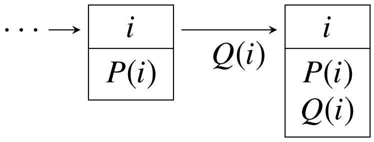
Splitting hairs

Besides conjoining an assertion into the common ground (Stalnaker), another kind of move that we often propose and accept in discourse is to change our notion of what is a thing and what is the same thing. For example, depending on how a conversation goes, in the middle of it we might start treating ‘a’ and ‘A’ as different letters, while preserving our common knowledge that it is a vowel. (After all, the English alphabet has 26 letters—or is it 52?) Or we might start to distinguish ‘a’ from ‘a’, or even ‘a’ from ‘a’. (After all, the word ‘lava’ contains two vowels—or is it just one?) Similarly in the case of copredication: after agreeing that today ‘lunch was delicious but took forever’ (Asher), we might start to distinguish the food from the event, in order to clarify how we plan to have ‘the same lunch’ tomorrow.

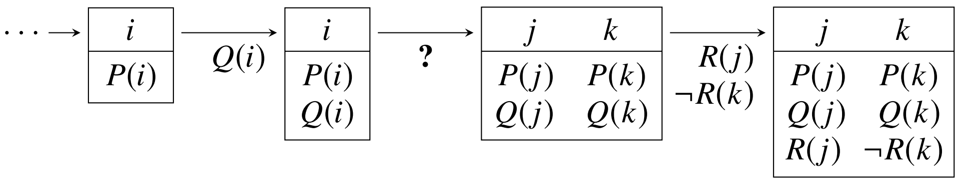
This paper begins a study of these moves. They are made by interlocutors such as linguists, meteorologists, and radiologists as they work together to conjure things like grammatical constructions, cold fronts, and tumors from the stuff that is our shared reality. More formally, suppose you and I introduce a discourse marker i in conversation and agree to ascribe to it various properties—i is P, and moreover i is Q—by successive updates to the discourse state:

(1)

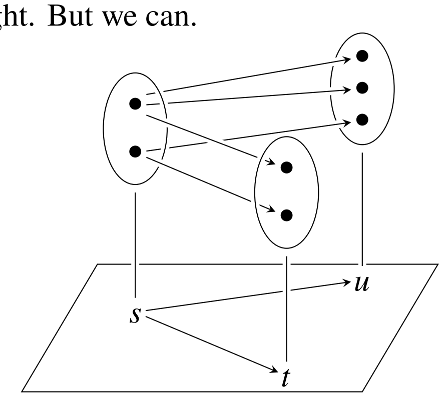


Then we turn to the topic of whether i is R and find upon investigation that we need to distinguish between two refinements of i, namely j, which is R, and k, which is not. So we make that distinction—we split the hair i into j and k—while preserving our common knowledge that it is P and Q. In other words, j and k are both P and Q:

(2)



The update marked ‘**?**’ above cannot be modeled as a relation on standard possible worlds (that is, possible worlds that each amount to a first-order model with a domain of individuals), because the valuation in each world is total and conflates propositions whose truth values persist after a split (like $P(i)$ and $Q(i)$) with propositions whose truth values may vary after a split (like $R(i)$). Thus, a discourse marker cannot naively refer to an individual in a standard possible-world semantics. On that naive view, there is a fact as to how many letters are in the word ‘lava’, so if you count 4 letters and I count 3, then we can’t both be right. And there is a fact as to how many things are on the dining table, so if you include the coasters under the glasses whereas I pair up each coaster and glass as one thing, then we can’t both be right. But we can.



To model how discourse markers split and merge in the course of a conversation, we introduce a modal logic in which accessibility relates not possible worlds but discourse states, and crucially, the individuals at each state as well. Formally, a *frame* for us is a quadruple $F = \langle S, R_S, D, R_D \rangle$ consisting of

- a set S of *states* (depicted to the right as $\{s, t, u\}$);
- a relation $R_S \subseteq S \times S$ on states, called the *(state) accessibility relation* (the arrows at the lower right);
- a function D mapping each state $s \in S$ to a set $D(s)$, called the *domain of individuals* at s (the balloons);
- this is what’s new: a function R_D mapping each pair $\langle s, t \rangle \in R_S$ to a relation $R_D(s, t) \subseteq D(s) \times D(t)$, called the *(individual) accessibility relation* (the arrows at the upper right).

If $\langle s, t \rangle \in R_S$, then we say that the state t is *accessible* from the state s, and notate the relationship infix as $s\,R\,t$. If moreover $\langle x, y \rangle \in R_D(s, t)$, then we say that the individual y at t is *accessible* from the individual x at s, and notate the relationship infix as $x\,{}_sR_t\,y$. We visualize individual accessibility as lifting state accessibility to a covering space. It can be implemented by a sort of counterpart relation (Lewis), with postulates modified to suit the difference that our states are discourse states, not possible worlds.

We define formulas ϕ, terms t, and models M as usual in first-order modal logic. But to define truth, we need not only the notion of *valuations at a state* but also the notion of *valuation accessibility*. A valuation v at a state s is a function that maps each variable name x to an individual at s. If the state t is accessible from the state s, then we say that a valuation w at t is *accessible* from a valuation v at s just in case v and w (have the same set of variable names for their domains and) map each variable name to an accessible pair of individuals. We notate this relationship infix as $v\,{}_sR_t\,w$. In short,

(3) $\quad v\,{}_sR_t\,w \quad$ just in case $\quad \forall x.\, v(x)\,{}_sR_t\,w(x)$.

We can finally define the *truth* of a formula ϕ in a model M under a valuation v at a state s, notated $M, s, v \Vdash \phi$. The definition is standard except for the modal operators $\Box$ and $\Diamond$, which are dual. We define

(4) $\quad M, s, v \Vdash \Box\phi \quad$ just in case $\quad \forall t.\, (s\,R\,t \to \forall w.\, (v\,{}_sR_t\,w \to M, t, w \Vdash \phi))$.

Two hallmark consequences distinguish our logic from standard modal logic. On one hand, $(x = y)$ does not entail $\Box(x = y)$, because one individual at the current state may be related to multiple individuals at future states. On the other hand, if ϕ and ϕ' are the same formula except with free variables x and y exchanged throughout, then $(x = y) \wedge \Box\phi$ does entail $\Box\phi'$. In particular, ϕ could be $P(x) \wedge Q(x)$, so to continue the example (2), the formula $(x = y) \wedge \Box(P(x) \wedge Q(x))$ does entail $\Box(P(y) \wedge Q(y))$. Both of these consequences are desirable for modeling our discourse moves of interest, where discourse markers split and merge.

Like standard modal logic, our logic enjoys natural frame conditions corresponding to modal axioms. The reflexivity ('T') axiom $\Box\phi \to \phi$ is valid for a frame if and only if

(5) $\quad \forall s.\, (s\,R\,s \wedge \forall x.\, x\,{}_sR_s\,x)$.

And the transitivity ('4') axiom $\Box\phi \to \Box\Box\phi$ is valid for a frame if and only if

(6) $\quad \forall s, t, u.\, (s\,R\,t \wedge t\,R\,u) \to (s\,R\,u \wedge \forall x, y, z.\, (x\,{}_sR_t\,y \wedge y\,{}_tR_u\,z) \to x\,{}_sR_u\,z)$.

The difference from standard modal logic in the two conditions are the subformulas $\forall x \ldots$ constraining individual accessibility.

Both of these axioms are intuitive, but they only begin to chip away at a myriad of discourse possibilities made available by individual accessibility: as conversation proceeds, discourse markers may split or merge as well as go in and out of existence. Investigating how interlocutors navigate this sea of states leads us back to the question: what does a discourse marker refer to in our shared reality, if not an individual in a standard possible-world semantics? In short, what is a thing? Our logic suggests that a possible world is a path along state accessibility and a thing is a path along individual accessibility.

References

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