CSCI 241:

Midterm

Please be concise, but make sure you have all the steps. Show your work. Each question is 17 points. Good luck!

1. $f(x) = x^2 g(x)$. Is it possible to come up with a function $g(x)$ so that $f(x) = O(g(x))$? Both f and g should always be returning positive (ie, strictly greater than 0) values. Prove your answer.

2. (a) Give a logic formula for: "the jury won't acquit the accused unless either new evidence shows up or a jury member is not honest." Don't use quantifiers.

   (b) Consider the following: $\forall x(\exists y P(x,y) \vee \exists z Q(x,z))$ and $\forall x \exists y P(x,y) \vee \forall x \exists z Q(x,z)$. Are these necessarily the same? Prove your answer.

   (c) Repeat the above for $\exists x(\forall y P(x,y) \vee \forall z P(x,z))$ and $\exists x \forall y P(x,y) \vee \exists x \forall z P(x,z)$. Prove your answer.

3. Prove using induction that

$$1/2 + 1/4 + 1/8 + \cdots 1/2^n = 1 - 1/2^n$$

4. Given three nonempty sets A, B, and C, is the following always true?

$$A - (B - C) = (A - B) - C$$

   Prove.

5. Prove by contradiction that the product of three consecutive numbers is divisible by 3.

6. You are given that $f(x) = O(g(x))$ and $g(x) = O(h(x))$. Argue formally (with constants c, k, etc) that $3f(x) + 5g(x) + 7h(x) = O(2h(x))$.