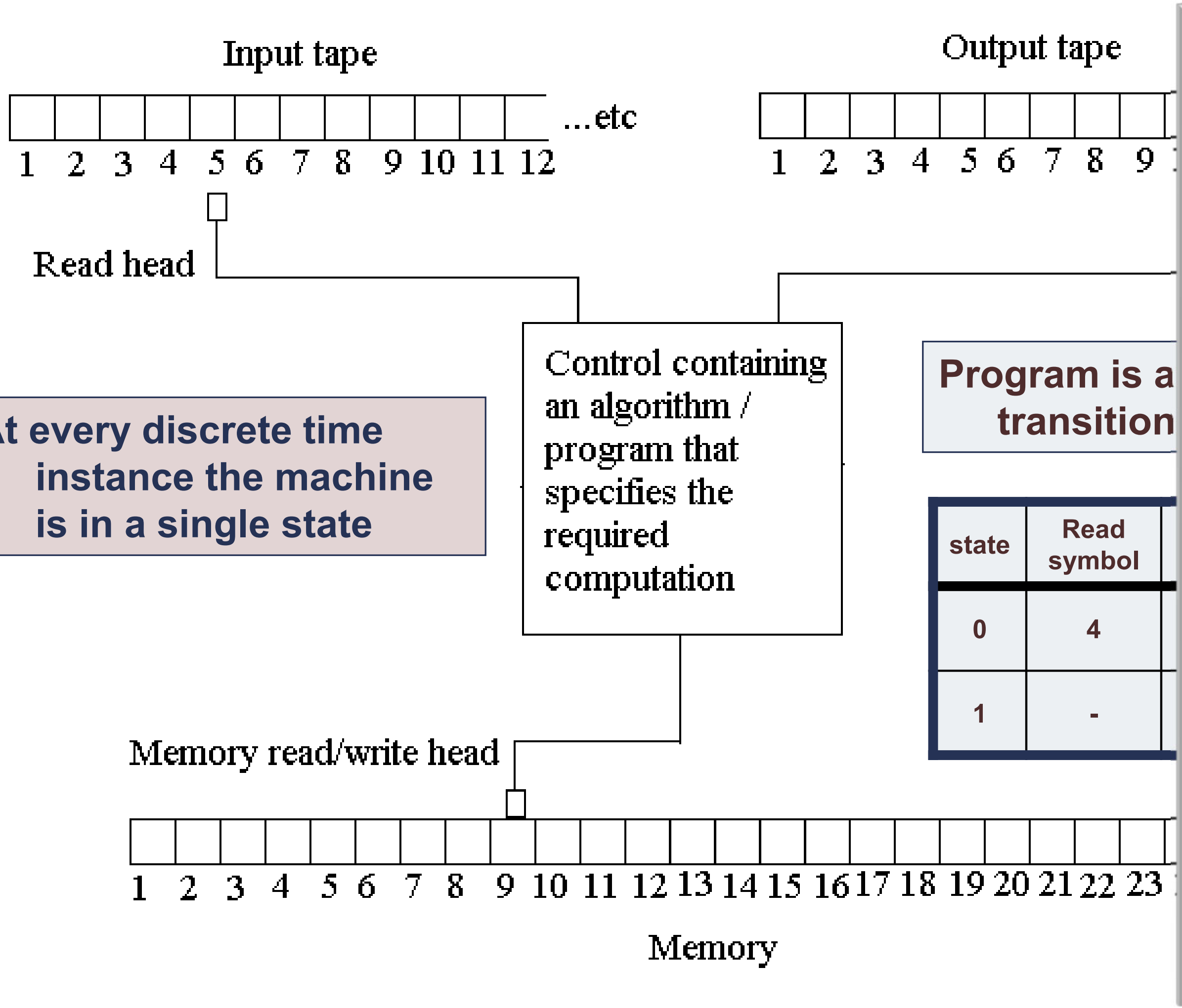
Control containing an algorithm / program that specifies the required computation

At every discrete time instance the machine is in a single state

Program is a state transition table

state	Read symbol	Next state	Write symbol	Tape move
0	4	1	-	left
1	-	0	1	right

A Turing Machine



Input tape

...etc

Output tape

Read head

Control containing an algorithm / program that specifies the required computation

Memory read/write head

Memory

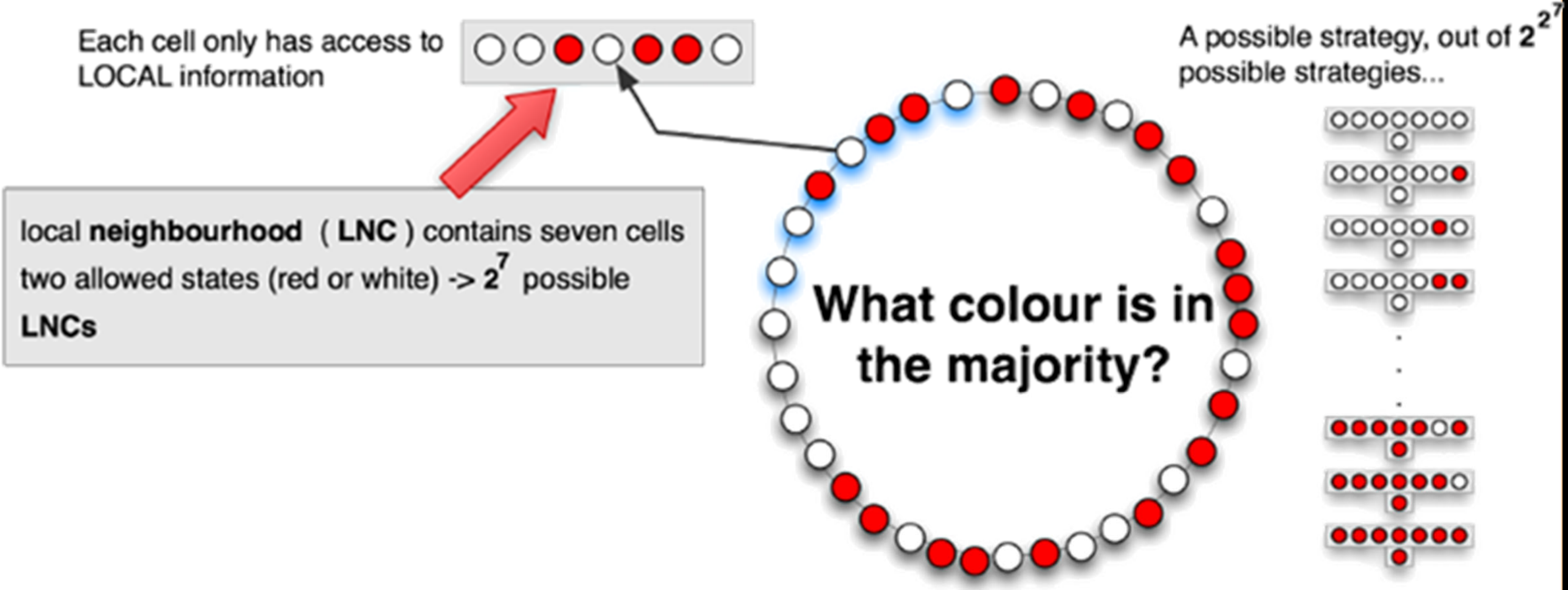
At every discrete time instance the machine is in a single state

Program is a transition

state	Read symbol
0	4
1	-

quorum sensing or what decision to take? (Density Classification)

$$K^{|N|} = 2^{7} = 128$$



Each cell only has access to LOCAL information

local **neighbourhood** (**LNC**) contains seven cells two allowed states (red or white) -> 2^7 possible **LNCs**

What colour is in the majority?

A possible strategy, out of 2^{2^7} possible strategies...

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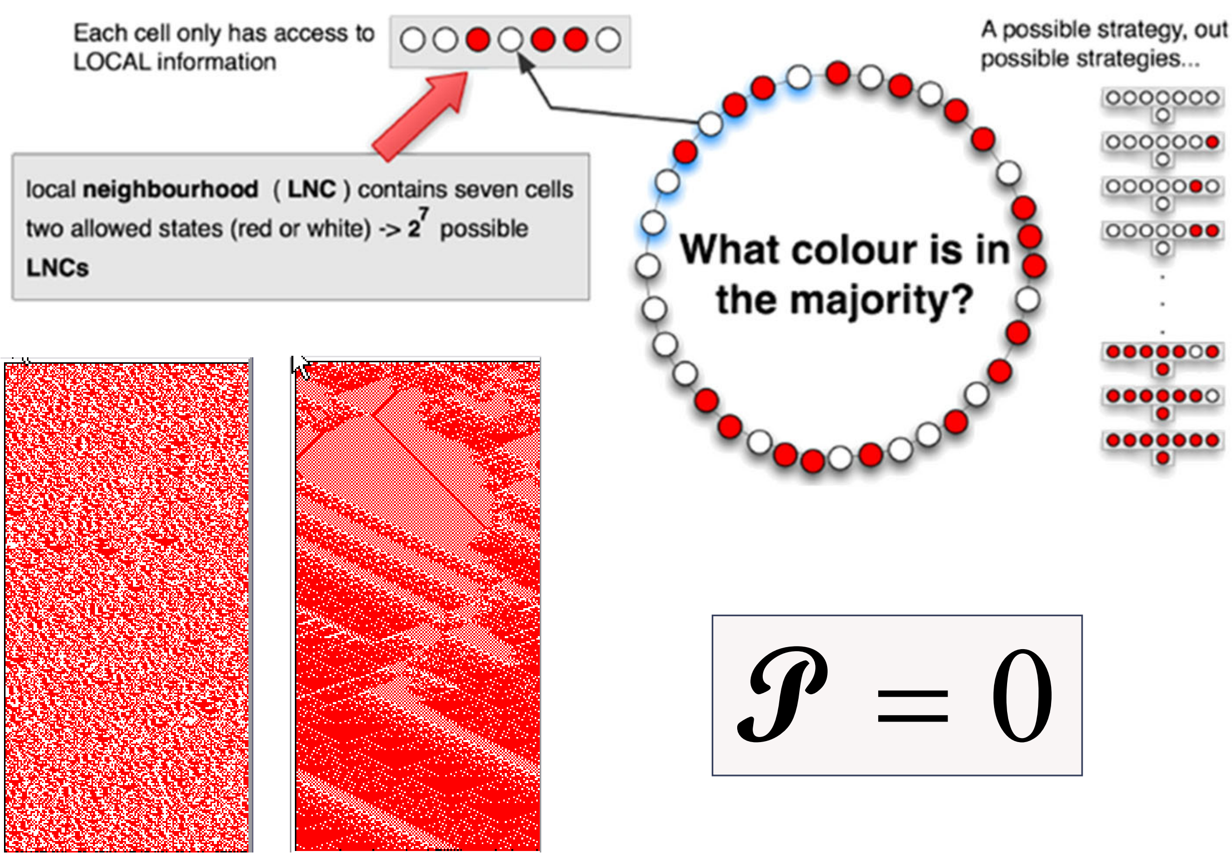
density classification task



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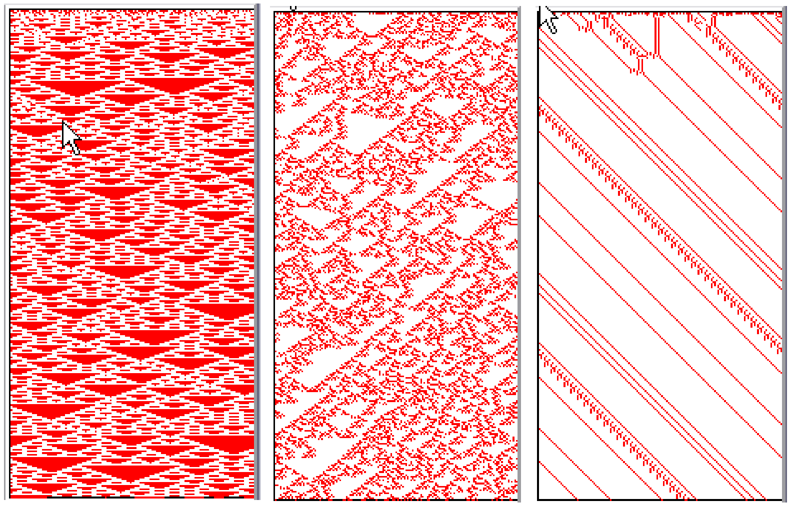
A possible strategy, out of 2^{2^7} possible strategies...

$$K^{|N|} = 2^7 = 128$$

Typically chaotic behavior
No convergence

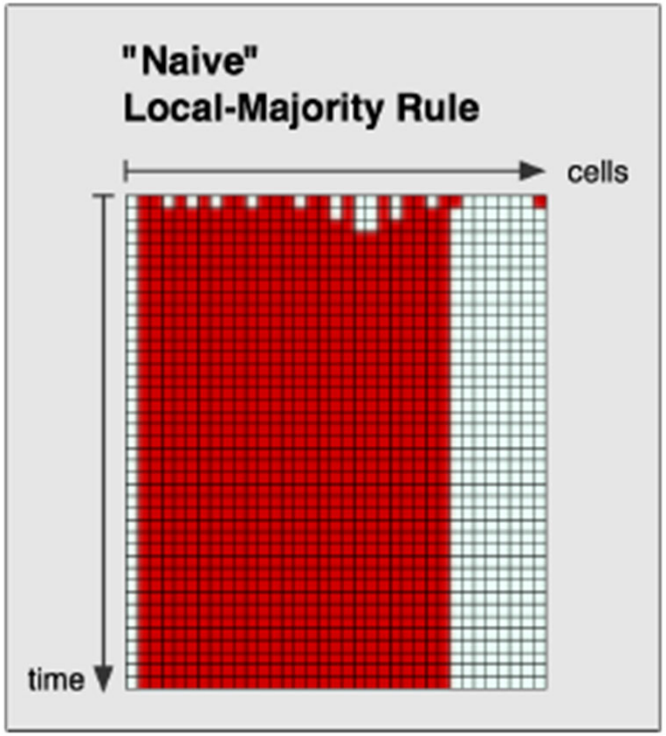
$$\mathscr{P} = 0$$

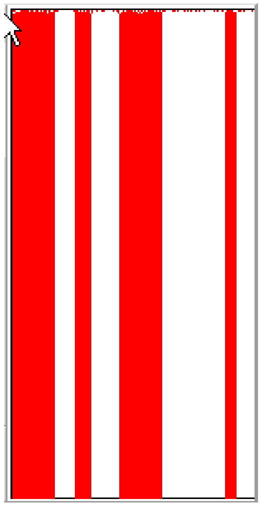


local strategy: majority rule

density classification task

$$K^{|N|} = 2^7 = 128$$





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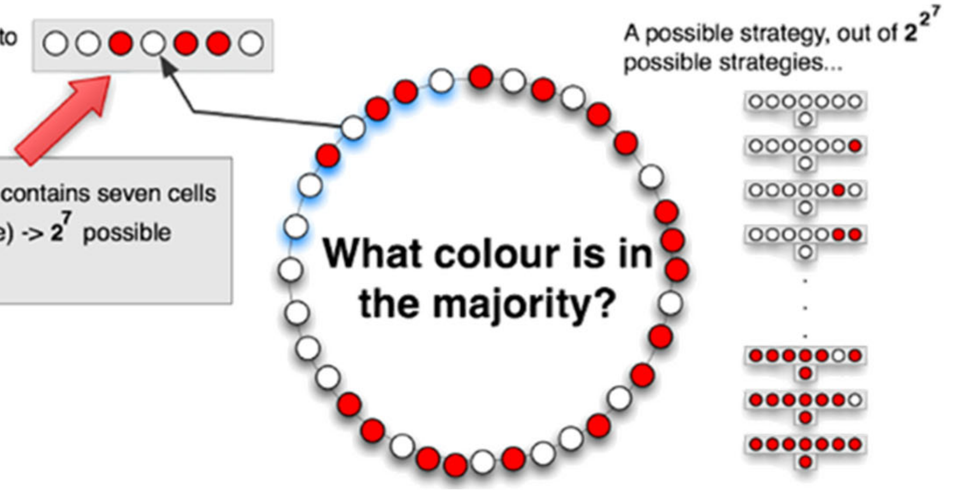


$$\mathscr{P} = 0$$

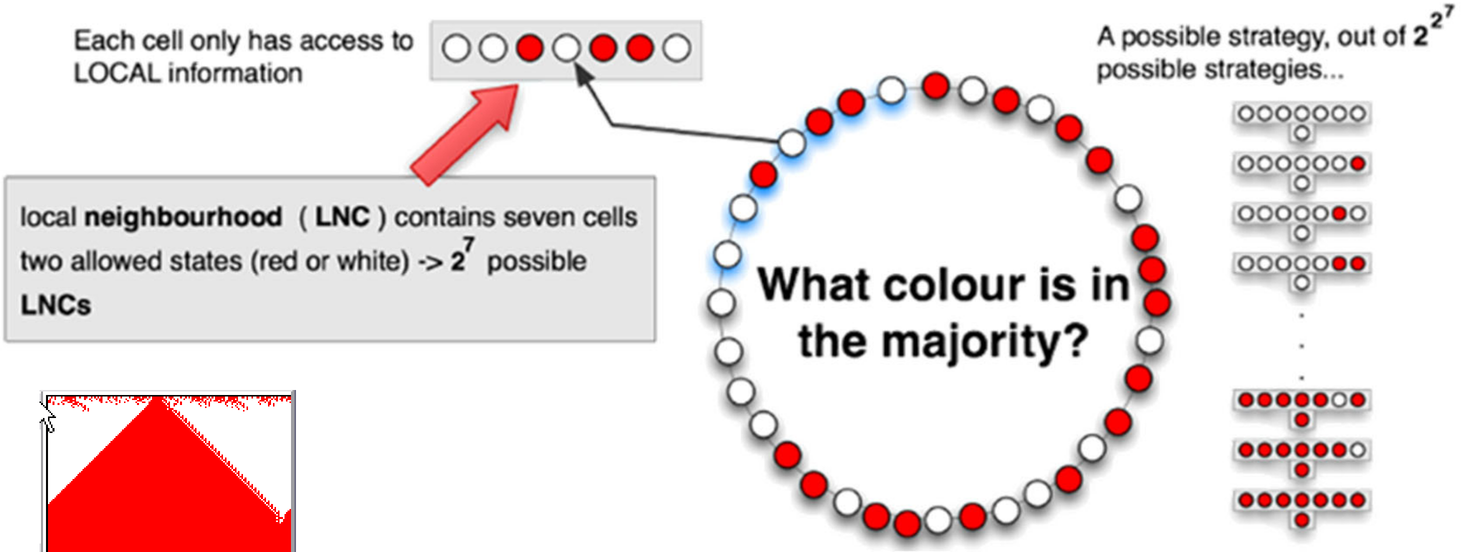
Isolated groups
No information transmission

block expansion strategy

density classification task

$$K^{|N|} = 2^7 = 128$$

$$\mathcal{P} \in [53\%, 60\%]$$



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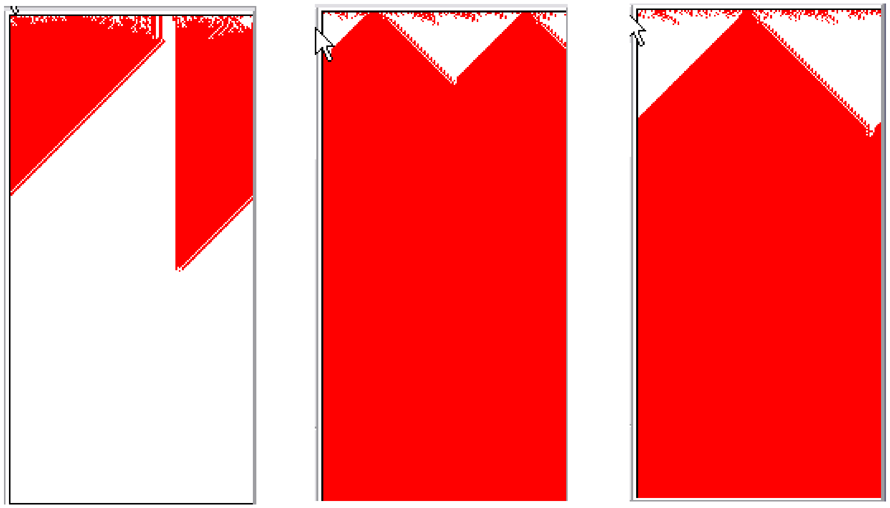
A possible strategy, out of 2^{2^7} possible strategies...



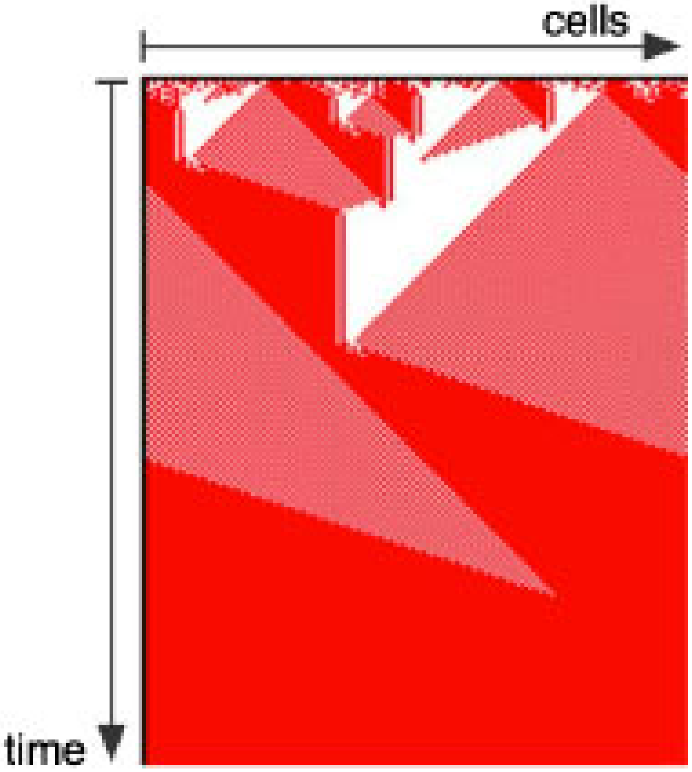
GA to evolve rules for DCT [1994]

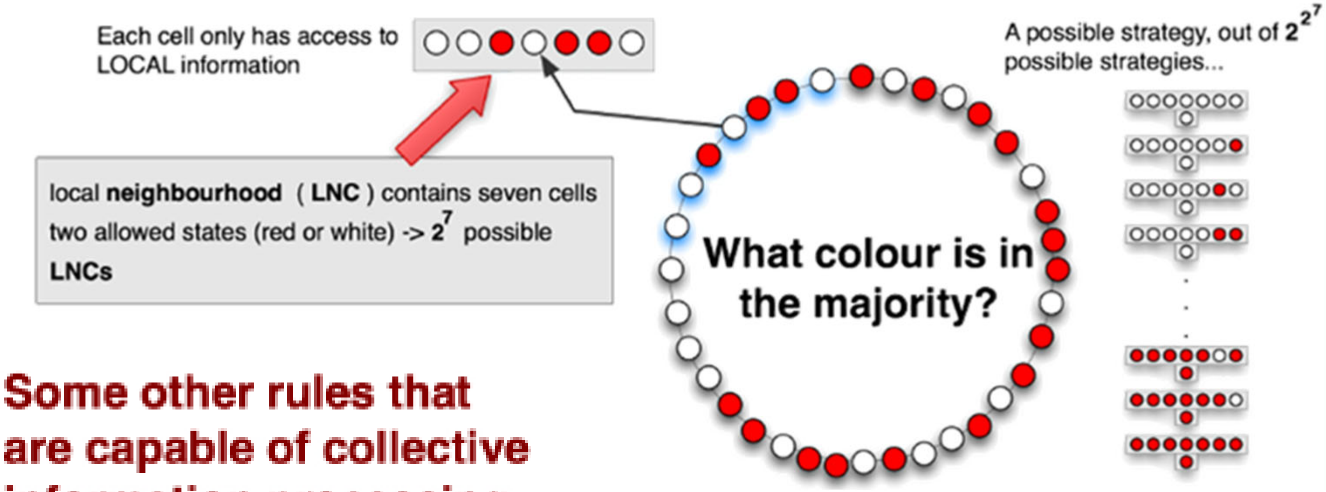
“blind” spreading of local information
No information integration
Not much better than random choice

emergent computation strategies

density classification task

$$K^{|N|} = 2^7 = 128$$

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A possible strategy, out of 2^{2^7} possible strategies...

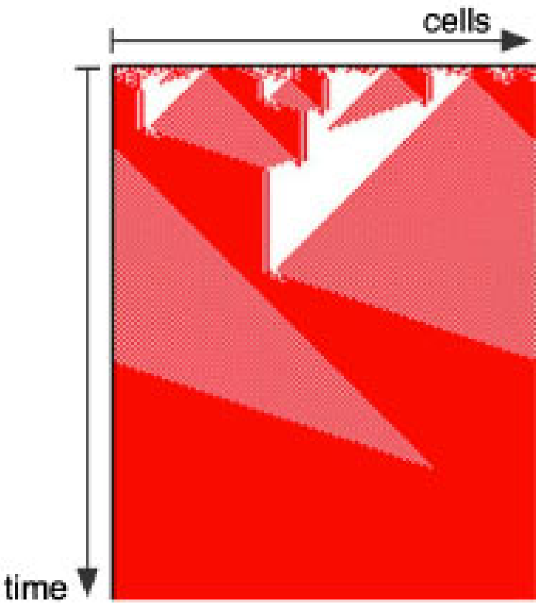
Some other rules that are capable of collective information processing over time and space can solve this task with a range of performances where

$$P_{149}^{10^5} > 80\ \%$$

Integration and transmission of information across population

best CA rules

for DST



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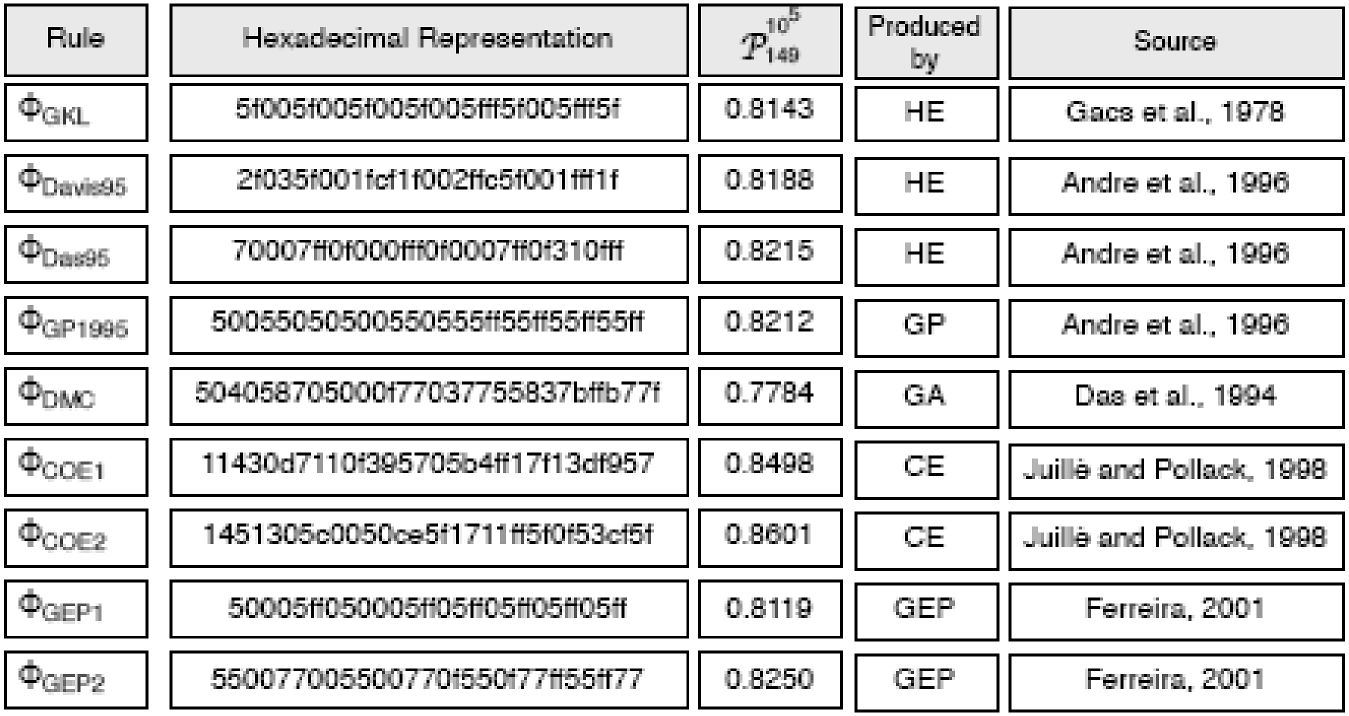


Rule	Hexadecimal Representation	$\mathcal{P}_{149}^{10^5}$	Produced by	Source
Φ_{GKL}	5f005f005f005f005fff5f005fff5f	0.8143	HE	Gacs et al., 1978
$\Phi_{Davis95}$	2f035f001fcf1f002ffc5f001fff1f	0.8188	HE	Andre et al., 1996
Φ_{Das95}	70007ff0f000fff0f0007ff0f310fff	0.8215	HE	Andre et al., 1996
Φ_{GP1995}	50055050500550555ff55ff55ff55ff	0.8212	GP	Andre et al., 1996
Φ_{DMC}	504058705000f77037755837bffb77f	0.7784	GA	Das et al., 1994
Φ_{COE1}	11430d7110f395705b4ff17f13df957	0.8498	CE	Juillè and Pollack, 1998
Φ_{COE2}	1451305c0050ce5f1711ff5f0f53cf5f	0.8601	CE	Juillè and Pollack, 1998
Φ_{GEP1}	50005ff050005ff05ff05ff05ff	0.8119	GEP	Ferreira, 2001
Φ_{GEP2}	550077005500770f550f77ff55ff77	0.8250	GEP	Ferreira, 2001

Integration and transmission of information across population

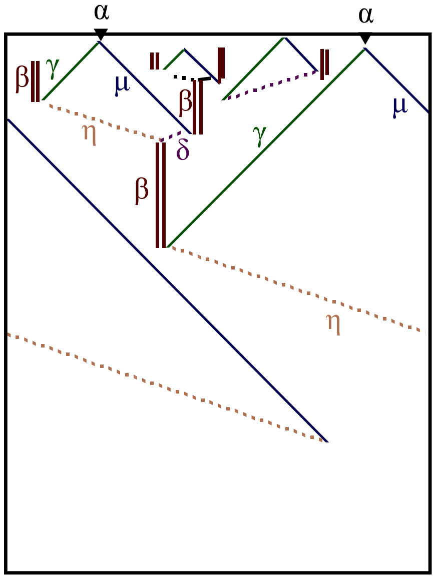
collective (emergent) computation via computational mechanics

GA to evolve rules for DCT [1994]

- Particle interaction scheme
  - Rules like a production grammar
    - the presence (collision) of two particles produces other particles
  - Transfer information accross the lattice
    - Loci of information processing
    - integrate local information globally to solve the nontrivial density task
  - Higher performance than block expansion

Crutchfield & Mitchell [1995]. *PNAS* **92**: 10742-10746

Das, Mitchell & Crutchfield [1994]. In: *Parallel Problem Solving from Nature-III*: 344-353.



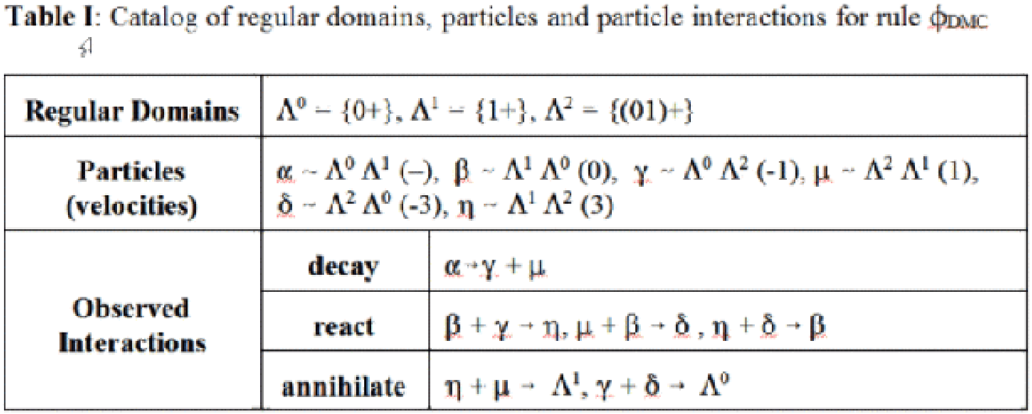


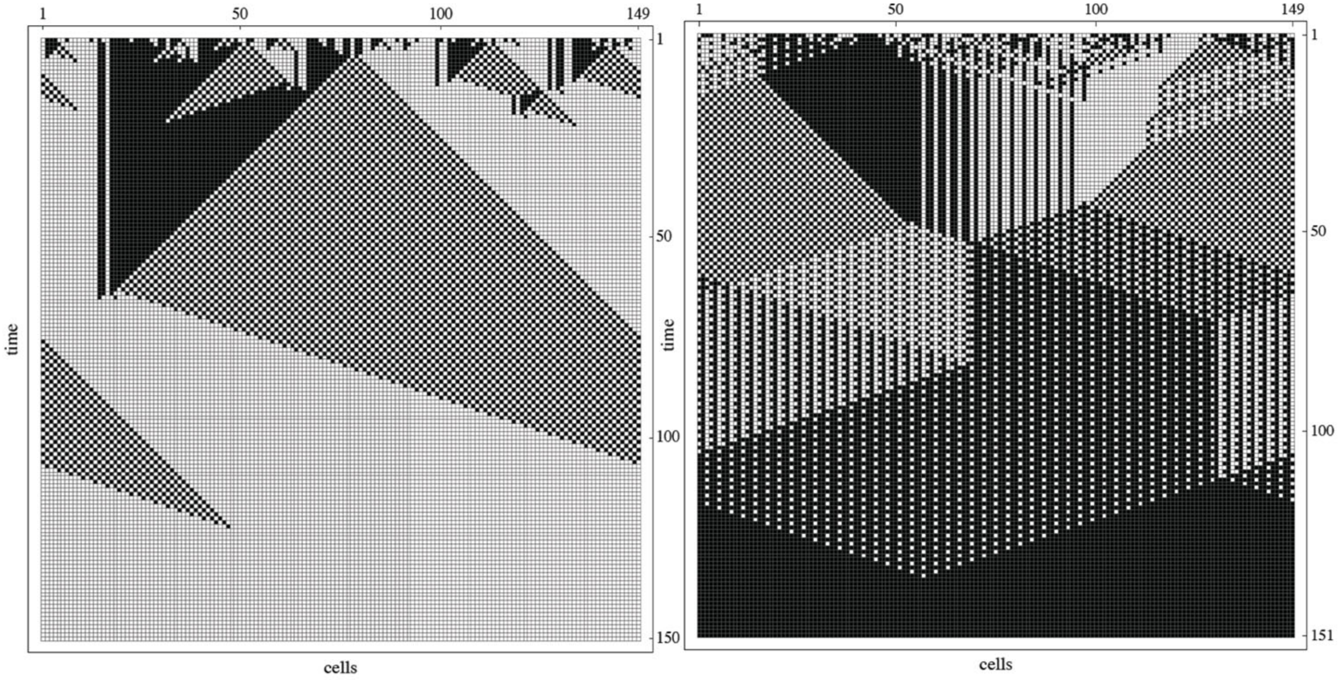
Table I: Catalog of regular domains, particles and particle interactions for rule ϕ_{DMC}

Regular Domains	$\Lambda^0 = \{0+\}, \Lambda^1 = \{1+\}, \Lambda^2 = \{(01)+\}$	
Particles (velocities)	$\alpha \sim \Lambda^0 \Lambda^1$ (–), $\beta \sim \Lambda^1 \Lambda^0$ (0), $\gamma \sim \Lambda^0 \Lambda^2$ (-1), $\mu \sim \Lambda^2 \Lambda^1$ (1), $\delta \sim \Lambda^2 \Lambda^0$ (-3), $\eta \sim \Lambda^1 \Lambda^2$ (3)	
Observed Interactions	**decay**	$\alpha \to \gamma + \mu$
	react	$\beta + \gamma \to \eta$, $\mu + \beta \to \delta$, $\eta + \delta \to \beta$
	annihilate	$\eta + \mu \to \Lambda^1$, $\gamma + \delta \to \Lambda^0$

Hanson,J.E., Crutchfield,J.P., [1992]. *Journal of Statistical Physics.* **66** (5/6), 1415-1462.
Crutchfield,J.P., Hanson,J.E., [1993]. *Physica D*. **69**, 279-301.

How do best rules solve the problem?

comparison of different automata



GKL Rule

- 3 domains
- 6 particles

$$p = \frac{d!}{(d-2)!}$$

GP Rule

- > 10 domains
- > 90 particles!!!

How do best rules solve the problem?

comparison of different automata

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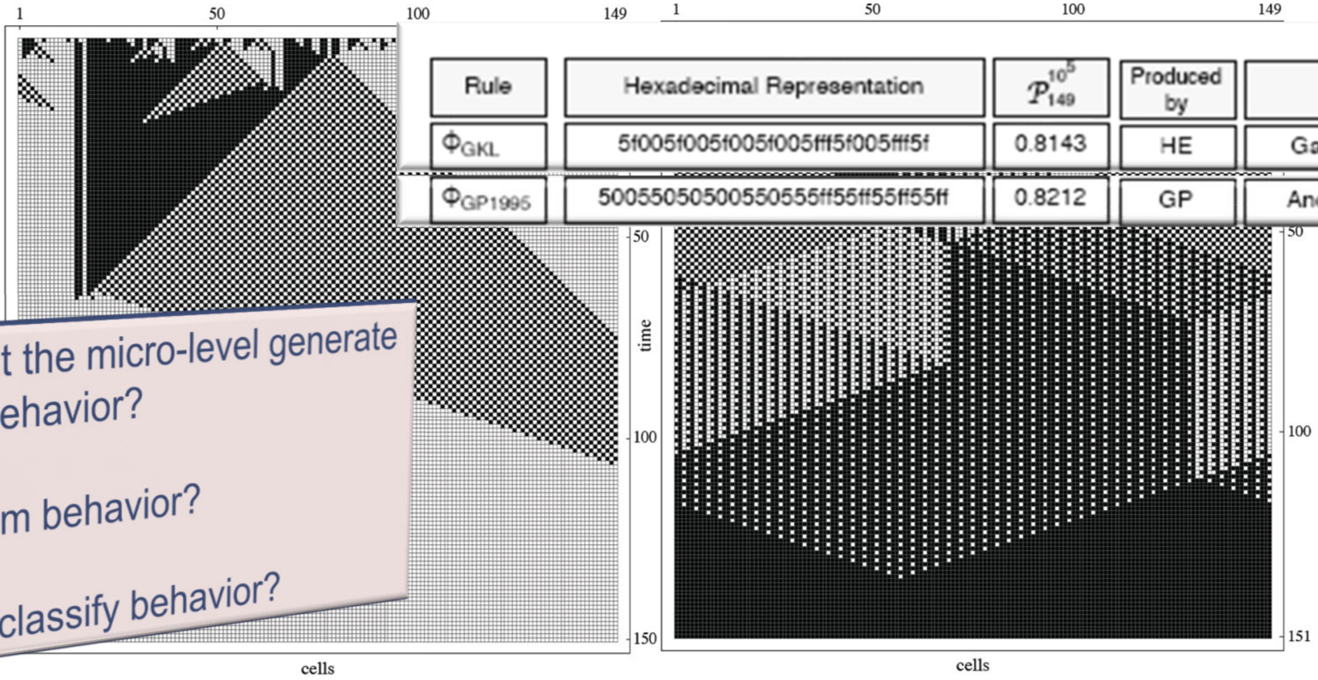


How do interactions at the micro-level generate desired macro-level behavior?

How to control/program behavior?

How to compare and classify behavior?

GKL Rule

- 3 domains
- 6 particles

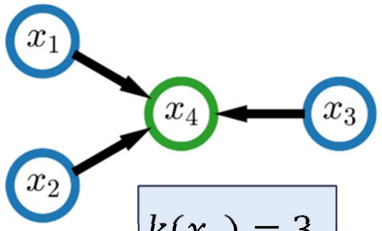
$$p = \frac{d!}{(d-2)!}$$

GP Rule

- > 10 domains
- > 90 particles!!!

redundancy in causal logic of automata (canalization)

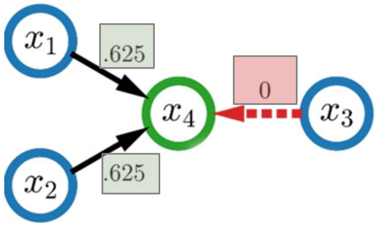
effective graph: nonlinear measure of effective connectivity

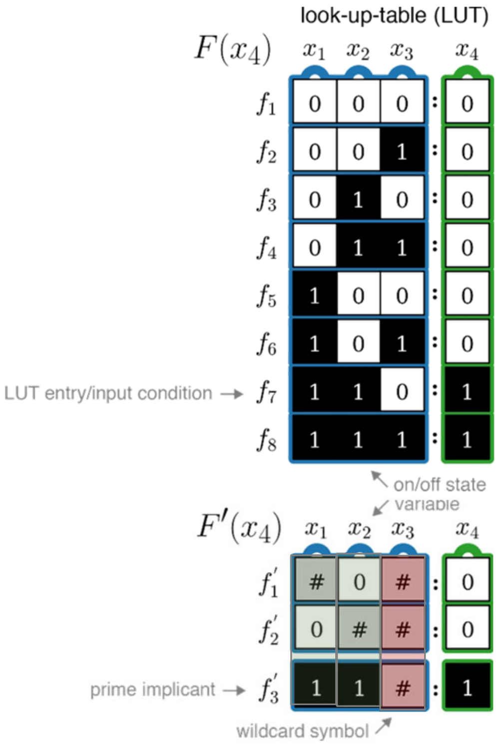


$k(x_4) = 3$

$x_4 = x_1 \wedge x_2$

$k_e(x_4) = 1.25$





$F(x_4)$	x_1	x_2	x_3		x_4
f_1	0	0	0	:	0
f_2	0	0	1	:	0
f_3	0	1	0	:	0
f_4	0	1	1	:	0
f_5	1	0	0	:	0
f_6	1	0	1	:	0
f_7	1	1	0	:	1
f_8	1	1	1	:	1

$F'(x_4)$	x_1	x_2	x_3		x_4
f'_1	#	0	#	:	0
f'_2	0	#	#	:	0
f'_3	1	1	#	:	1

Measuring dynamical ***redundancy***
and its dual ***effectiveness***

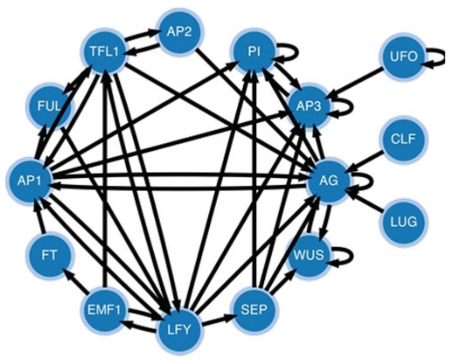
input redundancy:
$k_r(x)$ = mean number of "#" in LUT

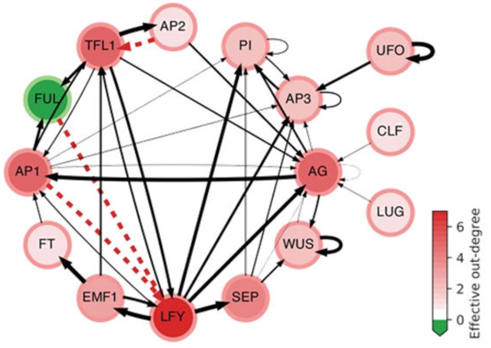
$k_r(x_4) = 1.75$

effective connectivity:
$k_e(x) = k(x) - k_r(x)$

$p(x) = 2/8 = 0.25$

p: bias, ratio of "1's" in output





Prime Implicants (Quine-McCluskey)

github.com/CASCI-lab/CANA

Correia, Gates, Wang & Rocha [2018]. *Frontiers in Physyology* **9:** 1046.
Gates, Correia, Wang & Rocha [2021]. *PNAS*. **118** (12): e2022598118.
Marques-Pita & Rocha, [2013]. *PLoS ONE*, **8**(3): e55946.

Chaos et al [2006]. *J. of Plant Growth Regulation*. **25**(4): 278-289.

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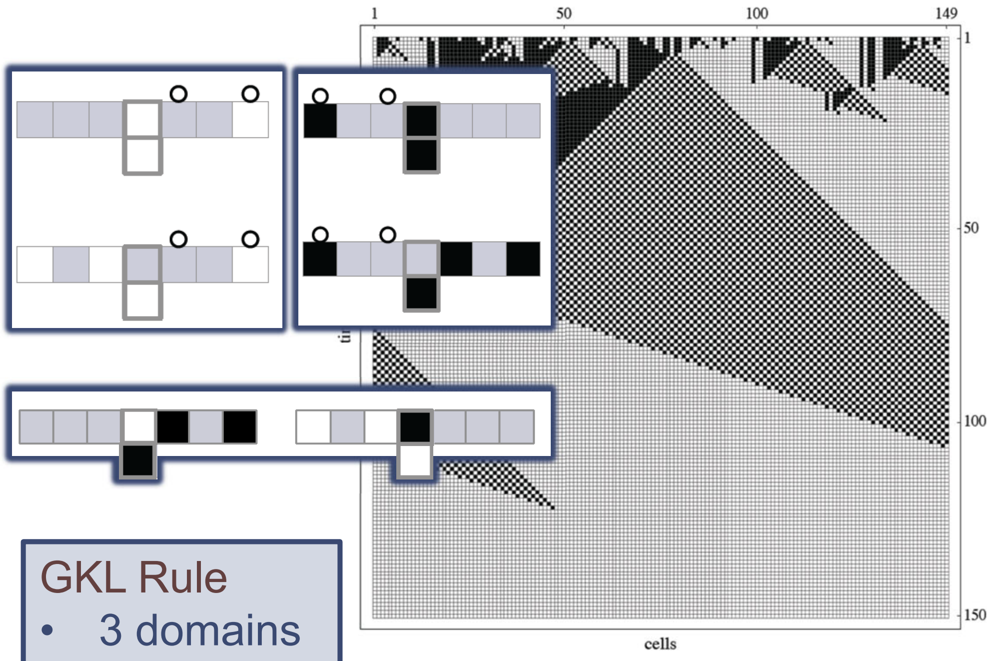
It takes redundancy to solve

Ignores most incoming information

- Solving by schemata
  - Each automaton ignores most inputs



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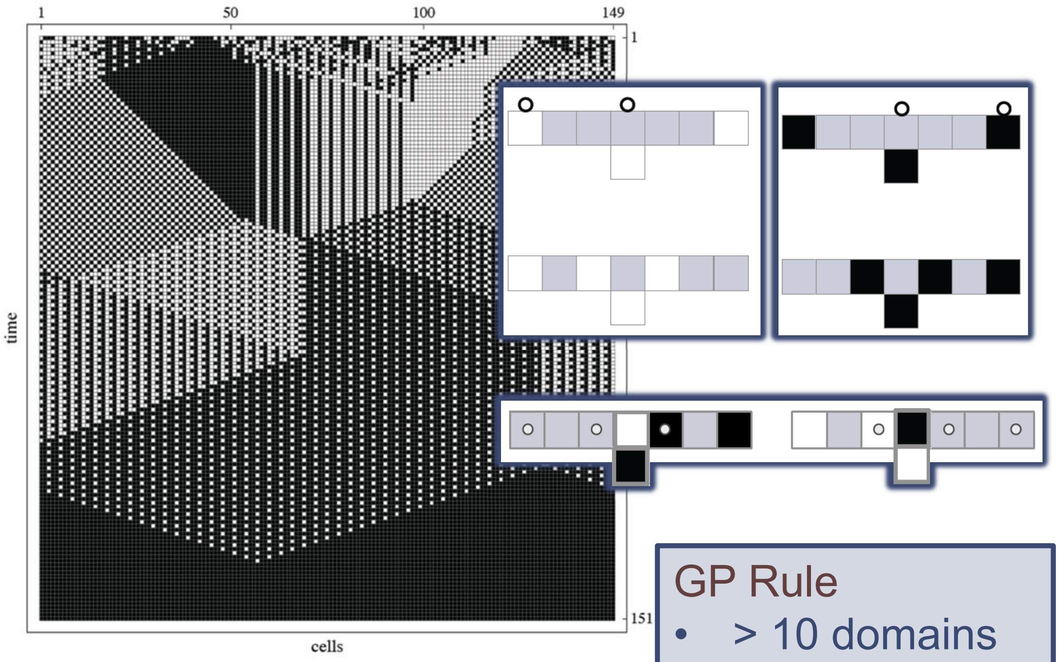


GP Rule

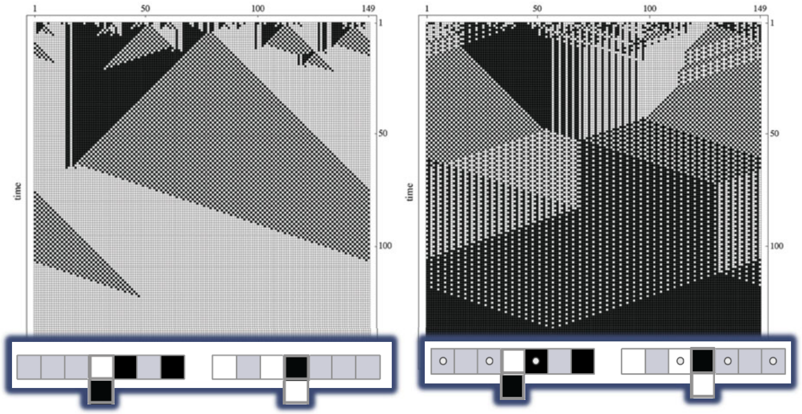
- > 10 domains
- > 90 particles!!!

Marques-Pita & Rocha, [2011]. *IEEE Alife*: 233-240.

search in redescription (canalization) space

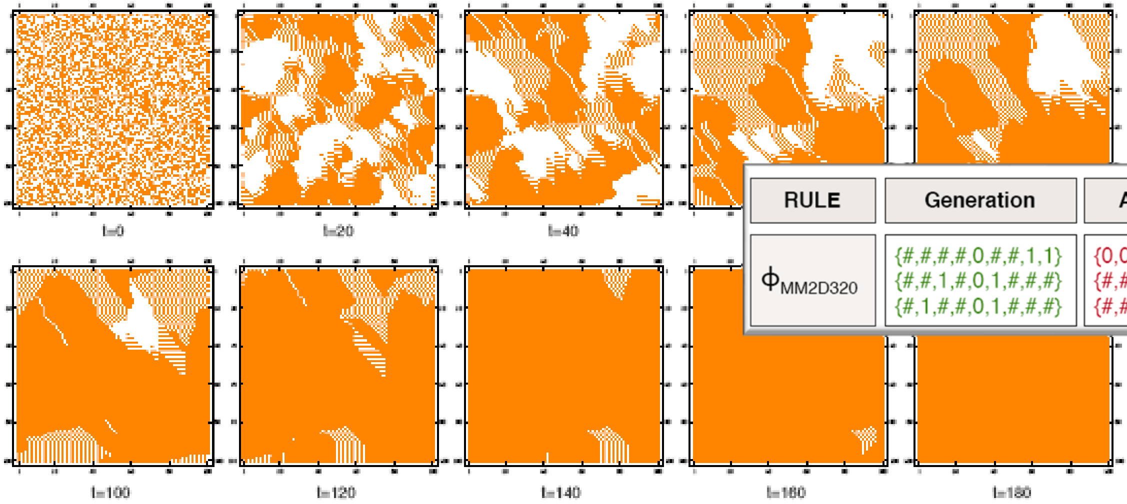
canalization (redundancy) improves evolutionary search

- Created much smoother search space
  - Allows more focused search of rules
    - Canalization, neutrality, robustness?
  - Second best rule in 1-D CA (best-known PS rule)
    - Best split-performance
  - Best rule in 2-D CA
- reason about emergent computation in new ways
  - Process-symmetry



Marques-Pita & Rocha. [2008]. *ALIFE XI.* MIT Press: 390-397.

RULE	Generation	Annihilation
ϕ_{MM0802}	{1, 0, 1, 0, #, #, #} {1, 0, #, 0, #, 1, 1} {1, 1, #, 0, 1, #, #} {1, #, 1, 0, 1, #, #} {1, #, 1, 0, #, 0, #} {1, #, #, 0, 1, 1, #} {1, #, #, 0, 1, #, 1} {#, 0, 0, 0, 0, 1, 1} {#, 1, 0, 0, 1, #, #} {#, 1, #, 0, 1, 0, #} {#, 1, #, 0, 1, #, 0} {#, #, 0, 0, 1, 0, 1}	{0, 0, 1, 1, 1, 1, #} {0, 0, #, 1, #, 1, 0} {0, 1, 0, 1, 1, #, #} {0, #, 0, 1, #, #, 0} {1, #, 0, 1, #, 0, #} {#, 0, 0, 1, #, #, 0} {#, 1, 0, 1, #, 0, #} {#, 1, #, 1, 0, #, 0} {#, #, 0, 1, 0, #, 0} {#, #, 0, 1, 1, 0, #} {#, #, 0, 1, #, 0, 0} {#, #, #, 1, 0, 1, 0}



RULE	Generation	Annihilation
$\phi_{MM2D320}$	{#,#,#,#,0,#,#,1,1} {#,#,1,#,0,1,#,#,#} {#,1,#,#,0,1,#,#,#}	{0,0,#,#,1,#,#,#,#} {#,#,#,0,1,#,0,#,#} {#,#,#,0,1,#,#,0,#}

Marques-Pita , Mitchell & Rocha. [2008]. *UC08.* LNCS. **5146**-163. 204:

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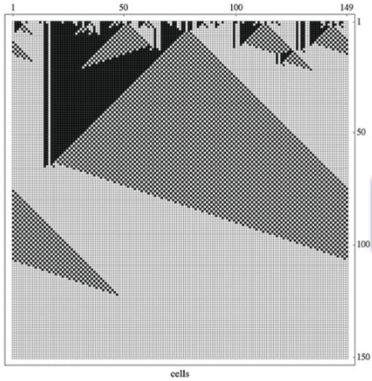
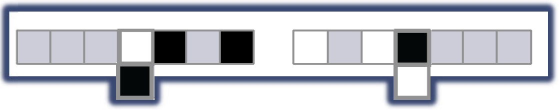
Studying emergence

linking local and global/collective behavior

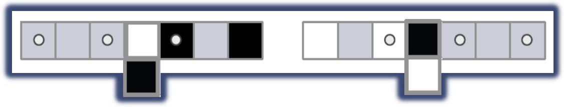
- Are emergent patterns good for explanation?
  - Do stripes or spots explain the “system”?
- Canalization (dynamical redundancy) is a powerful idea
  - Capture loci of control and building blocks of information transmission

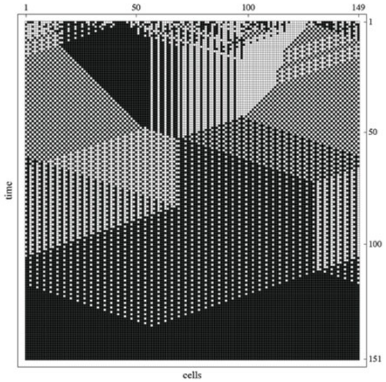


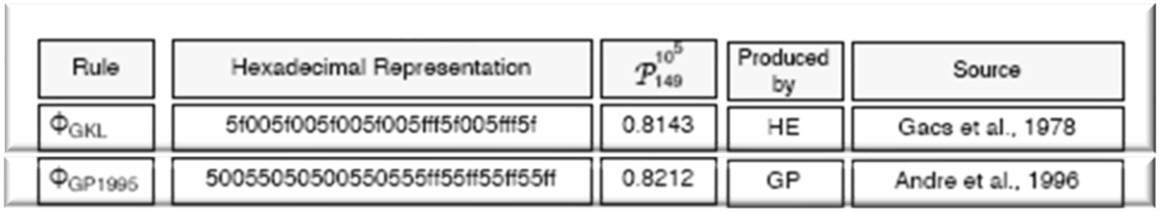
GKL Rule
- 3 domains
- 6 particles



GP Rule
- > 10 domains
- > 90 particles!!!







Rule	Hexadecimal Representation	$\mathcal{P}_{149}^{10^5}$	Produced by	Source
Φ_{GKL}	5f005f005f005f005fff5f005fff5f	0.8143	HE	Gacs et al., 1978
Φ_{GP1995}	5005505050055055ff55ff55ff55ff	0.8212	GP	Andre et al., 1996

Marques-Pita & Rocha, [2011]. *IEEE Alife.*

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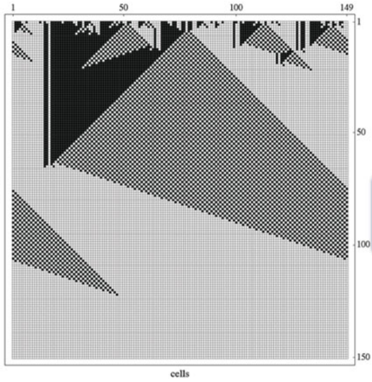
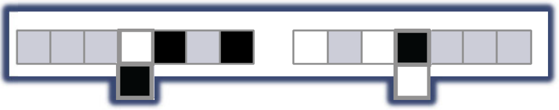
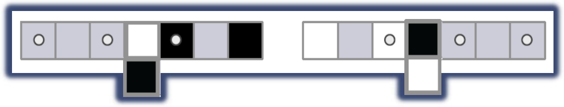


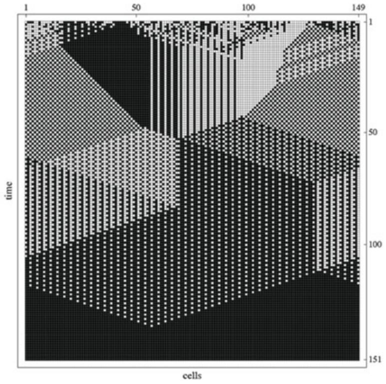
GKL Rule
- 3 do
- 6 pa

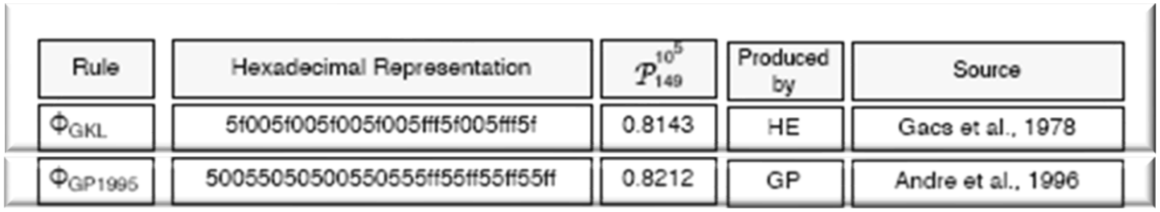
GP Rule
- > 10 domains
- es!!!

(mechanistic) reductionism vs emergence:
what is the best explanation?









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the game of life

John Horton Conway

$$x_{i,j} = \{0,1\}$$

2-D

Sum N^8	0	1	2	3	4	5	6	7	8
$x_{i,i}= 0$	0	0	0	1	0	0	0	0	0
$x_{i,i}= 1$	0	0	1	1	0	0	0	0	0

1) Any living cell with fewer than two neighbors dies of loneliness.
2) Any living cell with more than three neighbors dies of crowding.
3) Any dead cell with exactly three neighbors comes to life.
4) Any living cell with two or three neighbors lives, unchanged, to the next generation

Introduced in Martin Gardner's *Scientific American* "Mathematical Games" Column in 1970.

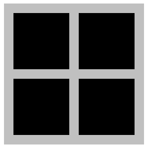
Conway was interested in a rule that for certain initial conditions would produce patterns that grow without limit, and some others that fade or get stable.

Popularized CAs.

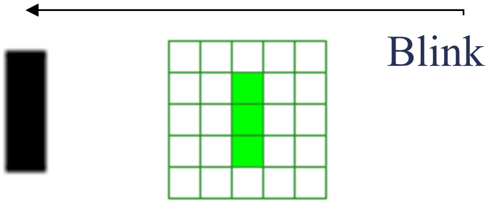
game of life

wide dynamic range

Simple Attractors



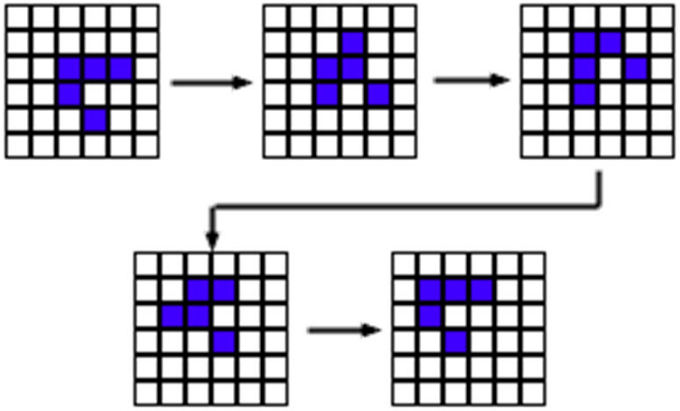
block

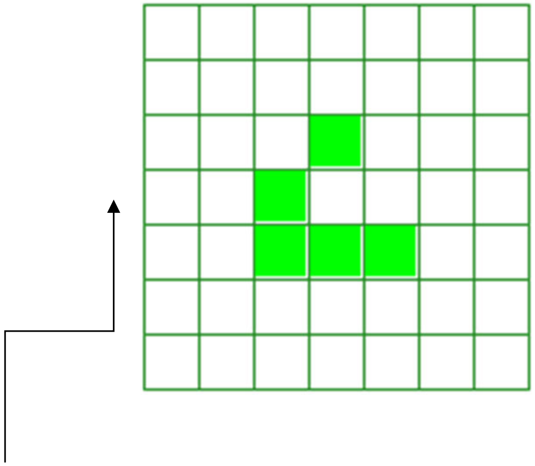


Blinkers

More complicated attractors

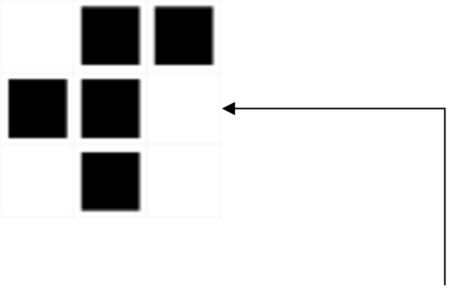
moving patterns



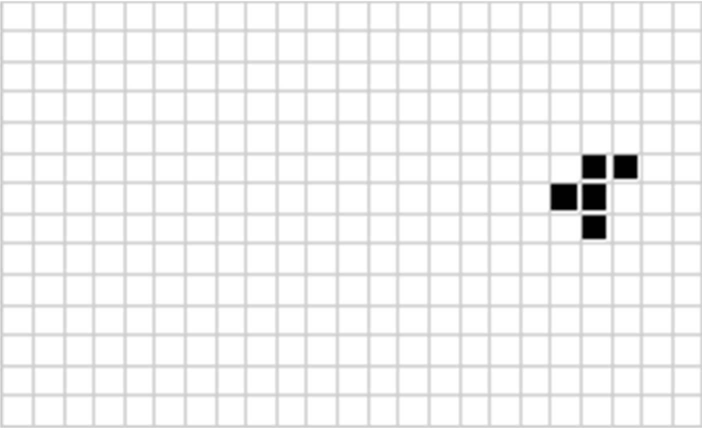


unbounded growth

a threshold of complexity?



R-pentomino

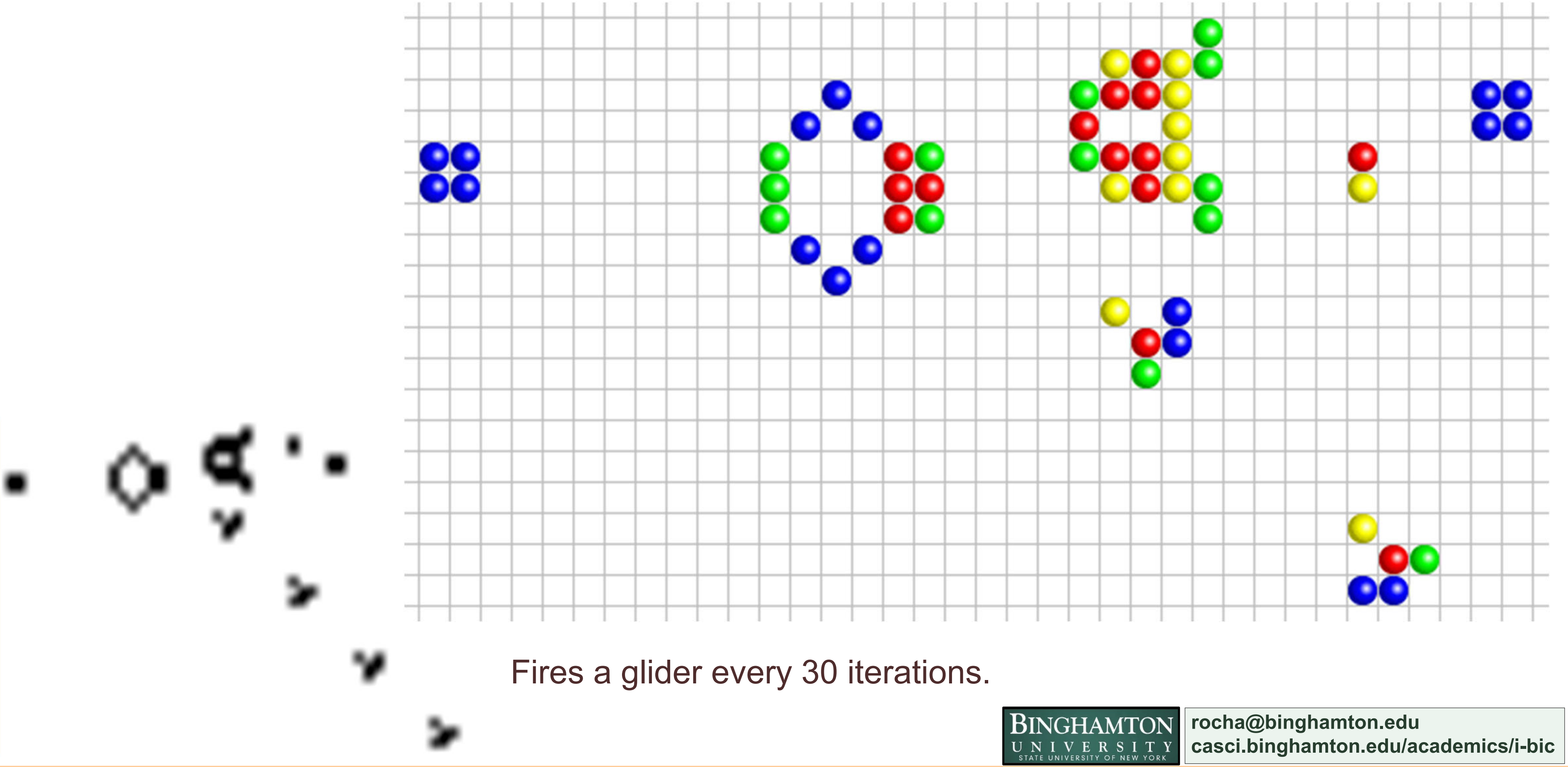


runs 1103 steps before settling down into 6 gliders, 8 blocks, 4 blinkers, 4 beehives, 1 boat, 1 ship, and 1 loaf.

something that lasts forever?

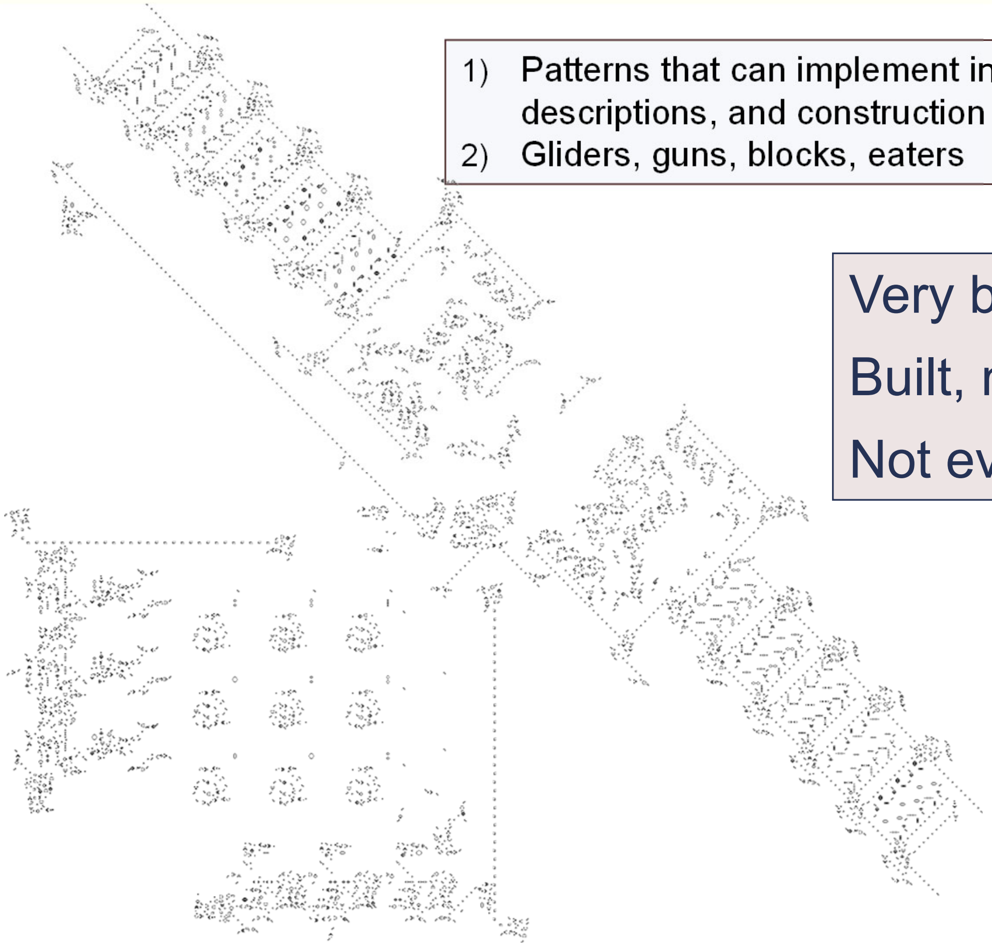
the glider gun

Unbounded growth but not complexity



Fires a glider every 30 iterations.

unbounded complexity requires information

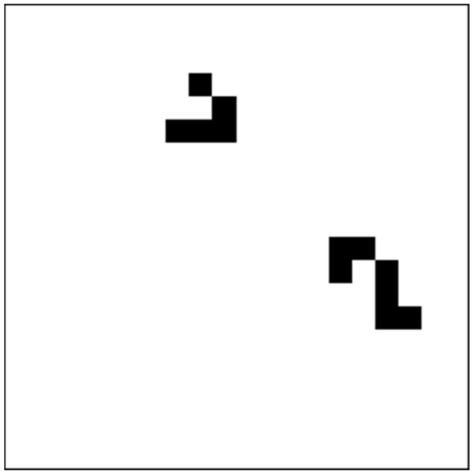


1) Patterns that can implement information, descriptions, and construction
2) Gliders, guns, blocks, eaters

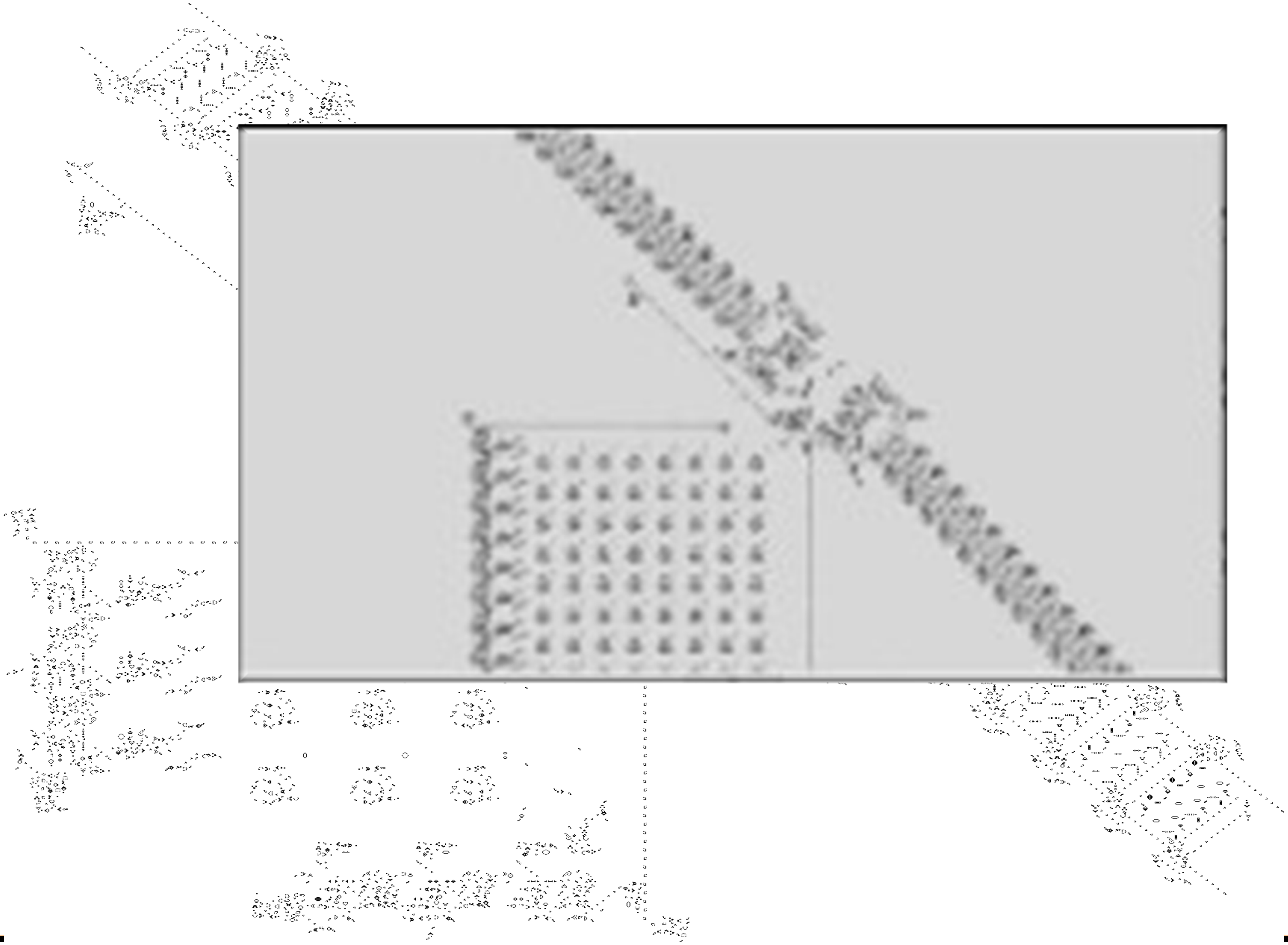
Very brittle

Built, not evolved

Not evolving



Universal Turing Machine on game of life!!!

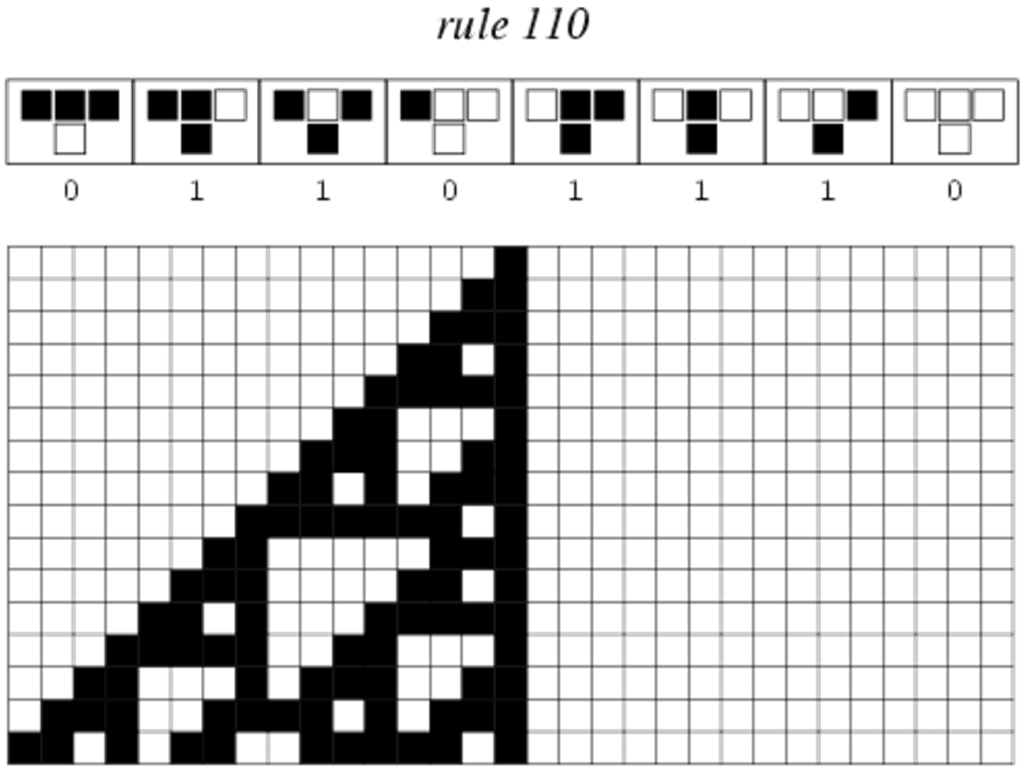


Rule 110

information in attractor patterns

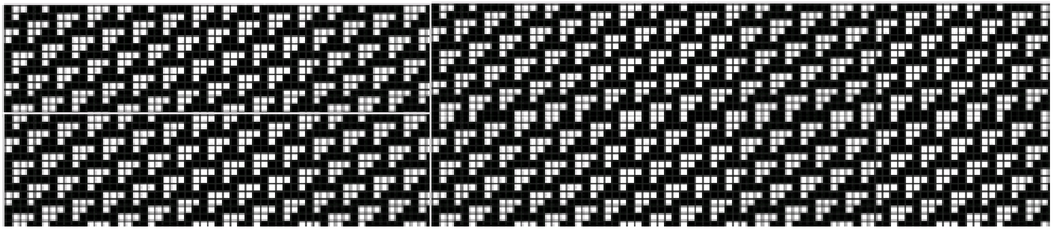
- Radius 1
  - Neighborhood =3
- Binary
  - $2^3 = 8$ input neighborhoods
  - $2^8 = 256$ rules

Universal Computation

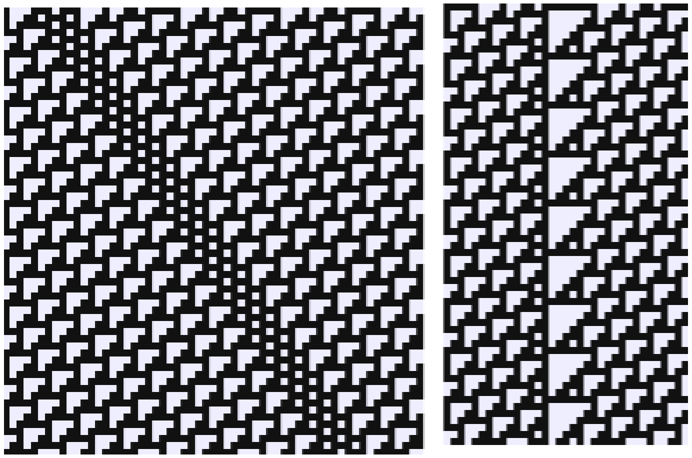


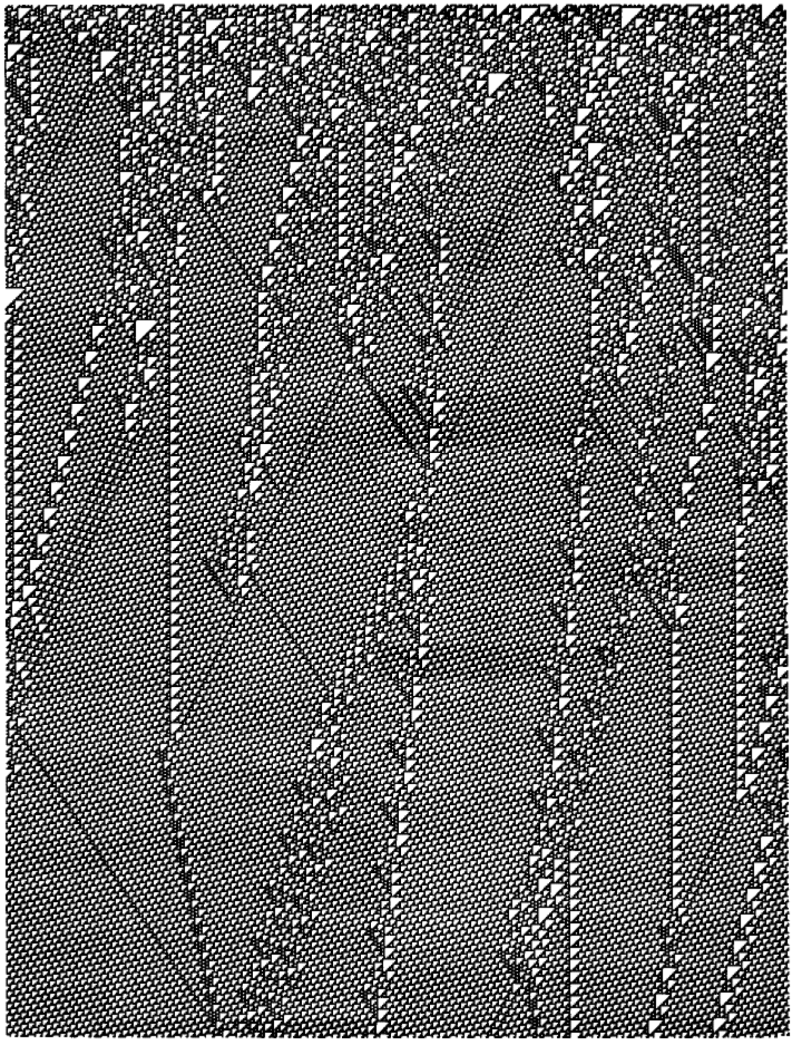
http://mathworld.wolfram.com/Rule110.html

computing structures in rule 110



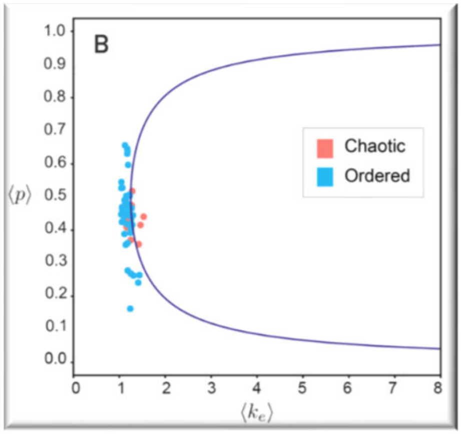
- Universal Computation
  - Identification of gliders, spaceships, and other long-range or self-perpetuating patterns
    - On the background domain produced by rule 110
      - 14 cells repeat every seven iterations: **00010011011111**
  - Collisions and combinations of glider patterns are exploited for computation.

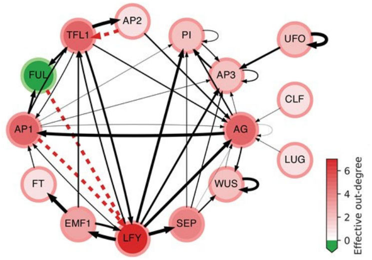


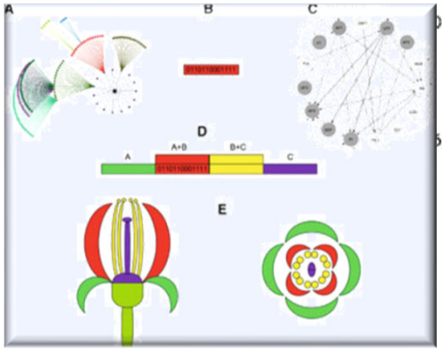


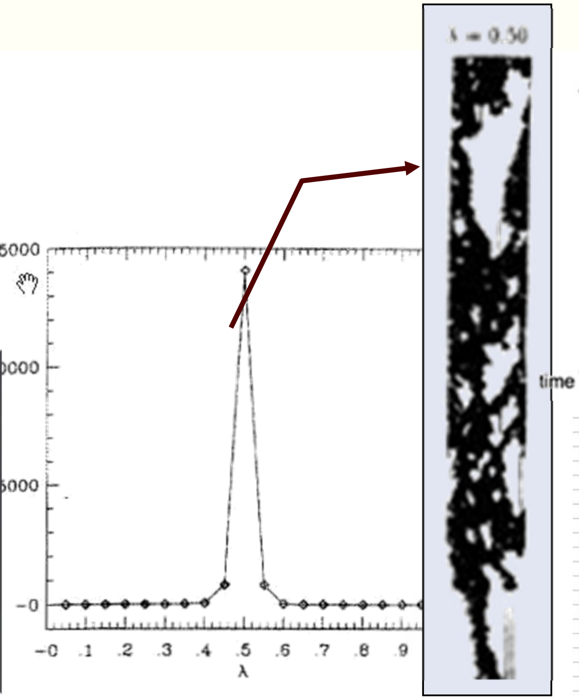
computation and the edge of chaos

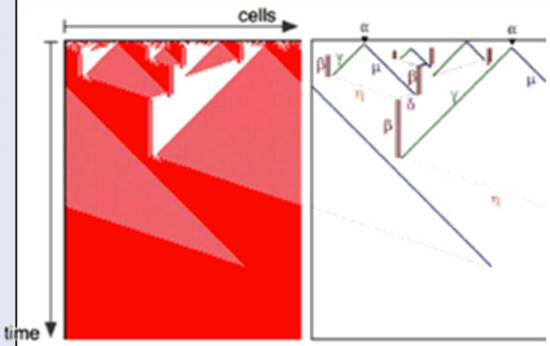
is self-organization enough?

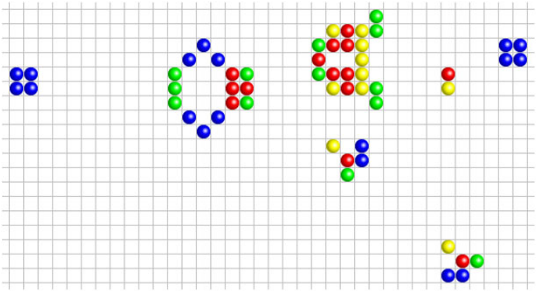












- systems biology models operate in near critical regime, though many are ordered
- Dynamical systems capable of computation exist before the edge of chaos
  - A wider transition due to redundancy?
- Most important information transmission and computation in Biology an altogether different process than self-organization
  - Turing/Von Neumann memory

readings

- Class Book
  - Floreano, D. and C. Mattiussi [2008]. *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies*. MIT Press.
    - **Chapter 2**.
- Lecture notes
  - Chapter 1: What is Life?
  - Chapter 2: The logical Mechanisms of Life
  - Chapter 3: Formalizing and Modeling the World
  - Chapter 4: Self-Organization and Emergent Complex Behavior
    - posted online @ http://informatics.indiana.edu/rocha/i-bic
- Papers and other materials
  - Optional
    - Nunes de Castro, Leandro [2006]. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Applications*. Chapman & Hall.
      - Chapter 2, all sections
      - Chapter 7, sections 7.3 – Cellular Automata
      - Chapter 8, sections 8.1, 8.2, 8.3.10
    - Flake's [1998], *The Computational Beauty of Life*. MIT Press.
      - Chapters 10, 11, 14 – Dynamics, Attractors and chaos