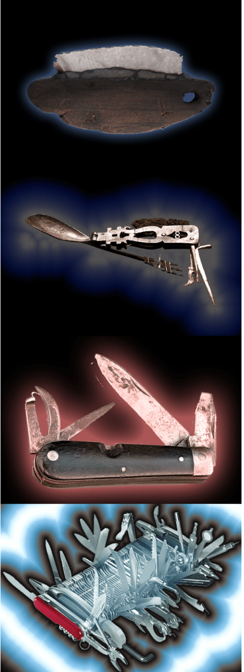
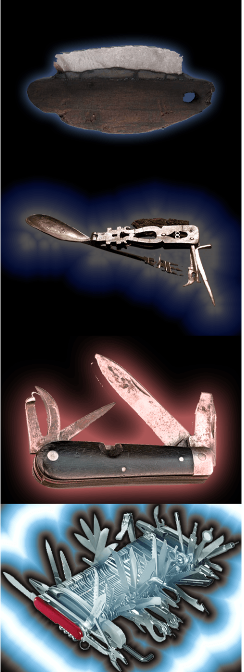
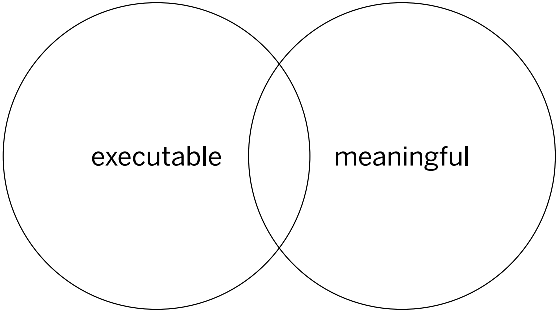


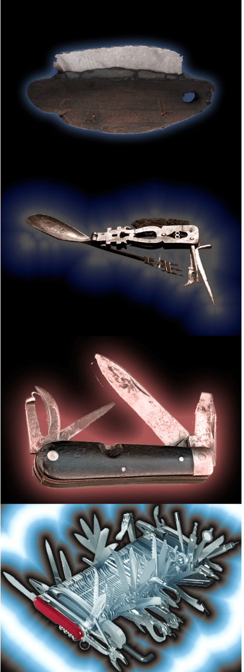
Calculating distributions

Chung-chieh Shan
Indiana University
2018-09-21

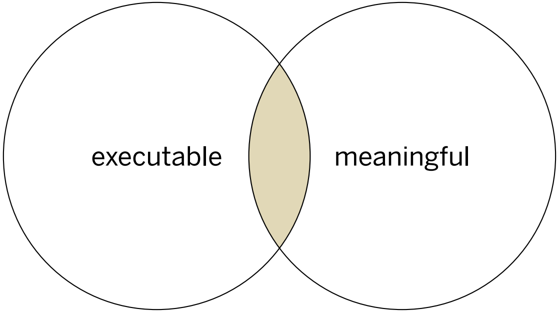


Calculating distributions





Calculating distributions

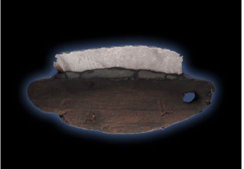


I'd also like to address this concept of being "fake" or "calculating."

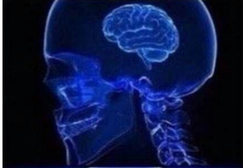
If being "fake" means not thinking or feeling the same way in one moment than you thought or felt in a different moment, then lord help us all.

If being "calculating" is thinking through your words and actions and modeling the behavior you would like to see in the world, even when it is difficult, then I hope more of you will become calculating.

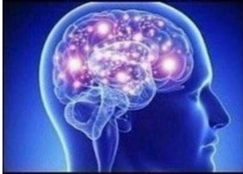
—BenDeLaCreme



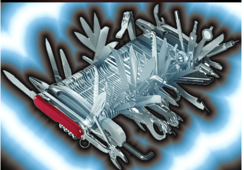
Creative definitions
and reasoning from first
principles



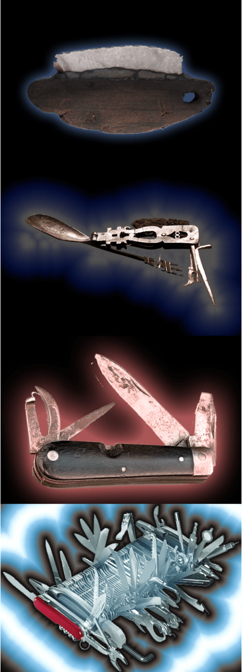
Symbolic representations
of common definition
patterns



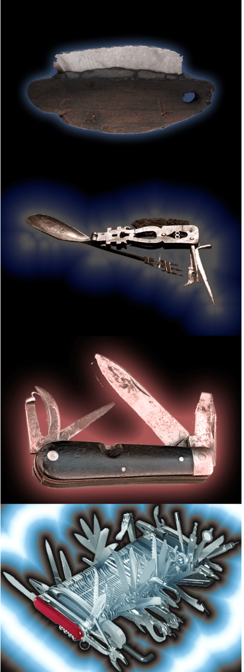
Mechanical operations
for common reasoning
patterns



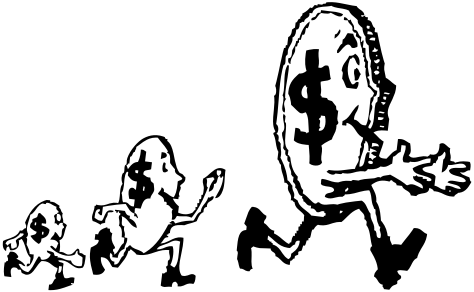
Virtuous cycle
of automation
and exploration
(Buchberger)



Creative definitions and reasoning from first principles	natural numbers
Symbolic representations of common definition patterns	unary, binary
Mechanical operations for common reasoning patterns	$<, +, \div$
Virtuous cycle of automation and exploration (Buchberger)	rationals, reals, polynomials



Creative definitions and reasoning from first principles	natural numbers	probability distributions
Symbolic representations of common definition patterns	unary, binary	table, Bayes net, probabilistic program
Mechanical operations for common reasoning patterns	$<, +, \div$	recognize, integrate, disintegrate
Virtuous cycle of automation and exploration (Buchberger)	rationals, reals, polynomials	inference, learning, optimization



An unknown random process yields a stateless coin that can be flipped repeatedly to produce heads (H) or tails (T).

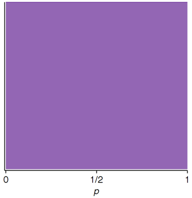
We assume that the probability p that the coin produces H each time is distributed uniformly between 0 and 1 by the process.

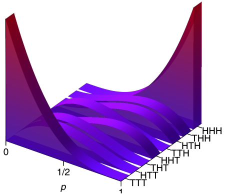
We flip the coin 3 times and observe THH.

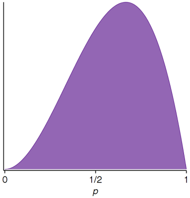
What is the probability that the next flip produces H versus T?

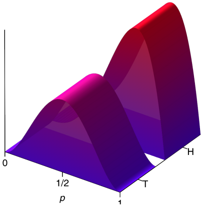
(adapted from Eddy)

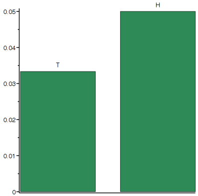
$$\Pr(p) \xrightarrow{\text{bind}} \Pr(p, \vec{x}) \xrightarrow{\text{disintegrate}} \Pr(p \mid \vec{x}) \xrightarrow{\text{bind}} \Pr(p, y \mid \vec{x}) \xrightarrow{\text{integrate}} \Pr(y \mid \vec{x})$$











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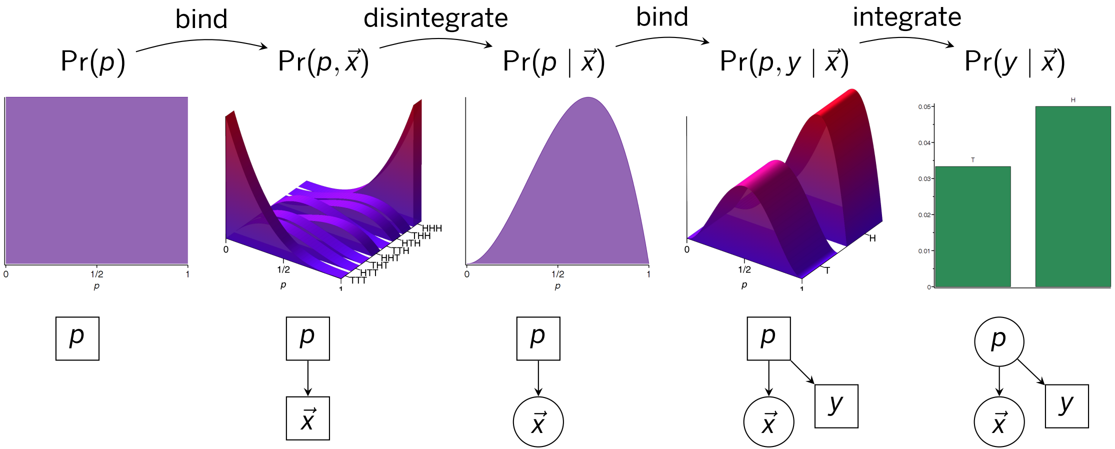
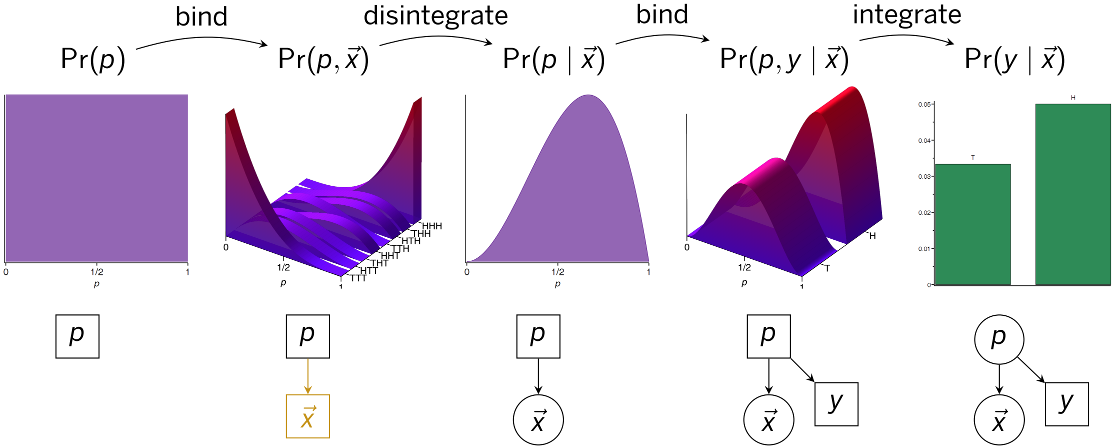

bind
disintegrate
bind
integrate
$\Pr(p)$
$\Pr(p, \vec{x})$
$\Pr(p \mid \vec{x})$
$\Pr(p, y \mid \vec{x})$
$\Pr(y \mid \vec{x})$
HHH
THH
HTH
TTH
HHT
THT
HTT
TTT
H
T
0
1/2
1
p
0.05
0.04
0.03
0.02
0.01
p
$\vec{x}$
y


bind
disintegrate
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integrate
$\Pr(p)$
$\Pr(p, \vec{x})$
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THH
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0.05
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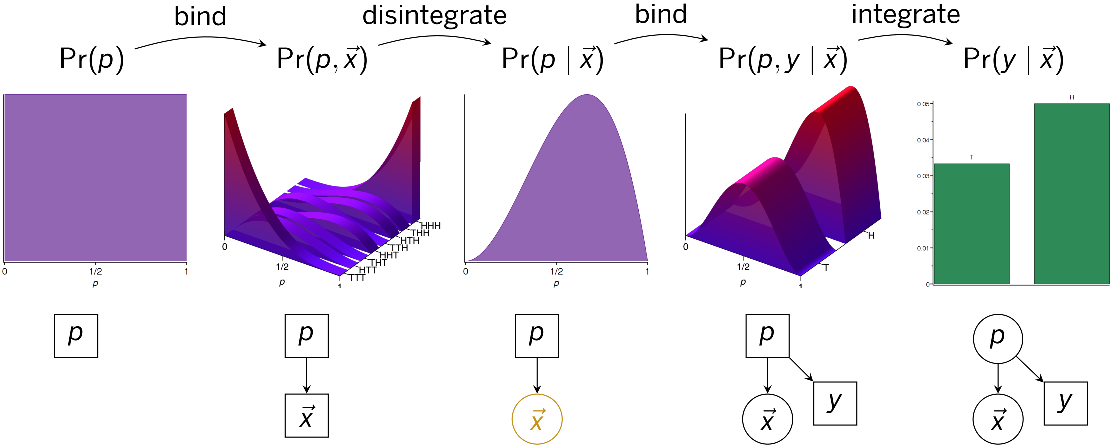
0.03
0.02
0.01
0
p
$\vec{x}$
y


bind
disintegrate
bind
integrate
Pr(p)
Pr(p, x⃗)
Pr(p | x⃗)
Pr(p, y | x⃗)
Pr(y | x⃗)
0
1/2
1
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HHH
THH
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T
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0.04
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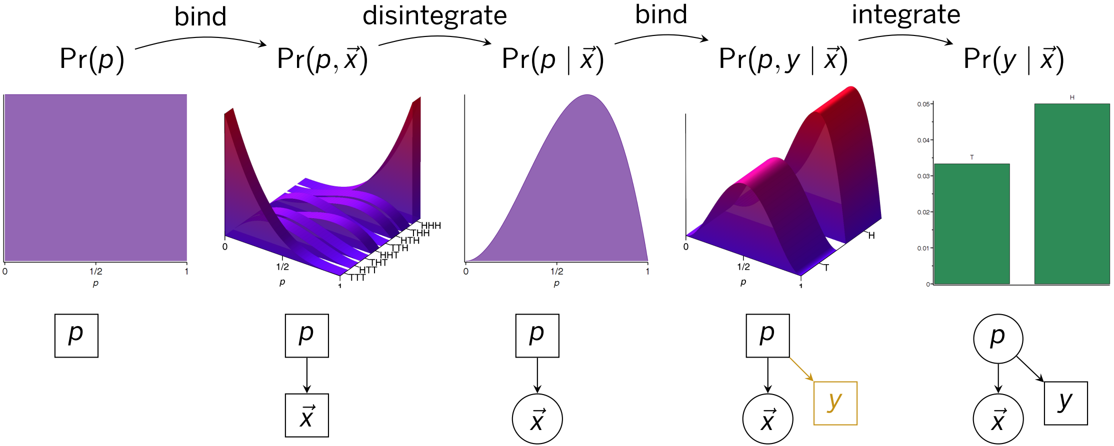
0.02
0.01
p
x⃗
y


bind
disintegrate
bind
integrate
Pr(p)
Pr(p, x⃗)
Pr(p | x⃗)
Pr(p, y | x⃗)
Pr(y | x⃗)
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1/2
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HHH
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0.02
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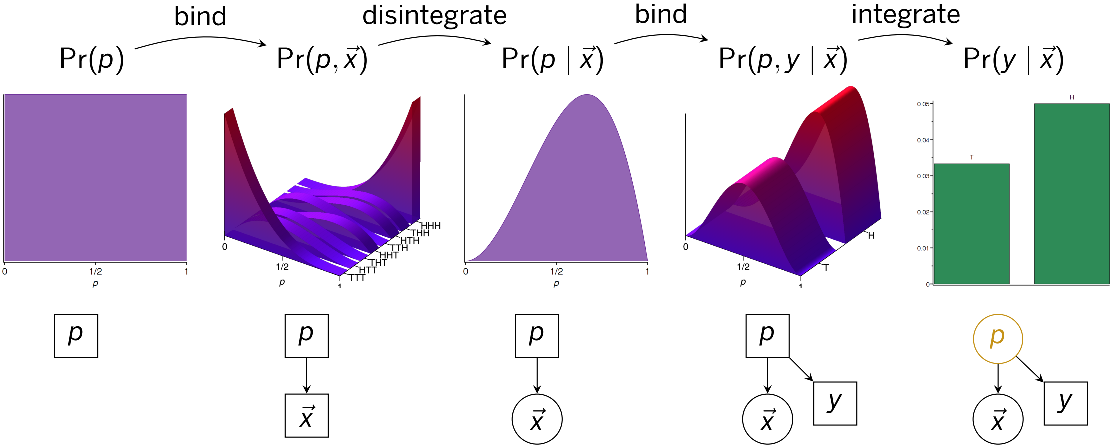
p
x⃗
y


bind
disintegrate
bind
integrate
$\Pr(p)$
$\Pr(p, \vec{x})$
$\Pr(p \mid \vec{x})$
$\Pr(p, y \mid \vec{x})$
$\Pr(y \mid \vec{x})$
0
1/2
1
p
HHH
THH
HTH
TTH
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T
0.05
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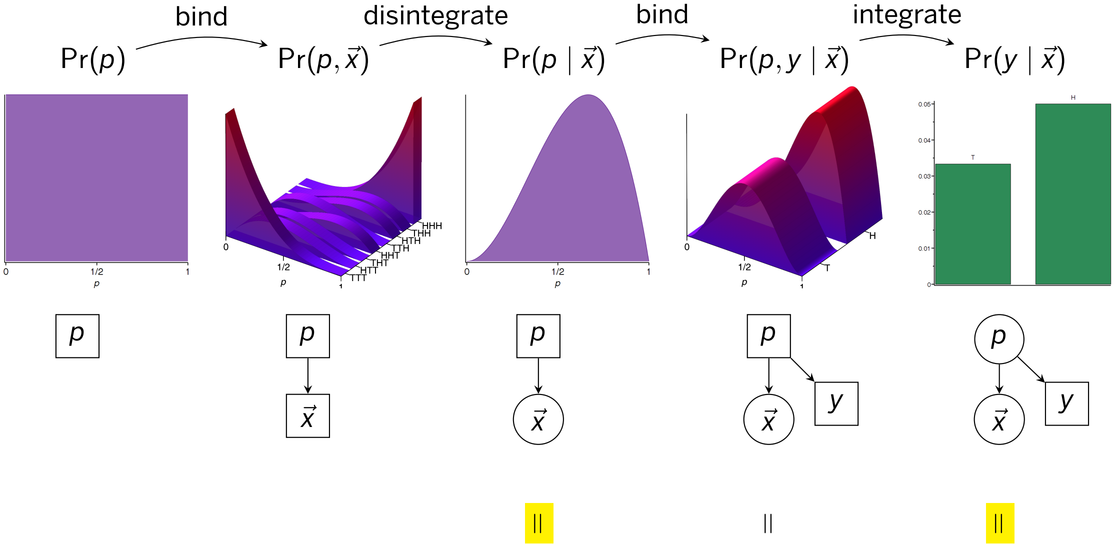
0.02
0.01
0
p
$\vec{x}$
y


bind
disintegrate
bind
integrate
Pr(p)
Pr(p, x⃗)
Pr(p | x⃗)
Pr(p, y | x⃗)
Pr(y | x⃗)
0
1/2
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p
HHH
THH
HTH
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THT
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p
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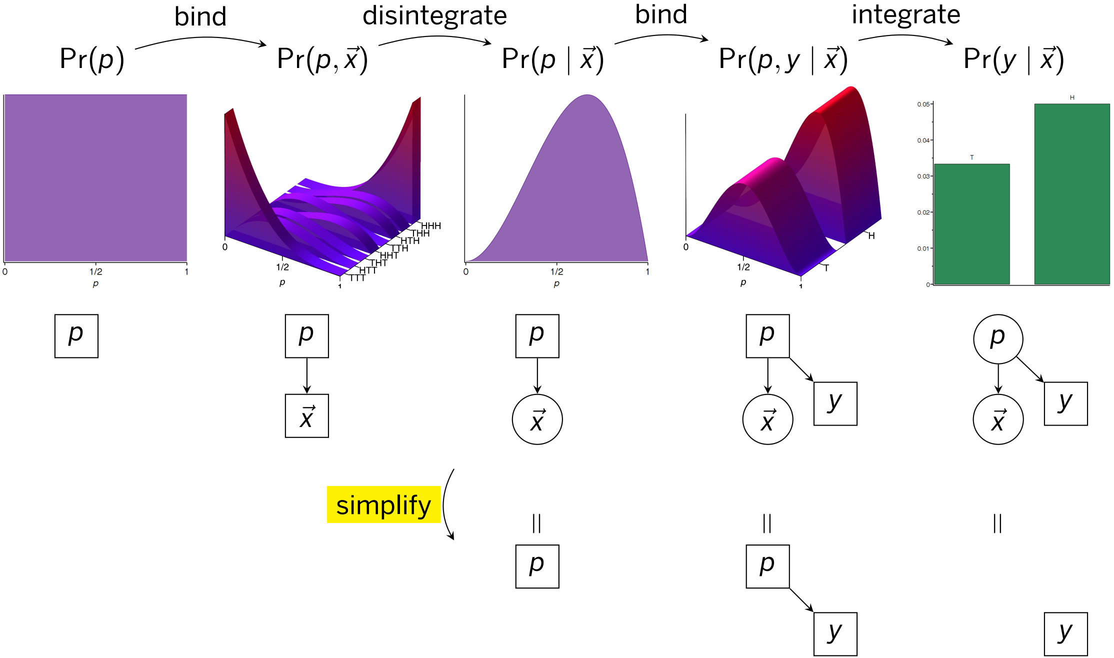
bind
disintegrate
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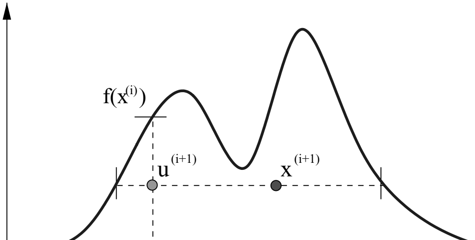
integrate
Pr(p)
Pr(p, x⃗)
Pr(p | x⃗)
Pr(p, y | x⃗)
Pr(y | x⃗)
0
1/2
1
p
HHH
THH
HTH
TTH
HHT
THT
HTT
TTT
H
T
0.05
0.04
0.03
0.02
0.01
0
p
x⃗
y
simplify
=
=
=

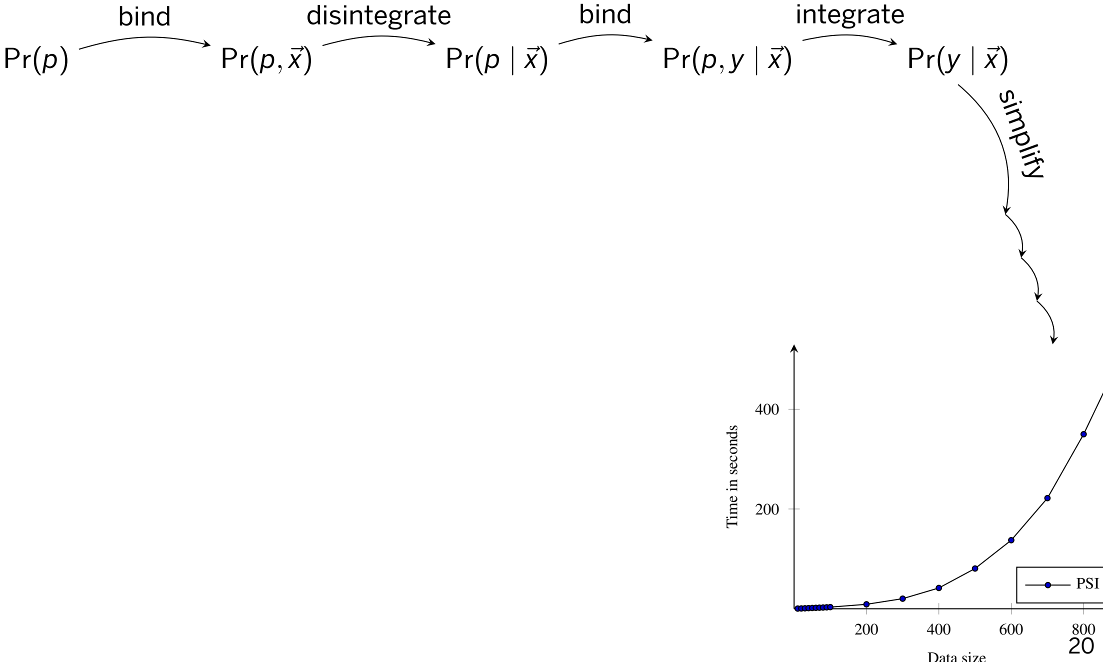
Approximations calculated exactly

Approximations calculated exactly

$$
\begin{aligned}
x_1^{(t+1)} &\sim P(x_1 | x_2^{(t)}, x_3^{(t)}, \ldots x_K^{(t)}) \\
x_2^{(t+1)} &\sim P(x_2 | x_1^{(t+1)}, x_3^{(t)}, \ldots x_K^{(t)}) \\
x_3^{(t+1)} &\sim P(x_3 | x_1^{(t+1)}, x_2^{(t+1)}, \ldots x_K^{(t)}), \text{etc.}
\end{aligned}
$$

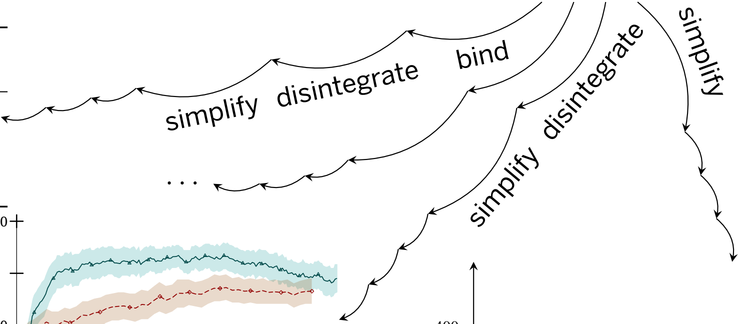


$$\Pr(p) \xrightarrow{\text{bind}} \Pr(p, \vec{x}) \xrightarrow{\text{disintegrate}} \Pr(p \mid \vec{x}) \xrightarrow{\text{bind}} \Pr(p, y \mid \vec{x}) \xrightarrow{\text{integrate}} \Pr(y \mid \vec{x})$$



bind
disintegrate
bind
integrate
$\Pr(p)$
$\Pr(p, \vec{x})$
$\Pr(p \mid \vec{x})$
$\Pr(p, y \mid \vec{x})$
$\Pr(y \mid \vec{x})$
simplify
400
200
Time in seconds
200
400
600
800
Data size
PSI

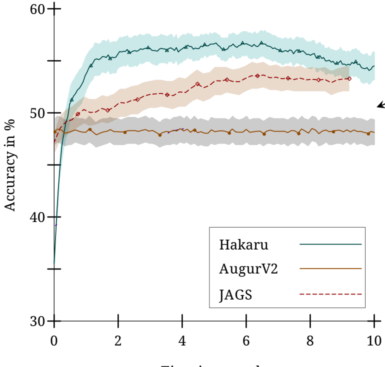
$$\Pr(p) \xrightarrow{\text{bind}} \Pr(p, \vec{x}) \xrightarrow{\text{disintegrate}} \Pr(p \mid \vec{x}) \xrightarrow{\text{bind}} \Pr(p, y \mid \vec{x}) \xrightarrow{\text{integrate}} \Pr(y \mid \vec{x})$$

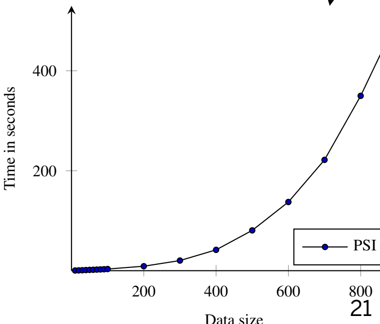


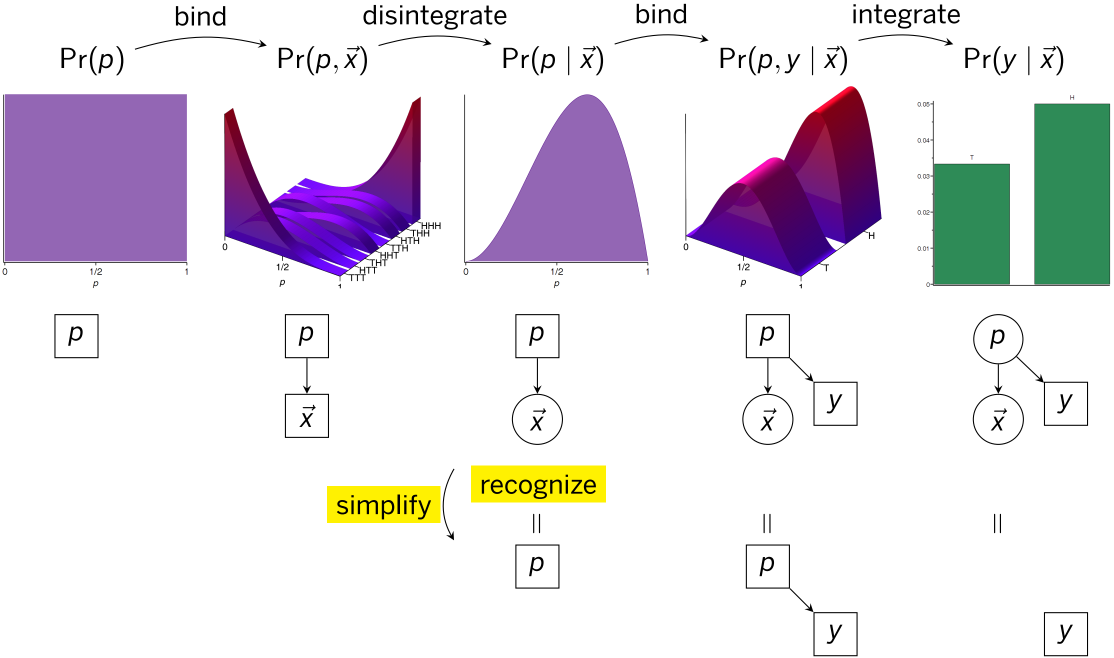
Inference method	Run time (msecs) Mean	SD
WebPPL	1078	16
Hakaru without simplifications	1321	93
Hakaru with simplifications	269	10
Handwritten	207	4

Put approximations in the language!

(FLOPS 2016, UAI 2017)






bind
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HHH
THH
HTH
TTH
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THT
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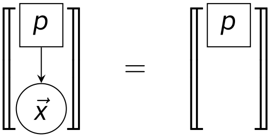
T
0.05
0.04
0.03
0.02
0.01
simplify
recognize

Recognizing a density function

Program denote measures:



Recognizing a density function

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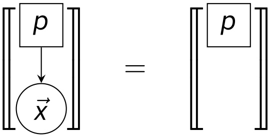
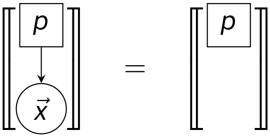


$$\in \quad \mathbb{M}\,[0,1]$$

Recognizing a density function

Program denote measures:



$$\in \quad \mathbb{M}\,[0,1] \quad \subseteq \quad ([0,1] \to \mathbb{R}_+) \to \mathbb{R}_+$$

Measures compute expectations:

Recognizing a density function

Program denote measures:

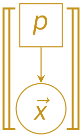
$$\left[\!\left[\begin{array}{c}\boxed{p}\\\downarrow\\\bigcirc\!\!\!\!\!\!\vec{x}\end{array}\right]\!\right] = \left[\!\left[\begin{array}{c}\boxed{p}\\ \\ \end{array}\right]\!\right] \in \mathbb{M}\,[0,1] \subseteq ([0,1] \to \mathbb{R}_+) \to \mathbb{R}_+$$

Measures compute expectations:

$$f \quad \mapsto$$

Recognizing a density function

Program denote measures:



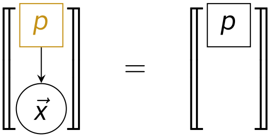
$$= [\![\, \boxed{p} \,]\!] \in \mathbb{M}\,[0,1] \subseteq ([0,1] \to \mathbb{R}_+) \to \mathbb{R}_+$$

Measures compute expectations:

$$f \mapsto \int_0^1 \sum_{\vec{x} \in \{\mathrm{H},\mathrm{T}\}^3} p^{\sum_i \langle x_i = \mathrm{H} \rangle} (1-p)^{\sum_i \langle x_1 = \mathrm{T} \rangle} \langle \vec{x} = \mathrm{THH} \rangle \cdot f(p)\, dp$$

Recognizing a density function

Program denote measures:



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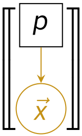
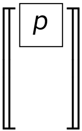


$=$



$$\in \quad \mathbb{M}\,[0,1] \quad \subseteq \quad ([0,1] \to \mathbb{R}_+) \to \mathbb{R}_+$$

Measures compute expectations:

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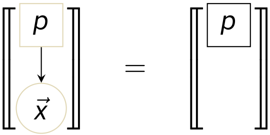
Recognizing a density function

Program denote measures:



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Recognizing a density function

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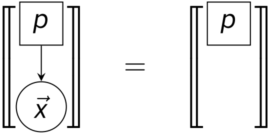

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Measures compute expectations:

$$f \quad \mapsto \quad \int_0^1 \sum_{\vec{x} \in \{\mathrm{H},\mathrm{T}\}^3} p^{\sum_i \langle x_i = \mathrm{H} \rangle} (1-p)^{\sum_i \langle x_1 = \mathrm{T} \rangle} \langle \vec{x} = \mathrm{THH} \rangle \cdot f(p)\, dp$$

$$= \quad \int_0^1 p^2 (1-p) \cdot f(p)\, dp$$

Recognizing a density function

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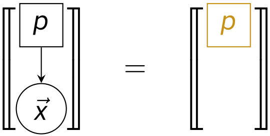


$$\in \quad \mathbb{M}\,[0,1] \quad \subseteq \quad ([0,1] \to \mathbb{R}_+) \to \mathbb{R}_+$$

Measures compute expectations:

$$f \quad \mapsto \quad \int_0^1 \sum_{\vec{x} \in \{\mathrm{H},\mathrm{T}\}^3} p^{\sum_i \langle x_i = \mathrm{H} \rangle} (1-p)^{\sum_i \langle x_1 = \mathrm{T} \rangle} \langle \vec{x} = \mathrm{THH} \rangle \cdot f(p)\, dp$$

$$= \quad \int_0^1 p^2 (1-p) \cdot f(p)\, dp$$

Need to recognize simplified denotation as Beta distribution ...

Recognizing a density function

Goal: recognize $h(p) = p^2(1-p)$ as the *density* of **beta** 3 2

Robustness challenge: many equivalent ways to write $p^2(1-p)$ arise

Modularity challenge: many distribution families (**beta**, **normal**, ...) known

Recognizing a density function

Goal: recognize $h(p) = p^2(1-p)$ as the *density* of **beta** 3 2

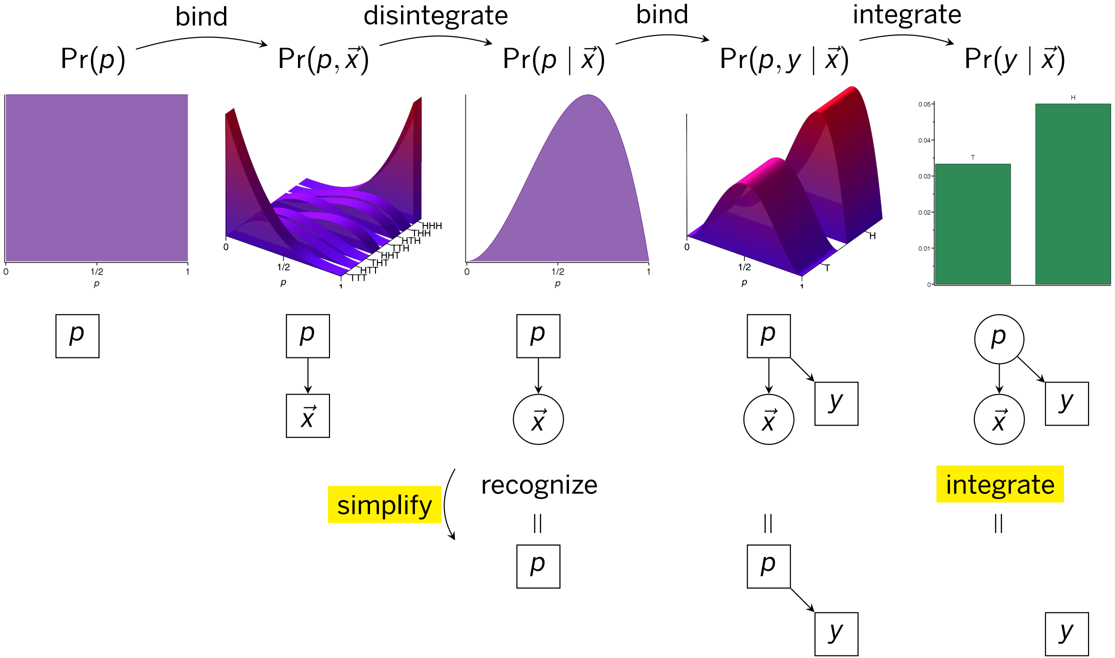
Robustness challenge: many equivalent ways to write $p^2(1-p)$ arise

Modularity challenge: many distribution families (**beta**, **normal**, ...) known

Solution: characterize density functions by their **holonomic representation**, a *homogeneous linear differential equation* such as

$$p(1-p) \cdot h'(p) + \big(p - 2(1-p)\big) \cdot h(p) = 0$$

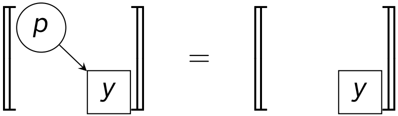
computed *compositionally*!



bind
disintegrate
bind
integrate
Pr(p)
Pr(p, x⃗)
Pr(p | x⃗)
Pr(p, y | x⃗)
Pr(y | x⃗)
0
1/2
1
p
HHH
THH
HTH
TTH
HHT
THT
HTT
TTT
H
T
0.05
0.04
0.03
0.02
0.01
p
x⃗
y
simplify
recognize
integrate
||

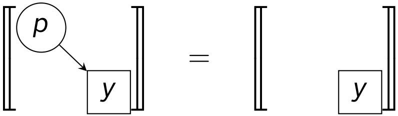
Eliminating a random variable

Program denote measures:



Eliminating a random variable

Program denote measures:



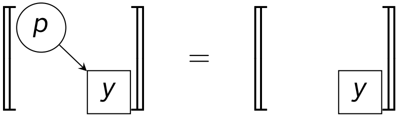
$$\in \quad \mathbb{M}\,\{\mathsf{H},\mathsf{T}\} \quad \subseteq \quad (\{\mathsf{H},\mathsf{T}\} \to \mathbb{R}_+) \to \mathbb{R}_+$$

Measures compute expectations:

$$f \quad \mapsto \quad \int_0^1 p^2(1-p) \sum_{y\in\{\mathsf{H},\mathsf{T}\}} p^{\langle y=\mathsf{H}\rangle}(1-p)^{\langle y=\mathsf{T}\rangle} \cdot f(y)\,dp$$

Eliminating a random variable

Program denote measures:



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Measures compute expectations:

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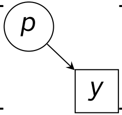
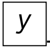
Eliminating a random variable

Program denote measures:

$[\![$  $]\!]$ $=$ $[\![$  $]\!]$ $\in \mathbb{M}\{\mathsf{H},\mathsf{T}\} \subseteq (\{\mathsf{H},\mathsf{T}\} \to \mathbb{R}_+) \to \mathbb{R}_+$

Measures compute expectations:

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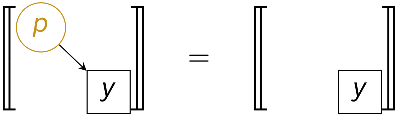
Eliminating a random variable

Program denote measures:



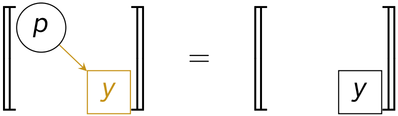
$$\in \quad \mathbb{M}\{\mathsf{H},\mathsf{T}\} \quad \subseteq \quad (\{\mathsf{H},\mathsf{T}\} \to \mathbb{R}_+) \to \mathbb{R}_+$$

Measures compute expectations:

$$f \quad \mapsto \quad \int_0^1 p^2(1-p) \sum_{y\in\{\mathsf{H},\mathsf{T}\}} p^{\langle y=\mathsf{H}\rangle}(1-p)^{\langle y=\mathsf{T}\rangle} \cdot f(y)\, dp$$

Eliminating a random variable

Program denote measures:



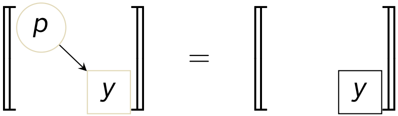
$$\in \quad \mathbb{M}\{\mathrm{H},\mathrm{T}\} \quad \subseteq \quad (\{\mathrm{H},\mathrm{T}\} \to \mathbb{R}_+) \to \mathbb{R}_+$$

Measures compute expectations:

$$f \quad \mapsto \quad \int_0^1 p^2(1-p) \sum_{y\in\{\mathrm{H},\mathrm{T}\}} p^{\langle y=\mathrm{H}\rangle}(1-p)^{\langle y=\mathrm{T}\rangle} \cdot f(y)\, dp$$

Eliminating a random variable

Program denote measures:



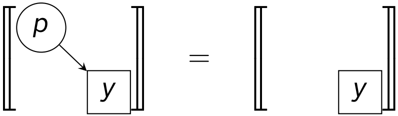
$$\in \quad \mathbb{M}\,\{\mathsf{H},\mathsf{T}\} \quad \subseteq \quad (\{\mathsf{H},\mathsf{T}\} \to \mathbb{R}_+) \to \mathbb{R}_+$$

Measures compute expectations:

$$f \quad \mapsto \quad \int_0^1 p^2(1-p) \sum_{y\in\{\mathsf{H},\mathsf{T}\}} p^{\langle y=\mathsf{H}\rangle}(1-p)^{\langle y=\mathsf{T}\rangle} \cdot f(y)\,dp$$

Eliminating a random variable

Program denote measures:



$$\in \quad \mathbb{M}\,\{\mathsf{H},\mathsf{T}\} \quad \subseteq \quad (\{\mathsf{H},\mathsf{T}\} \to \mathbb{R}_+) \to \mathbb{R}_+$$

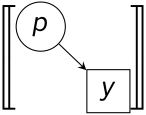
Measures compute expectations:

$$f \quad \mapsto \quad \int_0^1 p^2(1-p) \sum_{y\in\{\mathsf{H},\mathsf{T}\}} p^{\langle y=\mathsf{H}\rangle}(1-p)^{\langle y=\mathsf{T}\rangle} \cdot f(y)\, dp$$

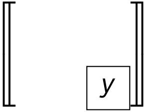
$$= \quad \sum_{y\in\{\mathsf{H},\mathsf{T}\}} \left(\int_0^1 p^2(1-p)p^{\langle y=\mathsf{H}\rangle}(1-p)^{\langle y=\mathsf{T}\rangle}\, dp \right) \cdot f(y)$$

Eliminating a random variable

Program denote measures:



$=$



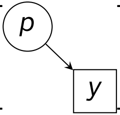
$\in \quad \mathbb{M}\,\{\mathrm{H},\mathrm{T}\} \quad \subseteq \quad (\{\mathrm{H},\mathrm{T}\} \to \mathbb{R}_+) \to \mathbb{R}_+$

Measures compute expectations:

$$
\begin{aligned}
f \quad \mapsto \quad & \int_0^1 p^2(1-p) \sum_{y\in\{\mathrm{H},\mathrm{T}\}} p^{\langle y=\mathrm{H}\rangle}(1-p)^{\langle y=\mathrm{T}\rangle} \cdot f(y)\,dp \\
= \quad & \sum_{y\in\{\mathrm{H},\mathrm{T}\}} \left(\int_0^1 p^2(1-p)p^{\langle y=\mathrm{H}\rangle}(1-p)^{\langle y=\mathrm{T}\rangle}\,dp \right) \cdot f(y) \\
= \quad & \sum_{y\in\{\mathrm{H},\mathrm{T}\}} \left(\frac{1}{20}\right)^{\langle y=\mathrm{H}\rangle} \left(\frac{1}{30}\right)^{\langle y=\mathrm{T}\rangle} \cdot f(y)
\end{aligned}
$$

Eliminating a random variable

Program denote measures:

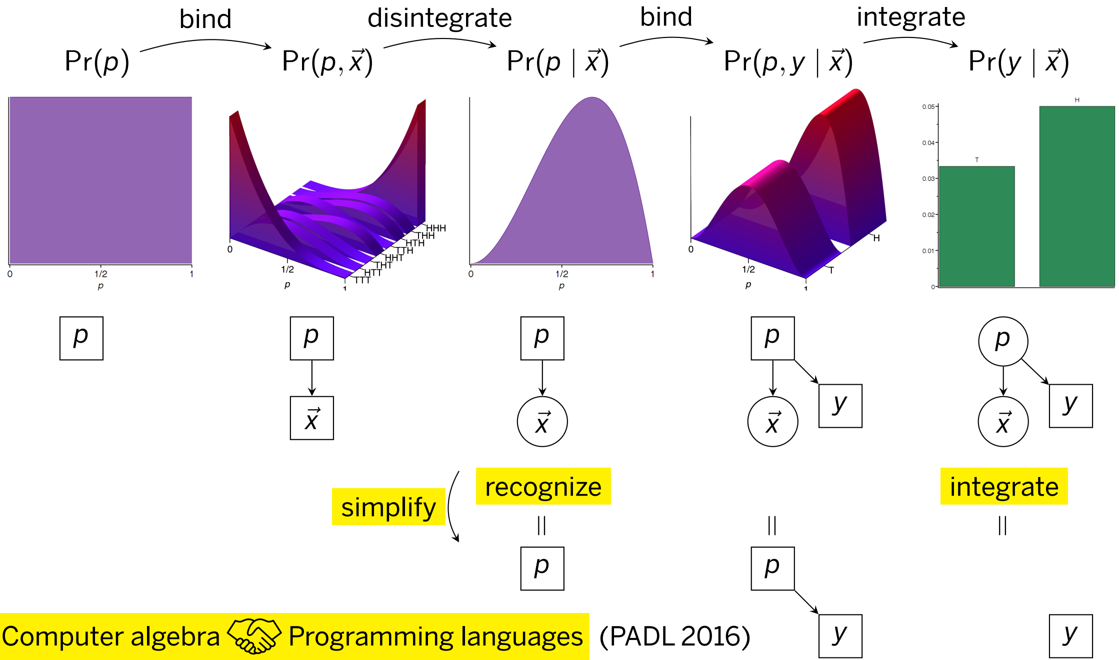
$[\![$  $]\!]$ $=$ $[\![$  $]\!]$ $\in \quad \mathbb{M}\{\mathrm{H},\mathrm{T}\} \quad \subseteq \quad (\{\mathrm{H},\mathrm{T}\} \to \mathbb{R}_+) \to \mathbb{R}_+$

Measures compute expectations:

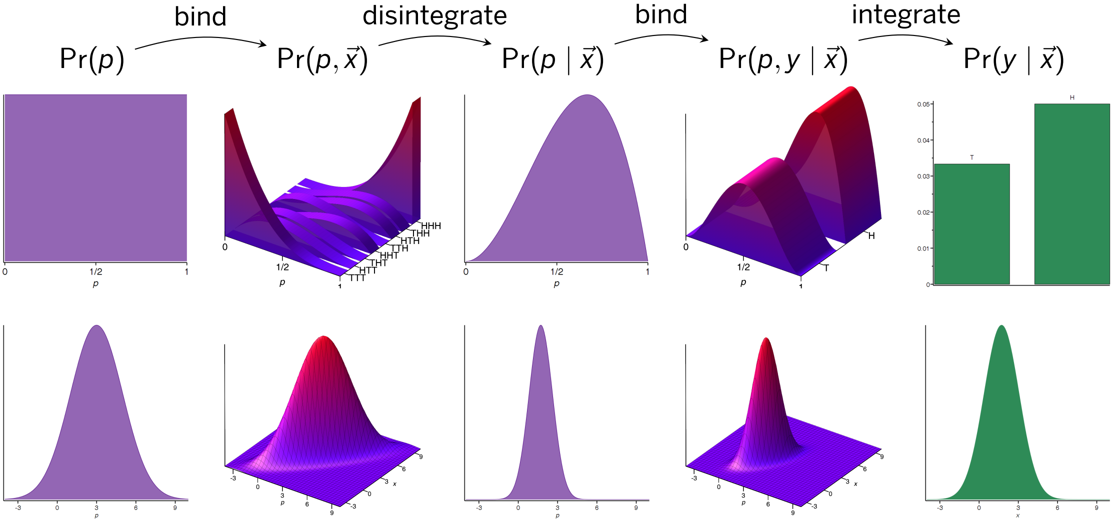
$$f \quad \mapsto \quad \int_0^1 p^2(1-p) \sum_{y\in\{\mathrm{H},\mathrm{T}\}} p^{\langle y=\mathrm{H}\rangle}(1-p)^{\langle y=\mathrm{T}\rangle} \cdot f(y)\, dp$$

$$= \sum_{y\in\{\mathrm{H},\mathrm{T}\}} \left(\int_0^1 p^2(1-p)p^{\langle y=\mathrm{H}\rangle}(1-p)^{\langle y=\mathrm{T}\rangle}\, dp \right) \cdot f(y)$$

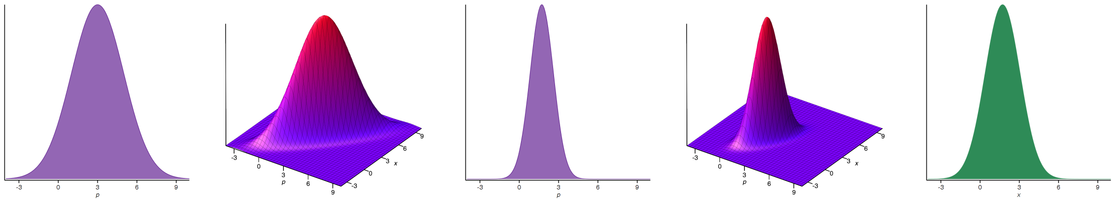
$$= \sum_{y\in\{\mathrm{H},\mathrm{T}\}} \left(\frac{1}{20}\right)^{\langle y=\mathrm{H}\rangle} \left(\frac{1}{30}\right)^{\langle y=\mathrm{T}\rangle} \cdot f(y)$$



Computer algebra 🤝 Programming languages (PADL 2016)


bind
disintegrate
bind
integrate
$\Pr(p)$
$\Pr(p, \vec{x})$
$\Pr(p \mid \vec{x})$
$\Pr(p, y \mid \vec{x})$
$\Pr(y \mid \vec{x})$
HHH
THH
HTH
TTH
HHT
THT
HTT
TTT
H
T
p
x

$$\Pr(p) \xrightarrow{\text{bind}} \Pr(p, \vec{x}) \xrightarrow{\text{disintegrate}} \Pr(p \mid \vec{x}) \xrightarrow{\text{bind}} \Pr(p, y \mid \vec{x}) \xrightarrow{\text{integrate}} \Pr(y \mid \vec{x})$$

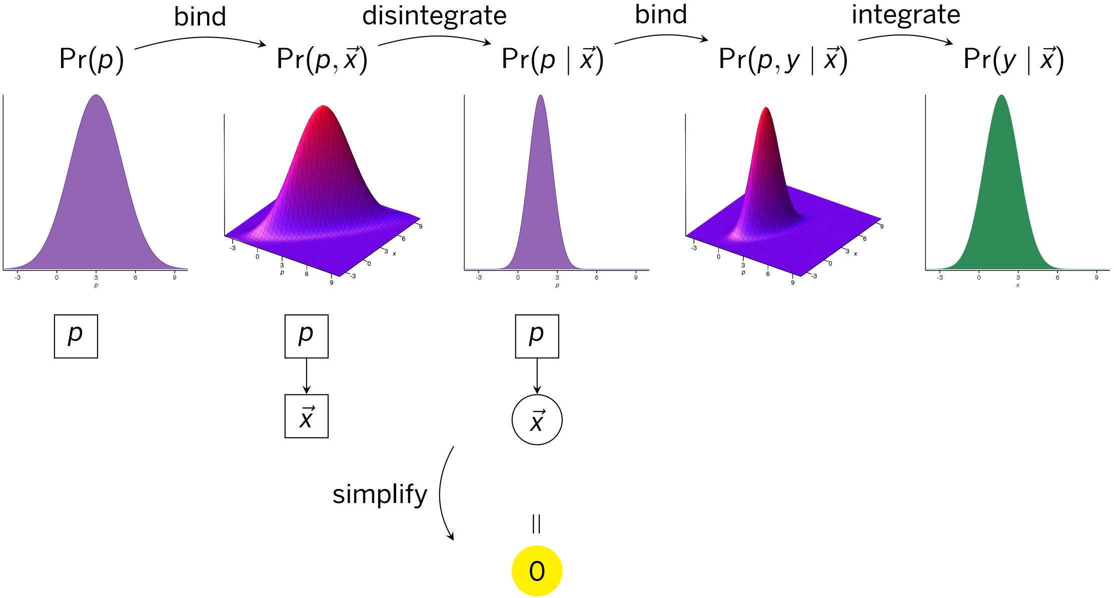


An unknown random process yields a stateless particle whose one-dimensional position can be measured repeatedly to produce a real number.

We assume that the position p of the particle is distributed normally with mean 3 and standard deviation 2.

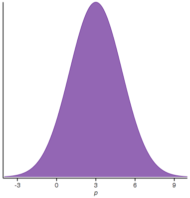
We measure the particle 3 times, each time drawing independently from the normal distribution with mean p and standard deviation 1, and observe $-1.4, +1.0, -0.2$.

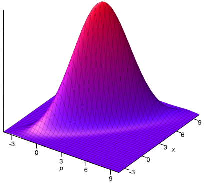
What is the distribution of the next measurement?

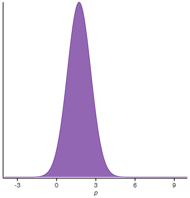

bind
disintegrate
bind
integrate
Pr(p)
Pr(p, x⃗)
Pr(p | x⃗)
Pr(p, y | x⃗)
Pr(y | x⃗)
p
p
x⃗
p
x⃗
simplify
||
0

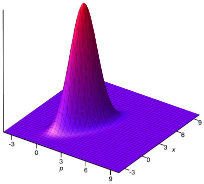
bind → **disintegrate** → bind → integrate

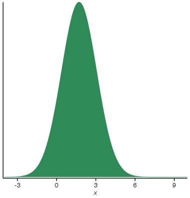
$\Pr(p)$ → $\Pr(p, \vec{x})$ → $\Pr(p \mid \vec{x})$ → $\Pr(p, y \mid \vec{x})$ → $\Pr(y \mid \vec{x})$









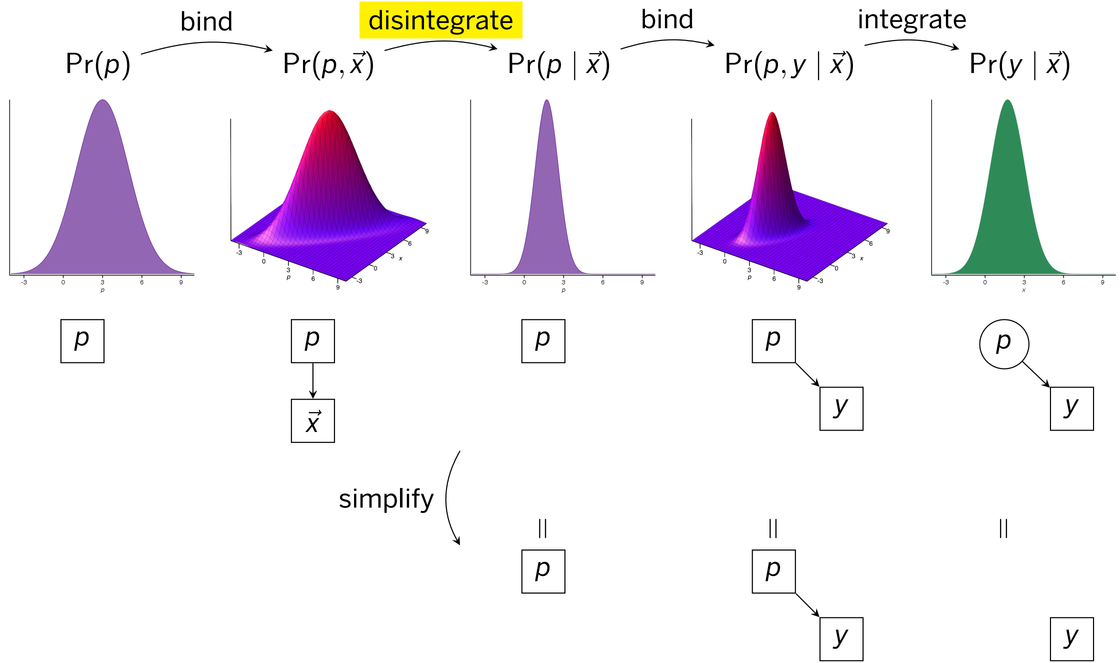


$\boxed{p}$ $\quad$ $\boxed{p} \rightarrow \boxed{\vec{x}}$ $\quad$ $\boxed{p}$

simplify

$\|$

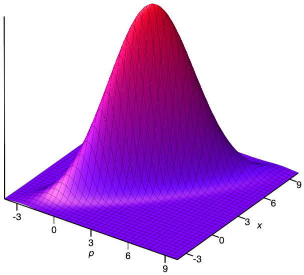
$\boxed{p}$

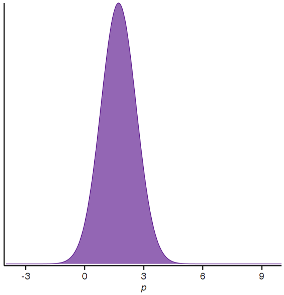

bind
disintegrate
bind
integrate
$\Pr(p)$
$\Pr(p, \vec{x})$
$\Pr(p \mid \vec{x})$
$\Pr(p, y \mid \vec{x})$
$\Pr(y \mid \vec{x})$
p
$\vec{x}$
y
simplify
||

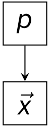
Disintegrating a joint measure

disintegrate

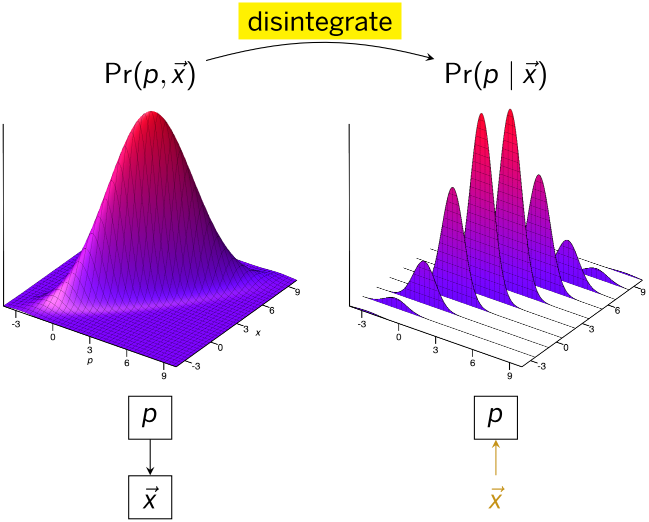
$\Pr(p, \vec{x})$ → $\Pr(p \mid \vec{x})$







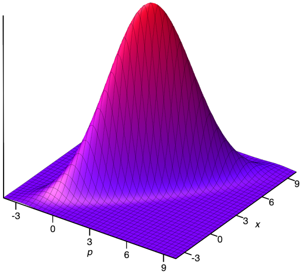
Disintegrating a joint measure

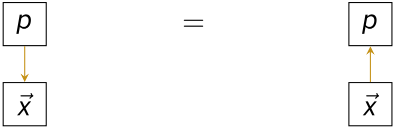


Disintegrating a joint measure

disintegrate

$\Pr(p, \vec{x}) \longrightarrow \Pr(p \mid \vec{x})$



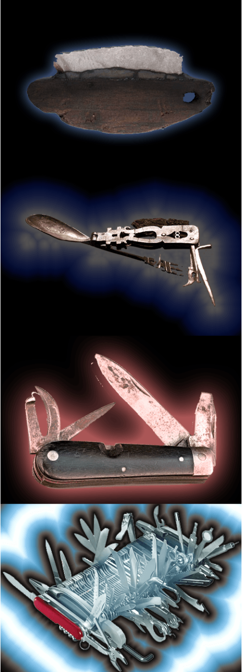


Program transformation derived from semantics.

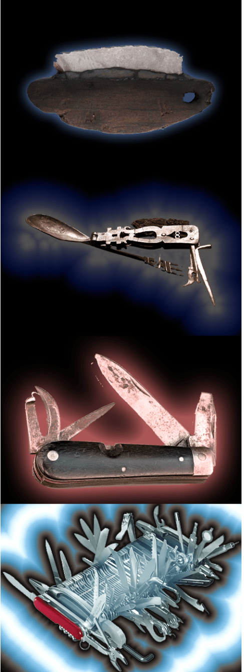
Tricky when $\vec{x}$ is not just drawn from a primitive distribution:

- total momentum
- loop over array
- clamped measurement
- coordinate-wise MCMC

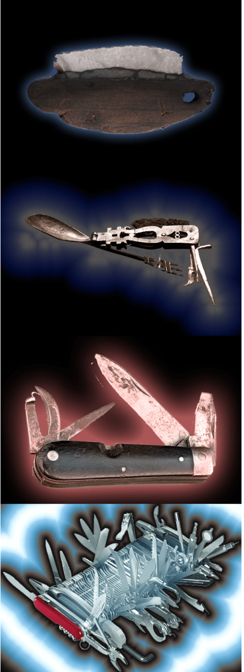
Addressed in recent work.
(ICFP 2016, POPL 2017,
ICFP 2017)



7	8	9	÷
4	5	6	×
1	2	3	−
0	.	=	+



7 *plate*	8 *lambda*	9 *apply*	÷ *disintegrate*
4 *pair*	5 *fst*	6 *snd*	× *bind*
1 *dirac*	2 *beta*	3 *normal*	− *gradient*
0 *mzero*	. *factor*	= *simplify*	+ *mplus*



Thanks!

Jacques Carette
Oleg Kiselyov
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Praveen Narayanan
Norman Ramsey
Wren Romano
Sam Tobin-Hochstadt
Rajan Walia
Robert Zinkov

÷ *disintegrate*

× *bind*

− *gradient*

0 *mzero*

. *factor*

= *simplify*

+ *mplus*