Meaningful Numbers and “Significant Figures”

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What Significant Figures Are

The discussion below is an excerpt from the Wikipedia article, “Significant figures”, as of the original version of this article (see http://en.wikipedia.org/wiki/Significant_figures). The article says considerably more, but the excerpt is sufficient for most purposes, and I don’t think what it says is at all controversial.

The concept of significant figures is often misunderstood, and one reason is surely that in this context the word *significant* has a specific and rather unusual meaning. No reasonable person would claim that the leading zeros in 0.00052, or the trailing zeros in 52,000,000, are unimportant and could be omitted!

From Wikipedia, the free encyclopedia

The **significant figures** (also called **significant digits**) of a number are those digits that carry meaning contributing to its precision. This includes all digits *except*:

- leading and trailing zeros where they serve merely as placeholders to indicate the scale of the number.
- spurious digits introduced, for example, by calculations carried out to greater accuracy than that of the original data, or measurements reported to a greater precision than the equipment supports.

The concept of significant digits is often used in connection with rounding. Rounding to *n* significant digits is a more general-purpose technique than rounding to *n* decimal places, since it handles numbers of different scales in a uniform way. For example, the population of a city might only be known to the nearest thousand and be stated as 52,000, while the population of a country might only be known to the nearest million and be stated as 52,000,000. The former might be in error by hundreds, and the latter might be in error by hundreds of thousands, but both have two significant digits (5 and 2). This reflects the fact that the significance of the error (its likely size relative to the size of the quantity being measured) is the same in both cases.

The rules for identifying significant digits when writing or interpreting numbers are as follows:

- All non-zero digits are considered significant. For example, 91 has two significant digits (9 and 1), while 123.45 has five significant digits (1, 2, 3, 4 and 5).
- Zeros appearing anywhere between two non-zero digits are significant. Example: 101.12 has five significant digits: 1, 0, 1, 1 and 2.
- Leading zeros are not significant. For example, 0.00052 has two significant digits: 5 and 2.
- Trailing zeros in a number containing a decimal point are significant. For example, 12.2300 has six significant digits: 1, 2, 2, 3, 0 and 0. The number 0.000122300 still has only six significant digits (the zeros before the 1 are not significant). In addition, 120.00 has five significant digits. This convention clarifies the accuracy of such numbers; for

example, if a result accurate to four decimal places is given as 12.23 then it might be understood that only two decimal places of accuracy are available. Stating the result as 12.2300 makes clear that it is accurate to four decimal places.

- The significance of trailing zeros in a number not containing a decimal point can be ambiguous. For example, it may not always be clear if a number like 1300 is accurate to the nearest unit (and just happens coincidentally to be an exact multiple of a hundred) or if it is only shown to the nearest hundred due to rounding or uncertainty. Various conventions exist to address this issue… However, these conventions are not universally used, and it is often necessary to determine from context whether such trailing zeros are intended to be significant.

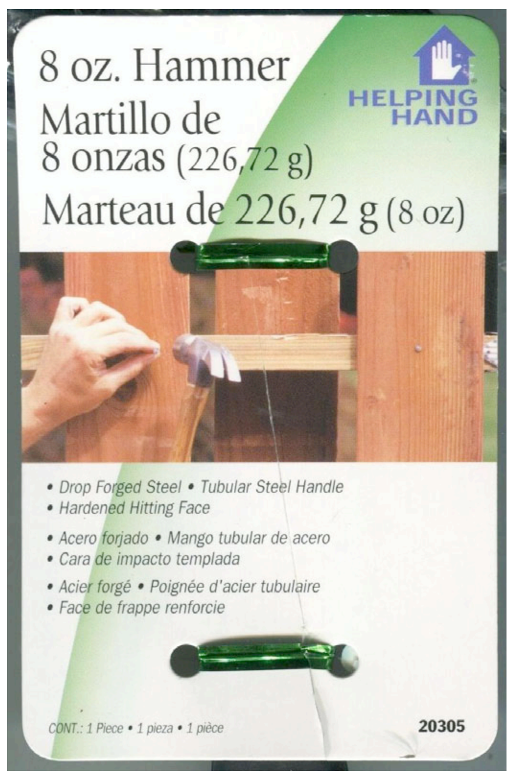
[end of the Wikipedia excerpt]

Significant Figures and Plus/Minus Limits

Since both the number of significant figures and plus/minus error limits express the accuracy of a number, there's a direct relationship between the two. For example, a value might be described as 131,500 ± 0.01%, i.e., the value 131,500 is within 13.15 (0.01% of 131,500) of the correct value. Therefore the actual value is at least 131,486.85 but no more than 131,513.15. This means that only the first three digits of 131,500 are known to be correct; in other words, it has three significant figures.

How Not to Handle Significant Figures

The label on the original package of a "Helping Hand®" hammer I have is trilingual, as shown in the images below.



The label gives the weight in English, Spanish, and French, in that order:

> "8 oz. Hammer
>
> Martillo de 8 onzas (226,72g)
>
> Marteau de 226,72 g (8 oz)"

Note that the French and Spanish texts describe its weight as both 8 oz. and 226.72 grams. Is the weight really accurate to *five* significant figures, i.e., to the nearest 100^{th} of a gram? I seriously doubt it, especially since the weight in ounces is given to only one significant figure! (Also, 8.0 oz. is actually about 226.796 grams, not 226.72; of course, the difference would be negligible in almost any real-world situation.)

The explanation is undoubtedly that the manufacturer took the weight in ounces and converted it to grams using the official conversion factor, but without a thought to the real accuracy of the weight. Similarly, it's not hard to find instructions that say things like "Take 5.08 or 7.62 cm of string and…" This is obviously a translation from English-system units; re-translated back, that's "Take 2.00 or 3.00 inches of string …" The unintended implication is that you should measure the length of string very carefully indeed, since, say, 5.13 or 7.65 cm (2.02 or 3.01 in.) might not work! (A Google search in January 2012 for the exact phrase "5.08 or 7.62" found 1,780 results. In six of the first ten, the statement is something like "It works with tape width of 2 or 3 inches (5.08 or 7.62 cm)", so the effect is not too misleading. But in the other four, there was no indication of the origin of the funny numbers.)