Theory of mind and bounded rationality without interpretive overhead

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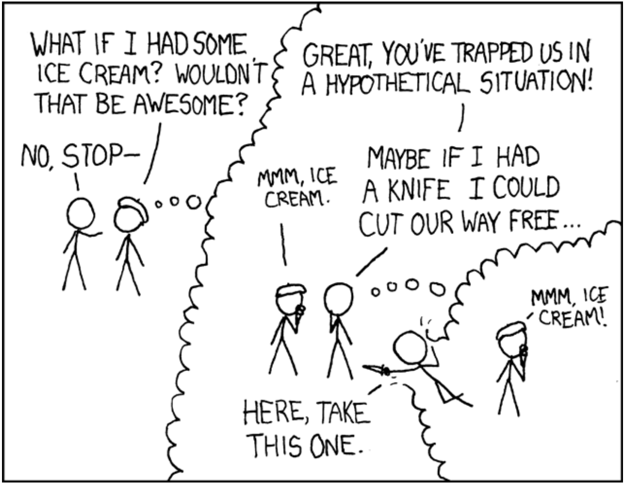

Theory of mind

- False-belief (Sally-Anne) task
- Gricean reasoning
- *p*-beauty contest
- Focal points in coordination games
- Information cascade
- Securities trading
- Plausibly deniable bribing (Pinker, Nowak, Lee)

Crucial for collaboration among human and computer agents!

Want executable models.

Modeling minds as programs

An agent's intention	A rational agent's beliefs / desires	Bounded rationality
	beliefs → A probability distribution; desires → A utility function	
A program	A stochastic program	Approximate inference

The amount of detail varies.

Modeling minds as programs

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	A probability distribution	A utility function	
A program	A stochastic program		Approximate inference

```
val random : random
val dist   : random -> (prob * 'a) list -> 'a
val fail   : random -> 'a
```

```
let flip random p = dist random [p, true; 1.-.p, false] in
let x = flip random 0.5 in
let y = flip random 0.5 in
if x || y then (x,y) else fail random
```

The amount of detail varies.

Modeling minds as programs

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A program	A probability distribution	A utility function	Approximate inference
	A stochastic program		

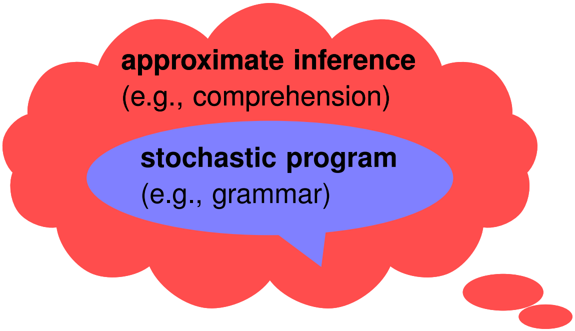
An agent's A rational agent's A bounded-rational agent's	theory of mind theory of rational mind theory of bounded-rational mind
A program A stochastic program Approximate inference	about programs about stochastic programs about approximate inference

The amount of detail varies. Encapsulated weighted search.

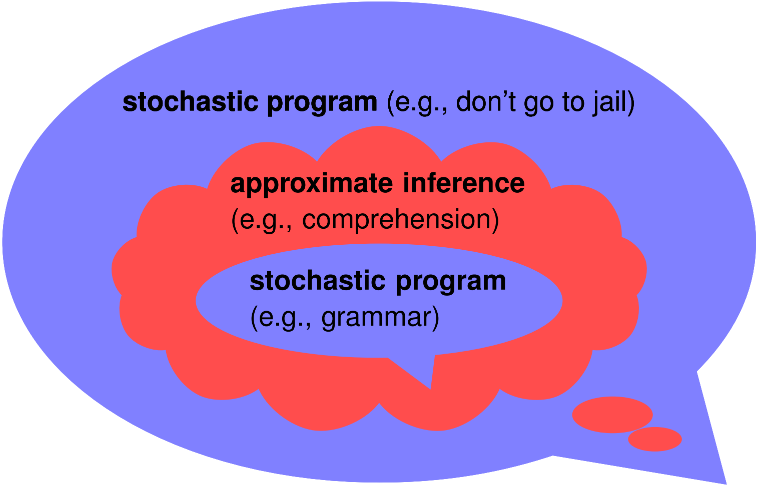
Marr's computational vs algorithmic models

stochastic program
(e.g., grammar)

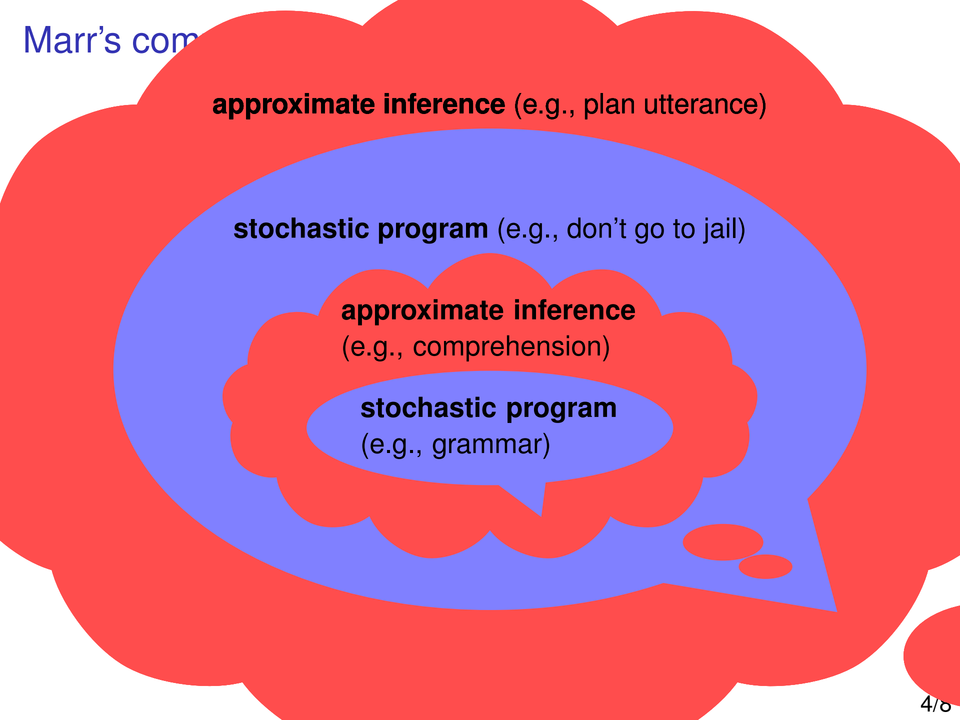
Marr's computational vs algorithmic models



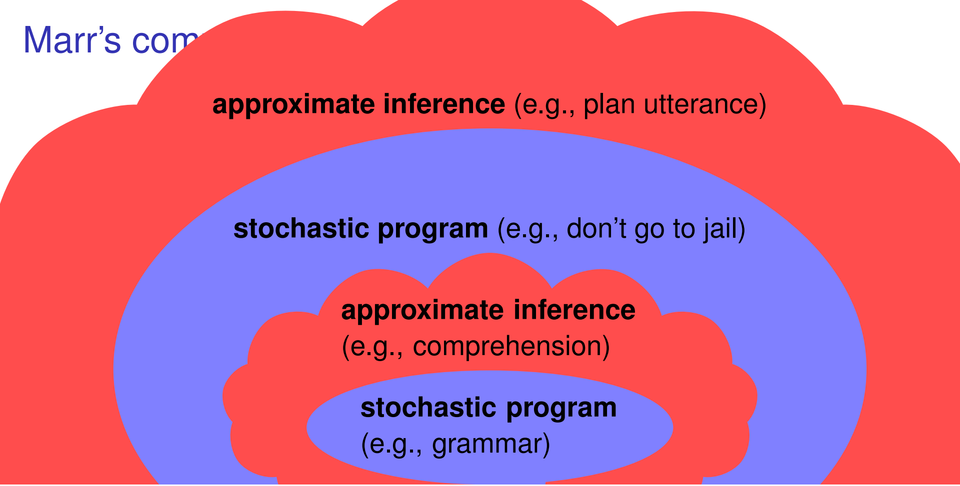
Marr’s computational vs algorithmic models



Marr's com


approximate inference (e.g., plan utterance)
stochastic program (e.g., don't go to jail)
approximate inference
(e.g., comprehension)
stochastic program
(e.g., grammar)

Marr's com



A computational model of the modeler nests an algorithmic model of the modelee.

For arbitrary nesting, implement inference as a stochastic program.

☞ Run deterministic code at full speed, to avoid slowdown exponential in the nesting depth (e.g., quantifier depth, plys).

How to eliminate interpretive overhead

Filinski: given a programming language with *delimited control*, add *layered side effects* (probabilities, memoization, etc.) while still running deterministic code at full speed.

- With delimited control, threads of execution can be suspended, resumed, copied, discarded.
- Represent stochastic programs not as data but as normal programs that suspend when they want randomness.
- Convert a stochastic program to a lazy tree of execution traces without interpretive overhead.
- Inference operates on this lazy tree. Implemented in OCaml.
- Inference is itself a stochastic program (e.g., importance sampling): it suspends when it wants randomness.
- Intuitions for nesting: sandboxes, virtualization, randomness adapters, mock objects.

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What comes in the box?

Represent stochastic programs
as normal programs
using `dist fail laze delay`

Represent approximate inference
as exploring a lazy tree of execution traces
using `sample_reify exact_reify collate at_least`

Represent theory of mind
as recursive invocations of approximate inference
using multiple randomness sources (values of type `random`)

Example: plausibly deniable bribing

```
# generate random ;;
.<fun random ->
  let make_boolean () =
    laze random (fun () -> flip random 0.5) in
  let q_1 = make_boolean () in
  let q_2 = make_boolean () in
  let q_3 = make_boolean () in
  let q_4 = make_boolean () in
  let q_5 = make_boolean () in
  if not (q_4 () <> q_3 () <> q_1 ()) &&
         (q_2 () <> q_5 () <> q_4 ()) &&
     not (q_2 () <> q_3 () <> q_2 ()) &&
     not (q_3 () <> q_3 () <> q_5 ()) &&
         (q_3 () <> q_4 () <> q_5 ())
  then q_5 () && q_4 ()
  else fail random>.
```

Example: plausibly deniable bribing

```
let predict random innocent problem =
  match collate
        (sample_reify random (Some 2) 5 problem)
  with
  | [] -> fail random               (* police rejects sentence *)
  | [_, false] ->
    if innocent then Ticketed                     (* naïve driver *)
    else          (* police perceives (unambiguous) bribe *)
    if flip random 0.5 then Bribe             (* corrupt police *)
    else                                         (* honest police *)
    if at_least 0.01 true                        (* criminal trial *)
        (sample_reify random (Some 4) 20 problem)
    then Ticketed           (* court finds reasonable doubt *)
    else Convicted                          (* court finds bribe *)
  | _ -> Ticketed          (* police does not perceive bribe *)
```

Example: plausibly deniable bribing

```
let prefer random = function
  | Ticketed  -> if flip random 0.2 then fail random
  | Bribe     -> ()
  | Convicted -> fail random

let analyze problem =
  List.map snd (exact_reify problem)

let driver innocent random =
  let problem = .!(generate random) in
  prefer random (predict random innocent problem);
  analyze problem
```

Example: r

```
(* innocent driver *)
# collate (sample_reify random (Some 3) 1000
                        (driver true)) ;;
[(0.0724, [true; false]);                    (* 21% *)
 (0.0498, [true]);                           (* 15% *)
 (0.2183, [false])]                          (* 64% *)

(* bribing driver *)
# collate (sample_reify random (Some 3) 1000
                        (driver false)) ;;
[(0.0768, [true; false]);                    (* 30% *)
 (0.0457, [true]);                           (* 18% *)
 (0.1327, [false])]                          (* 52% *)
```

Summary

People people model model

- Concise, concrete, composable, compilable
- Can model unknown nesting depth

Next steps

- Applications please!
- Faster inference: conditional independence; memoization; Markov chain Monte Carlo
- From probability to expected value and maximum utility
- Imperfect information: staging