4D Backface Culling

To carry out 4D backface culling, first assume we are given three ordered, non-degenerate vectors within each polyhedron, typically three edges computed from known vertex positions: $\vec{a} = \vec{x}_1 - \vec{x}_0$, $\vec{b} = \vec{x}_2 - \vec{x}_0$, and $\vec{c} = \vec{x}_3 - \vec{x}_0$. Then the 4-vector direction normal to the polyhedral face is the 4D cross-product

$$\vec{n} = \det \begin{vmatrix} a_x & b_x & c_x & \hat{x} \\ a_y & b_y & c_y & \hat{y} \\ a_z & b_z & c_z & \hat{z} \\ a_w & b_w & c_w & \hat{w} \end{vmatrix} \quad = \quad n_x\hat{x} + n_y\hat{y} + n_z\hat{z} + n_w\hat{w} \; ,$$

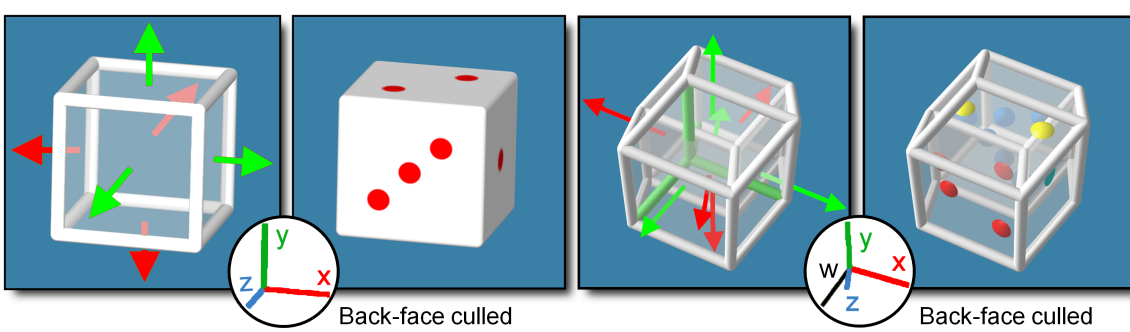
normalized as usual to $\hat{n} = \vec{n}/|\vec{n}|$. Note that our convention requires that the cross-product of the x, y, z unit vectors produces a vector in the positive w direction. The final step is to pick a 4D eye-point $\vec{P}$, and to remove from the rendering all polyhedral faces for which

$$(\vec{P} - \vec{x}_0) \cdot \hat{n} \leq 0 \; ,$$

where $\vec{x}_0$ is a representative point lying in the polyhedral face (see the Figure). We remark that the orientation of a 4D, possibly nonconvex, polyhedron projected into the 3D screen can be checked in the *3D screen*, without needing the full calculation given above, using an exact analogy to the standard 2D-projection check: $\sum_{i=0,n-1} (u_i v_{i+1} - u_{i+1} v_i) > 0$ where u and v are 2D coordinates of the projected polygon. We simply add the signed 3D projected-tetrahedron volumes, equivalent to computing only the w-component of the cross-product, yielding this test,

$$\sum_{i=0,n-1} \det \left| \; \vec{u}_i \quad \vec{u}_{i+1} \quad \vec{u}_{i+2} \; \right| > 0 \; ,$$

where the $\vec{u}_i$ are the projected vertex coordinates of the polyhedron in the 3D image. The result for a typical example such as the hypercube is that half *or more* of the polyhedra that would appear in a wire-frame rendering *disappear* in any given culled rendering, reappearing in an appropriate sequence when 4D rotations are applied to the object. That is, just as only 1, 2, or 3 square faces of a 3D die can be seen simultaneously, but never more, only 1, 2, 3, or *4* cubic faces of a 4D die can be seen simultaneously, but *never more*.



Green arrows are forward-facing normals. Left: 3D Back-face culling. Right: 4D Back-face culling.