Splitting hairs*

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1 Introduction

Besides conjoining an assertion into the common ground (Stalnaker 1978), another kind of move that we often propose and accept in discourse is to change our notion of what is a thing and what is the same thing. For example, depending on how a conversation goes, in the middle of it we might start treating ‘a’ and ‘A’ as different letters, while preserving our common knowledge that it is a vowel. (After all, the English alphabet has 26 letters—or is it 52?) Or we might start to distinguish ‘a’ from ‘a’, or even ‘a’ from ‘a’. (After all, the word ‘lava’ contains two vowels—or is it just one?) Similarly in the case of copredication: after agreeing that today ‘lunch was delicious but took forever’ (Asher 2011), we might start to distinguish the food from the event, in order to clarify how we plan to have ‘the same lunch’ tomorrow.

This paper begins a study of these moves. They are made by interlocutors such as linguists, meteorologists, and radiologists as they work together to conjure things like grammatical constructions, cold fronts, and tumors from the stuff that is our shared reality. These moves are useful because two interlocutors can agree completely on facts of the matter—be both omniscient, even—yet appear to disagree because they individuate the world into things differently. For example, two linguists may appear to disagree about whether Sita speaks Rama’s language, not because they disagree about how Sita and Rama speak, but because they group idiolects into languages differently. And two meteorologists may appear to disagree about how many cold fronts formed in this country today, only because they distinguish cold fronts, countries, or days differently.

I don’t know about you, but this kind of apparent disagreement happens all the time around me. When it does, instead of accusing each other of inaccurate perception, we interlocutors can use these moves to change and align our domains of individuals. These moves are essential for productive conversation, because what individuals we should distinguish and how (Aloni 2000) depends on the current context of the conversation, including what we are trying to do together. Indeed, I can refer to a thing without even myself having an ultimate refinement of it in mind (‘this letter is beautiful’). And as the examples in the previous paragraph suggest, the moves are not specific to words like English ‘same’ and ‘different’ (Beck 2000; Barker 2007).

2 Hair-splitting in action

To give a more formal example, suppose you and I introduce a discourse marker i in conversation and agree to ascribe to it various properties—i is P, and moreover i is Q—by successive updates to the discourse state:

$$\cdots \longrightarrow \begin{array}{|c|}\hline i \\ \hline P(i) \\ \hline \end{array} \xrightarrow{\quad Q(i) \quad} \begin{array}{|c|}\hline i \\ \hline P(i) \\ Q(i) \\ \hline \end{array} \tag{1}$$

*Thanks to Chris Barker, Philippe de Groote, Larry Moss, Reinhard Muskens, Valeria de Paiva, and the anonymous reviewers for many helpful comments.

Then we turn to the topic of whether i is R and find upon investigation that we need to distinguish between two refinements of i, namely j, which is R, and k, which is not. So we make that distinction—we split the hair i into j and k—while preserving our common knowledge that it is P and Q. In other words, j and k are both P and Q:

$$\cdots \longrightarrow \begin{array}{|c|}\hline i \\ \hline P(i) \\ \hline \end{array} \xrightarrow[Q(i)]{\qquad} \begin{array}{|c|}\hline i \\ \hline P(i) \\ Q(i) \\ \hline \end{array} \xrightarrow[\textbf{?}]{\qquad} \begin{array}{|cc|}\hline j & k \\ \hline P(j) & P(k) \\ Q(j) & Q(k) \\ \hline \end{array} \xrightarrow[\substack{R(j) \\ \neg R(k)}]{\qquad} \begin{array}{|cc|}\hline j & k \\ \hline P(j) & P(k) \\ Q(j) & Q(k) \\ R(j) & \neg R(k) \\ \hline \end{array} \tag{2}$$

It is impossible to model the update marked '**?**' above as a relation on standard possible worlds. (By 'standard possible world', I mean what amounts to a first-order model with a domain of individuals.) The reason is that the valuation in a standard possible world is a *total* function—it always returns either true or false. Such a valuation conflates propositions whose truth values persist after a split (like $P(i)$ and $Q(i)$) with propositions whose truth values may vary after a split (like $R(i)$). Now to see the impossibility, suppose w is a world possible in the common ground before the update '**?**' above. In each such w, we have not only both $P(i)$ and $Q(i)$, but also one of $R(i)$ and $\neg R(i)$. Consider one such w, where we have $R(i)$, say. If '**?**' above were a relation on worlds, then we would want it to relate w to only worlds where both $P(j) \wedge P(k)$ and $Q(j) \wedge Q(k)$ hold, but not only worlds where $R(j) \wedge R(k)$ holds. But w contains no information to help the relation treat R so differently from P and Q.

The upshot is that a discourse marker cannot naively refer to an individual in a standard possible-world semantics. On that naive view, there is a fact of the matter as to how many letters are in the word 'lava', so if you count 4 letters and I count 3, then we can't both be right. And there is a fact of the matter as to how many things are on the dining table, so if you include the coasters under the glasses whereas I pair up each coaster and glass as one thing, then we can't both be right. But we can. So when we agreed in (1) that there is a P that is Q, what did we existentially quantify over?

We could start to answer this question by blaming underspecification. We would then have to explain what kind of underspecification would let us equivocate between ascribing the properties P and Q to one individual i on one hand, and to two individuals j, k on the other hand. What *are* these underspecified individuals that an indefinite could existentially quantify over? (Perhaps plurals of some sort, with parts?)

As a first step, the rest of this paper defines a modal logic in which a point models not a possible world but a discourse state.

3 A modal logic with individual accessibility

To model how discourse markers split and merge in the course of a conversation, we introduce a modal logic in which accessibility relates not possible worlds but discourse states, and crucially, the individuals at each state as well.

3.1 Frames

Formally, a *frame* for us is a quadruple $F = \langle S, R_S, D, R_D \rangle$ consisting of

1. a set S of *states* (depicted in Figure 1 as $\{s, t, u\}$);

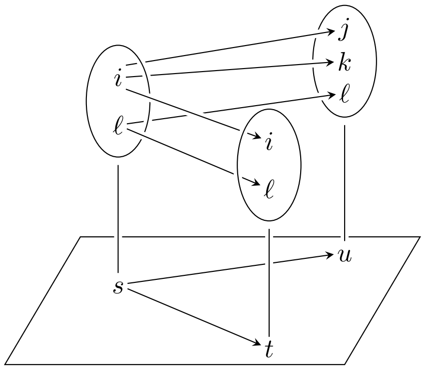

Figure 1: A frame

2. a relation $R_S \subseteq S \times S$ on states, called the *(state) accessibility relation* (the arrows at the bottom of Figure 1);
3. a function D mapping each state $s \in S$ to a set $D(s)$, called the *domain of individuals* at s (the balloons in Figure 1);
4. this is what's new: a function R_D mapping each pair $\langle s, t\rangle \in R_S$ to a relation $R_D(s,t) \subseteq D(s)\times D(t)$, called the *(individual) accessibility relation* (the arrows at the top of Figure 1).

If $\langle s, t\rangle \in R_S$, then we say that the state t is *accessible* from the state s, and notate the relationship infix as $s\,R\,t$. If moreover $\langle x, y\rangle \in R_D(s,t)$, then we say that the individual y at t is *accessible* from the individual x at s, and notate the relationship infix as $x\,{}_sR_t\,y$. We visualize individual accessibility as lifting state accessibility to a *covering space* (Munkres 2000). It can be implemented by a sort of counterpart relation (Lewis 1968), with postulates modified to suit the difference that our states are discourse states, not possible worlds. (For example, whereas according to Lewis 'it would not have been plausible to postulate that the counterpart relation was transitive', it *is* plausible to postulate that our accessibility relation be transitive, as discussed below.)

3.2 Truth

We define formulas ϕ, terms t, and models M as usual in first-order modal logic. But to define truth, we need not only the notion of *valuations at a state* but also the notion of *valuation accessibility*. A valuation v at a state s is just a function that maps each variable name x to an individual at s. If the state t is accessible from the state s, then we say that a valuation w at t is *accessible* from a valuation v at s just in case v and w (have the same set of variable names for their domains and) map each variable name to an accessible pair of individuals. We notate this relationship infix as $v\,{}_sR_t\,w$. In short,

$$v\,{}_sR_t\,w \quad \text{just in case} \quad \forall x.\, v(x)\,{}_sR_t\,w(x). \tag{3}$$

Finally, we define the *truth* of a formula ϕ in a model M under a valuation v at a state s, notated $M, s, v \Vdash \phi$. The definition is standard except for the modal operators $\Box$ and $\Diamond$, which are dual. We define

$$M, s, v \Vdash \Box\phi \quad \text{just in case} \quad \forall t.\, (s\,R\,t \to \forall w.\, (v\,{}_sR_t\,w \to M, t, w \Vdash \phi)). \tag{4}$$

3.3 Consequences

Two hallmark consequences distinguish our logic from standard modal logic. On one hand, $(x = y)$ does not entail $\Box(x = y)$, because one individual at the current state may be related to multiple individuals at future states. On the other hand, if ϕ and ϕ' are the same formula except with free variables x and y exchanged throughout, then $(x = y) \wedge \Box\phi$ does entail $\Box\phi'$. In particular, ϕ could be $P(x) \wedge Q(x)$, so to continue the example (2), the formula $(x = y) \wedge \Box(P(x) \wedge Q(x))$ does entail $\Box(P(y) \wedge Q(y))$. Both of these consequences are desirable for modeling our discourse moves of interest, where discourse markers split and merge.

These consequences also offer one way to relate our logic to the *modal logics of relations* that Marx and Venema (1997) study under the rubric of *multi-dimensional modal logic*. Starting at the 'local cube' condition on multi-dimensional frames (LC_n in their Definition 5.3.4), our logic amounts to extending their modal similarity type with a unary modality $\Box$. It would be enlightening if their metatheoretical results could be generalized—either to specify $\Box$ so that every dimension moves along the same individual accessibility relation, or to axiomatize our logic so that the two hallmark consequences fall out.

3.4 Frame correspondence

Like standard modal logic, our logic enjoys natural frame conditions corresponding to modal axioms. The reflexivity ('T') axiom $\Box\phi \to \phi$ is valid for a frame if and only if

$$\forall s.\ (s\,R\,s \wedge \forall x.\ x\,{}_sR_s\,x). \tag{5}$$

And the transitivity ('4') axiom $\Box\phi \to \Box\Box\phi$ is valid for a frame if and only if

$$\forall s, t, u.\ (s\,R\,t \wedge t\,R\,u) \to (s\,R\,u \wedge \forall x, y, z.\ (x\,{}_sR_t\,y \wedge y\,{}_tR_u\,z) \to x\,{}_sR_u\,z). \tag{6}$$

The difference from standard modal logic in the two conditions are the subformulas $\forall x \ldots$ constraining individual accessibility.

4 Outlook

The axioms (5) and (6) are both intuitive, but they only begin to chip away at a myriad of discourse possibilities made available by individual accessibility: as conversation proceeds, discourse markers may split or merge as well as go in and out of existence. Investigating how interlocutors navigate this sea of states leads us back to the question: what does a discourse marker refer to in our shared reality, if not an individual in a standard possible-world semantics? In short, what is a thing? Our logic suggests that a possible world is a path along the state accessibility relation and a thing is a path along the individual accessibility relation.

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