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"Math is Cool, Fun, Wild, etc." Teaching Ideas

Donald Byrd
Woodrow Wilson Indiana Teaching Fellow
and
School of Informatics, Indiana University Bloomington

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These ideas are all over the map, so to speak, both in terms of scale (5 minutes to term project) and topic. What's the point? Above all, to promote *intrinsic motivation* by getting students to see that math can be cool and fun, "wild and crazy"—and mind-expanding. I think most or all of these ideas could be presented in ways understandable to average high-school students, and many could even be understood by middle-school students. But I'm sure a lot of them are way, way off the mark; comments are very welcome! CAVEAT:

(1) This list is in a permanent "draft" state. It's not finished, and I doubt if it ever will be.

(2) A lot of the ideas are described in shorthand ways likely to be difficult for anyone but me to understand; please ask for clarification.

Key:

** : good for a quick change of pace during a lesson.

Boldface: has very broad applicability.

wild?: Y = yes, wild; Y! = very wild (and maybe crazy).

SD: potential for student discovery

Min grd: approximate minimum grade level the idea seems appropriate for (for average students who've followed a typical U.S. math curriculum).

wild ?	*SD*	*Area*	*Description*	*Min grd*	*Likely Use*
		Interesting constants; irrational numbers	Celebrate special days, e.g., Pi Day (3/14)	6	Engagement
		"	Pi (and e, etc.?) Memorizing Challenge: students vs. teacher	7	Engagement
		"	Pi memorizing: Greg Mongold's "American Pie" version(?), or Larry Lesser's; standard mnemonics in English, French, etc.; Keith's "Poe, E. Near a Raven"; Carol Meyers' Ode to Pi	7	Engagement
		"	Computing pi via power series: the mysterious connection between circles and various constants (e.g., in Machin's formula, 1/5 and 1/239; in one of Ramanujan's formulae, 9801, 1103, 26390); the Madhava-Gregory, Machin-like, and more recent formulations	9	
Y		"	Pi legislation: construction implying various values of pi (&	9	

			squaring the circle) in the "Indiana Pi Bill"		
		"	History of Pi: biblical value (3 vs. 3-1/7); ancient methods (rope & circle in the sand, circumscribed & inscribed polygons, etc.); development of calculus, power series, calculation via savants & mechanical calculators, digital computers. Cf. file HistoryOfComputingPi.pdf	9	
	B	General	**History of any mathematical topic: who developed it, when, and especially *why* (their motivation)**	6	
Y	B	Proofs: fallacious	Ask students, what *exactly* is wrong with this "proof"? (a) "Prove" by contradiction there's no smallest uninteresting number; (b) **the animated "proof" that 64 = 65 (on the Web); (c) **algebraic "proofs" (usually via dividing by zero) that 1=2, etc.; (d) the "proof" that all triangles are isosceles (in Greenberg, pp. 25-26, & on the Web); etc. Cf., e.g., the Wikipedia article "Mathematical fallacy".	9	
		Proofs; philosophy of mathematics	Discuss and give examples of proof methods; constructive vs. nonconstructive proofs; bring in completeness vs. consistency ala Godel	11	
		Proofs; philosophy of mathematics	"Lexicon of Mathematical Invective". Quote ?? on imaginary numbers, Saccheri on non-Euclidean (hyperbolic? elliptic?) geometry being "repugnant to the nature of the straight line"(?), etc. Cf. comment in GEB	9	
	B	Proofs	Logic puzzles ala Dr. M., Smullyan, etc.; cf. Khan Academy videos.	9	
		Proofs	Incredibly Simple, Elegant Proofs of Easy-To-Understand But Deep Things in Mathematics: file IncredibleButGenuineProofs.txt	9	
		Proofs; philosophy of mathematics	Hofstadter's demonstration of the uniqueness of math via his analogy (in *I Am a Strange Loop*, p. 126-27) between mathematicians' reaction to finding exactly three perfect nth powers in the Fibonacci sequence, and what would likely happen if exactly three precious stones were found in a truly exhaustive search (assuming that was possible!) of a huge lake.	10	
Y			Simple but surprising (more-or-less, pathological) functions and sets: "ruler" function (continuous at all irrationals, discontinuous at all rationals); Weierstrass' functions (continuous everywhere but differentiable nowhere); Dirichlet's function (continuous nowhere ??WHY IS THIS INTERESTING? IS IT REALLY DIRICHLET?); the Cantor set (uncountable but with measure = 0), etc.	12	
Y!		Geometry vs. analysis; infinity	Dimensionality; fractals & their precursors: what's the length of a coastline? Lebesgue's non-differentiable surface, etc.; Space-filling curves (e.g., Peano's), curves that self-intersect at every point, curves of infinite length enclosing a finite area (Von Koch's snowflake); the Cantor set (uncountable but with measure = 0); Gabriel's Horn, a geometric figure with infinite surface area but finite volume; etc. Cf. Stewart, Chap. 16.	10	
	A	Geometry & the arts	Geometric transformations in visual patterns (esp. ambigrams), music, dance, etc. Cf. NCTM 2011 talks: Scott	8	

			Kim's "Experiencing Symmetry: Geometric Transforms in Art, Music, and Dance", & Mark Jaffee's "Teach Math thru Music"; also the amazing Big Bang Rubette for Rubato Composer.		
Y	B	General; graphing & visualization	**My "Infinite Bottles of Beer" as a framework for discussing various ideas.** Potential for discovery by graphing, etc.	9	
Y		Sequences; extrapolating from limited data	Doug H.'s example of extrapolating the next term after 0, 1, 2: 720! (if the nth term is n-1 followed by n-1 factorial signs :-)) vs. the obvious 3 (if the nth term is n-1 :-)	10	
Y!		Proofs; infinite sets	The "dense" concept; prove the rationals are dense in the reals, and the irrationals in the rationals(?)	11	
Y!!		Infinite sets	Prove existence of (ala Cantor) and discuss different sizes of infinities. Use stuff from Infinite Bottles of Beer (file InfiniteBottlesOfBeer.doc) and Hilbert's hotel story (in Gamow's One, Two, Three... Infinity)	10	
Y!!		Infinity	The length of a diagonal line across the unit square is sqrt(2), but the Manhattan distance (length of a stairstep approximation to it with any finite number of steps) between (0, 0) and (1, 1) is 1. Thus, if len(n) is the length of the line, then for all finite n, len(n) = √2, and lim as n goes to infinity of len(n) = √2, but…	9	
Y!		Infinity, infinite series	Describe Zeno's paradox of Achilles & the tortoise; demonstrate and (if time/sophistication of audience allow) prove that 2 is an upper bound on sum from n=0 to infinity of (1/2)^n, or that 1.111111…. is on sum of (1/10)^n, and point out how the possibility of an infinite series converging resolves the paradox: file Zeno+InfiniteSeries.txt; also cf. Vi Hart's Infinite Elephants	8	
Y	A	Infinity	Functions whose graphs have solid black areas. In *any* neighborhood of $x = 0$, the real-valued function $\sin(1/x)$ takes on every value between –1 and +1 infinitely many times; $v \sin(1/x)$ takes on every value between $-v$ and $+v$ infinitely many times; and $1/x \sin(1/x)$ takes on *every* value infinitely many times! (Meromorphic functions do that for every *complex* value in any neighborhood of an essential discontinuity, but they're considerably more advanced; I don't even know what meromorphic functions are ☺ .) A space-filling curve is a graph of a function, defined by parametric equations, that consists entirely of a solid black area (normally the unit square). Cf. FunctionsBlackNearZero.gcx	11	
Y		Infinite sets	The "almost" concept in math, as in "almost surely" (in probability), "almost everywhere" (in analysis), "almost disjunct"; the Cantor set; Russell's paradox	11	
		Notation; powers of 2	Positional notation, base 10 vs. base 2, etc.; durations of notes with varying numbers of beams (up to 9 beams = 1/2^11 in Heinrich) and augmentation dots as values in base 2; **Vi Hart's "math class is boring" video	7	
		Probability theory;	A straightforward practical application of probability theory: Gamow's story about bread rationing and normal	9	

		Probability in the real world	distributions (file Gamow_BreadRationingStory.pdf)		
Y		Probability theory: conditional probability, etc.	Simple but highly counterintuitive things, e.g., the Monty Hall problem; **the "birthday paradox"; of two children, probability the 2nd is a girl given that (a) one child is a boy, (b) the 1st child is a boy, (c) the 1st child is a boy born on Tuesday, etc.	9	
Y		Probability in the real world: conditional probability, etc.	Simple but highly counterintuitive things that directly affect people: the Sally Clark murder trial; the Simpson effect; false positives & false negatives in medicine & data mining for terrorists, etc. Cf. Dave Richeson's blog		
		Probability and theory vs. reality	If a fair coin comes up heads 20 times in a row, what's the chance it'll come up heads on the 21st toss? If *this* coin (a real one) comes up heads 20 times in a row, what's the chance it'll come up heads on the 21st toss?	9	
	B	Probability in the real world	Computing *meaningful* odds that: your next poker hand will be a royal flush (easy); a specific person will get cancer within five years (fairly easy); the stock market will go up tomorrow (very difficult); a deep-water oil rig will spill a large amount of oil this year (extremely difficult); the space shuttle Challenger would have a fatal accident (almost impossible). Cf. file RealWorldProbability.txt	9	Project
Y		Probability; interesting constants	Estimate pi by dropping toothpicks on a surface with wide stripes and seeing how many cross a stripe boundary (ala Buffon, & Gamow)	9	
Y		Number theory, etc.	Periodical cicadas and their periods of 13 and 17 years: why the prime numbers? Cf. file BaltSun_PeriodicCicadas+Primes.html	10	Project
			Detecting alien civilizations a la Project SETI: Fourier analysis, etc.	10	Project
Y		Number theory, etc.	Communicating with alien civilizations (via prime nos., etc.; cf. Vi Hart(?) – is this on the Voyager golden record?)	9	Project
			T. Rex and The Crater of Doom -34. Project: Mythbusters	10	Project
		Exponential functions	"The End of the World (As We Know It)" from overpopulation (idea of L. Sitnikov?); exponential growth, etc. But is population growth really exponential? It *tends* to be; cf. Malthus (ca. 1798).	10	Project
		Exponential functions	Dramatic exx: the story about the reward for the inventor of chess (in Gamow, etc.); the *engaging* question, would you rather have $1M or 1 cent doubled every day for a month?	7	
		Numbers/number sense: large numbers & scientific notation	**E.g., how many elephants would it take to fill the sun? How many grains of sand would it take to fill the universe, how long to complete the 64-disk Tower of Hanoi puzzle, etc. How many elementary particles in the universe? (Davis: 10^{79} electrons, protons, & neutrons) How many would it take to fill the universe? (Davis: density of matter in the universe = 10^{-28} => 10^{107}) (Cf. Davis, Gamow) How many Planck cubes to fill the universe? (J. Sebens: 10^{120}) How many 4D Planck cubes to describe the universe over all time? (DAB: the age of	8	

			the universe is ca. 2 * 10^{62} Planck times => $10^{182 \text{ or } 183}$)		
Y	B	Numbers/number sense: large numbers	Ask what's the largest number you can write with three digits? (With standard operators, answer: 9^(9^9).) Show/discuss "Powers of 10" film(s), zoom into Mandelbrot set; talk about vigintillion, googol, googolplex, Skewes' number, etc. (and cf. to the smallest infinity)	8	
	B	Numbers/number sense: estimating	Estimate real-world numbers: in grocery stores or the kitchen (cf. Davis); large numbers in the school or environment, e.g., cars on the highway between Bloomington and Indianapolis, raindrops in the air at the moment in town, total weight of everyone in the school, length of the room or building via "Rule of Thumb". How many significant figures in your estimate? For "large numbers", zero significant figures might be fine!	8	
		Numbers/number sense: "innumeracy"	As in Hofstadter's term and Paulos' book by that title, Innumeracy is analogous to illiteracy	8	
		Numbers/number sense: accuracy vs. precision, etc.	How meaningful/accurate/precise is a number?, Formulas with *principled* vs. *empirical* constants. E.g., Swokowski & Cole, Sec. 5.4, problem 65 (children's weight via the Ehrenberg relation). It says "empirically based". Or -- but ??Metabolic rate is proportional to weight ^ (3/4) ??	7	
		Numbers/number sense: accuracy vs. precision, etc.	How meaningful/accurate/precise is a number?, Significant Figures Dept. The package of a Helping Hand® hammer I have is trilingual; the French and Spanish text give its weight as 226.72 grams. Five significant figures? Unlikely! (The English text says "8 oz.", i.e., one figure). Cf. MeaningfulNumbers+SignificantFigures.pdf	7	
		Numbers/number sense: rounding	Dramatic example: What was the price of my first house, rounded to the nearest quarter of a million dollars? Answer: zero—it cost $124,900. (That's zero significant figures: not very useful in this case, but often useful, e.g., for estimating.)		
		Math & physics	Mathematics of Juggling. Cf. Allen Knutson's talk at http://www.youtube.com/watch?v=38rf9FLhl-8	9	
Y?	B	Math & music	Challenge: What transformations on the Star-Spangled Banner are going on in my versions (the "Star-*Strangled* Banner")? Cf. geometric transformations a la Scott Kim.	9	
Y	B	Math & music	**Challenge: What's 'squared" in my "Happy Birthday Squared", and "cubed" in "Happy Birthday Cubed"? Instead of squaring a tune (multiplying by itself), could you multiply two different tunes?	9	
Y	B	Math & music; physics	Tuning & temperament: pure thirds, pure fifths, and pure octaves: choose any one—you can't even have two at the same time! => equal temperament: frequency ratios are irrational numbers. Cf. my article and *Musimathics*, vol. 1.	10	
		Math & music; physics	Pythagoras & music: the harmonic series, "pleasing" harmony, Fourier series, etc.	8	
	A	Math & music; logarithms & exponentials	Pitch is a logarithmic function of frequency: demonstrate by recording notes (preferably made by students) and looking at with audio editor; relate to wavelength & measure (e.g.)	10	

			string lengths to several frets on a guitar		
	A	Logarithms & exponentials	Discovery lesson: "This is a slide rule. You can do multiplication with it like this: … And you can find square roots with it like this: … How does it work?" NB: Could be done with computer-simulated slide rules, but much, much better with real, physical ones!	10	
		Math & music	Note durations & rational numbers: negative powers of 2, etc. What numbers can be represented with a single note, allowing augmentation dots? Allowing tuplets?	8	
		Math & music; physics	Fourier analysis as a way to understand tone quality, the chorus effect, interference, etc. Shouldn't be hard to demo live with a vibrating string & a strobe light; on computer with Audacity, SPEAR, FourierSeriesApplet, JG's Max/MSP demo, etc.	10	
Y		Math & music; physics	Acoustic wave phenomena. Reflection: demo w/ the Old Rope Trick. Show standing waves in musical instruments or rooms; the latter makes an incredibly dramatic demo, but isn't practical in a lot of rooms. For instruments, by far the best demo is a vibrating string & a strobe light.	10	
Y		Math & music; physics	Parabolas & "whispering galleries" (build & test one?)	10	
		Math & music; physics	"White", "pink", and "brown" noise. CF. Gardner, Martin (1978, April). Mathematical Games: White and brown music, fractal curves and one-over-f fluctations. Scientific American, pp. 16ff.	10	
	A	Math & music; probability	Composing with random numbers, e.g., with the MelodicWandering programs.	9	
	B	Math & music; analytic geometry, precalculus	Illustrate phase shift for periodic functions with words vs. notes: "Row, Row Your Boat" (*any* no. of "row"s but 3); "[Doe] A Deer, A FeMALE Deer, re; A golDEN ray of Sun, me;"; etc. However, these are periodic only at the level of verses and even then only for the music. Isorhythmic motets and delay between voice entrances in canons are more accurate examples, but probably less engaging.	10	
	B	Patterns, symmetry	Conway's Game of Life, groups (e.g., D4)	11	
Y		Geometry, non-Euclidean	In elliptic geometry, every line has finite length; similar triangles don't exist (except when they're congruent); a "point" is two points; there's more than one triangle with the same three "points" as vertices(?); etc.	10	
Y	B	Topology	Mobius strips & related phenomena (Klein bottles, etc.). E.g., make Mobius strips, draw a line down the middle on "one side", then cut down the middle two or three times, or cut 1/3 of the way across; show Vi Hart's Mobius strip music box or George Hart's bagel dissection	9	
Y?		Topology	Knots, wild knots, & Wilder knots	11	
		Geometry vs. analysis	Fractals: applications to generating realistic landscapes, vegetation, etc.	11	
Y		Geometry vs. analysis	Fractals: geometric a la Doug McKenna; his fractal cut in steel by a waterjet cutter	11	

Y		General	Equations for relativistic effects: time dilation demonstrated by flying an atomic clock around the world on airliners, etc. In practical situations: the design of particle accelerators and cellphones	10	
		Algebra; pure vs. applied math	$e/c^2 = m$ (in Einstein's 1st paper): "reasonable" amount of energy => microscopic amount of matter, yawn. $e = mc^2$ (in his 2nd paper): a small amount of matter => enormous amount of energy, WOW! Anyone who understood & remembers Algebra 1 can see the expressions are mathematically identical, but the implications even experts saw were *very* different.	10	
		Probability & statistics	Statistics: How to Lie With. "Contrary to popular belief, Norlandmark is not a socialist success story in which anyone who wants a job has one; unemployment went up 50% last year." [True, but from only 4 people (.0001% of the population) to only 6 (.00015%)!]	10	
		Rates of growth; asymptotes; music	In children's song "Bingo", substitute name of our dog "Antidisestablishmentarianism" (or their own pet): O(n^2) — though it seems like at least a bit less. Fast growth, but not as fast as exponential, e.g., of population!	8	
		General (mathematical thinking; number sense; graphing; analytic geometry; etc.)	**1D: Put a loooooong number line on floor (e.g., with masking tape); give Ss numbers (random or other) and have them put themselves at the right positions and in order. Depending on class, use unsigned or signed integers, rational number, or any real number. 2D: Put looooong axes on floor; for more advanced classes, use log or semilog axes, complex plane, or polar coords. Give Ss coordinate pairs, e.g., points on a function or geometric figure. Maybe have Ss pair off and act as vectors. Etc.**	6-12	
		Mathematical thinking; algebra	Continued fractions, e.g., as approximations for pi	10	
		Mathematical thinking; algebra	An article on a new study that links math, physics, da Vinci, and trees: http://news.sciencemag.org/sciencenow/2011/11/leonardos-formula-explains-why-t.html It might help teaching summations, but its main value in teaching math would likely be getting students comfortable with arbitrary exponents -- in this case, any value between 1.8 and 2.3, not just integers or rational numbers.	10	
		Graphing & visualization	"A tenth of a picture is worth 100 words": graphic and partially-graphic ways of conveying information	9	
		Mathematical thinking; number systems	Generalizing *operations* and *operands*, step by step: adding natural numbers => … multiplying (or dividing) real numbers => … raising complex numbers to complex powers. Or, exponents from **N** to **Z** to **Q** to **R** to **C**	10	Inquiry
		Mathematical thinking	Hyperoperations. It could reasonably be said that exponentiation is to multiplication as multiplication is to addition. If so, solve for *x* and *y* the "proportions" mult : add = add : *x* , and *y* : exp = exp : mult,. (Answer: *x* = the successor or increment operation; *y* = the nonstandard operation of tetration. Of course virtually no students will	10	Inquiry

			know the *term* tetration, and most probably won't know or think of the terms successor and increment.)		