CSCI 241H:
HOMEWORK 5

Show your work.

1. If I tell you that the running time of my program is at least $O(f(x))$, why is this statement meaningful?

   This statement is meaningful because it has a truth value: it's always true. However, you might have observed that it has no information value since it says nothing about the actual complexity of the problem, as any function is at least $O(f(x))$. (why?) You will get full points for noticing either case.

2. Let $f(x)$ be $O(g(x))$ and $g(x)$ be $O(h(x))$. Show that $k(x) = 3f(x) + 5g(x) + 8h(x)$ is $O(h(x))$.

   So there are k_1, k_2, c_1, c_2 such that for $x > k_1$ $f(x) \leq c_1 g(x)$ and for $x > k_2$ $g(x) \leq c_2 h(x)$. Let $k = max(k_1, k_2)$. Then, for $x > k$ $f(x) \leq c_1 c_2 h(x)$, which implies $k(x) \leq (3c_1c_2 + 5c_2 + 8)h(x)$. That's all we need, we can set $c = 3c_1c_2 + 5c_2 + 8$ and use k as our witnesses.

3. Consider the series $f(k) = \Sigma_{i=1}^{k} 2k - 1$. Show that $f(k) = k^2$. Don't look at any resources please.

   Let's say $A = 1 + 3 + \ldots 2k - 1$. This has k terms, if we add 1 to each term we get $A+k = 2+4+6+\ldots 2k = 2(1+2+\ldots k) = k(k+1) = k^2+k$. Thus $A = k^2$.

4. Let $0 < x < 1$. Show that the infinite summation $x + x^3 + x^5 + x^7 + \ldots$ evaluates to $x/(1 - x^2)$.

   Let $A = x+x^3+x^5+x^7+\ldots$ Then $Ax^2 = x^3+x^5+x^7+\ldots$ Subtracting the second from the first, we get $A(1 - x^2) = x$, so $A = x/(1 - x^2)$.