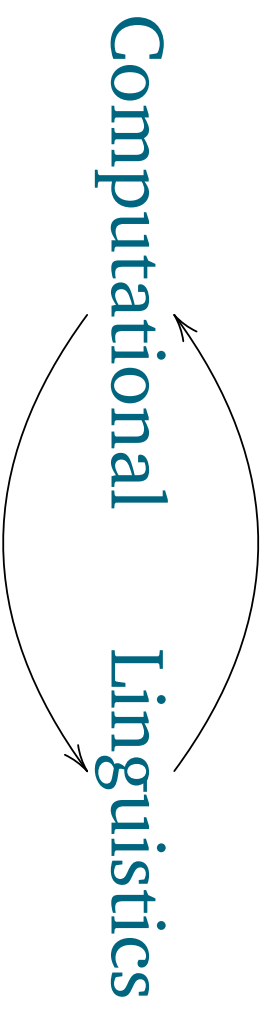


Delimited continuations in natural language: Quantification and polarity sensitivity

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What a linguist cares about

Entailment

every student enjoyed a conference	⊄	every student enjoyed POPL
no student enjoyed a conference	⊢	no student enjoyed POPL
a student enjoyed a conference	⊄	a student enjoyed POPL
most students enjoyed a conference	⊄	most students enjoyed POPL

Ambiguity

Did **some** student enjoy **every** conference?

$\exists x. \forall y. \text{enjoyed}(x, y)$

$\forall y. \exists x. \text{enjoyed}(x, y)$

Did **any** student enjoy **every** conference?

Acceptability

***every** student enjoyed any conference

no student enjoyed any conference

***a** student enjoyed any conference

***most** students enjoyed any conference

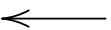
This talk deals with English, but the approach hopefully extends to other languages (which are different!).

Translation to a logical metalanguage

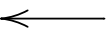
Every student enjoyed a conference

⊄

Every student enjoyed POPL



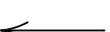
$\forall x. \text{student}(x) \Rightarrow \exists y. \text{conf}(y) \wedge \text{enjoyed}(x, y)$ ⊄ $\forall x. \text{student}(x) \Rightarrow \text{enjoyed}(x, \text{popl})$



⟨some truth condition on models⟩

⊄

⟨some truth condition on models⟩



The guiding analogy

Programming languages	Natural languages
desired behavior	speaker judgments
observations at ground type	truth conditions, etc.
type system	syntax
denotational semantics	denotational semantics

computational side effects	“linguistic side effects”
control effects	quantification, polarity, etc.

Computational side effects in the logical metalanguage ...

... handles “*linguistic side effects*”

State in the logical metalanguage ...

... handles pronouns and binding

Control operators in the logical metalanguage ...

... handles quantification and *polarity sensitivity*

Outline

- ✓ Overview
- ▶ A simple grammatical formalism
- Quantification with shift and reset
- Quantifier scope ambiguity
- Polarity sensitivity

Computational side effects in the logical metalanguage ...

... handles “*linguistic side effects*”

State in the logical metalanguage ...

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Control operators in the logical metalanguage ...

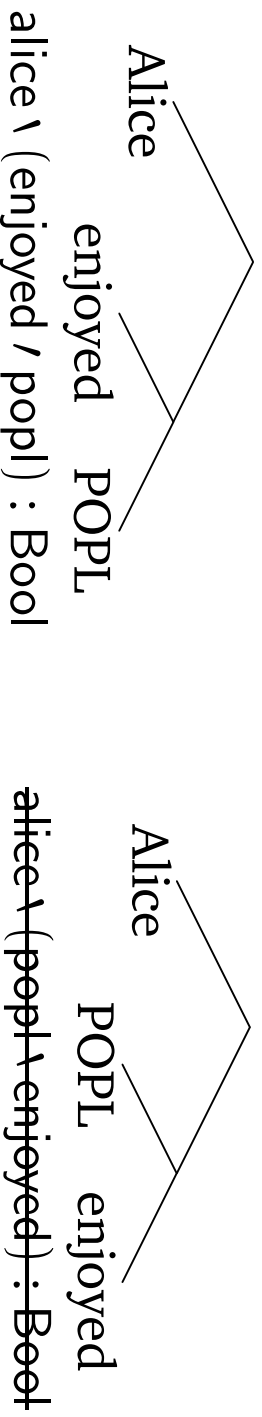
... handles quantification and *polarity sensitivity*

A simple grammatical formalism

Alice enjoyed POPL. *Alice enjoyed. *Alice enjoyed Bob POPL.

$\llbracket \text{Alice} \rrbracket = \text{alice} \quad : \text{Thing}$
 $\llbracket \text{POPL} \rrbracket = \text{popl} \quad : \text{Thing}$
 $\llbracket \text{enjoyed} \rrbracket = \text{enjoyed} : \text{Thing} \rightarrow (\text{Thing} \rightarrow \text{Bool})$

$f / x = f(x) \quad : \beta \quad \text{where } f : \alpha \rightarrow \beta, \quad x : \alpha$
 $x \setminus f = f(x) \quad : \beta \quad \text{where } f : \alpha \rightarrow \beta, \quad x : \alpha$



Right-associative by convention.

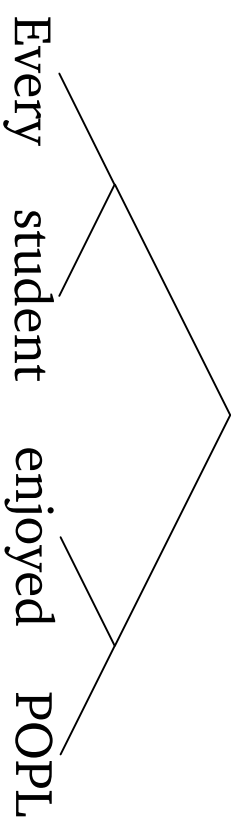
Notational variant of combinatory categorial grammar.

Quantification

Every student enjoyed POPL.

$\forall x. \text{student}(x) \Rightarrow \text{enjoyed}(\text{popl})(x)$

$\llbracket \text{Every student} \rrbracket = ??$

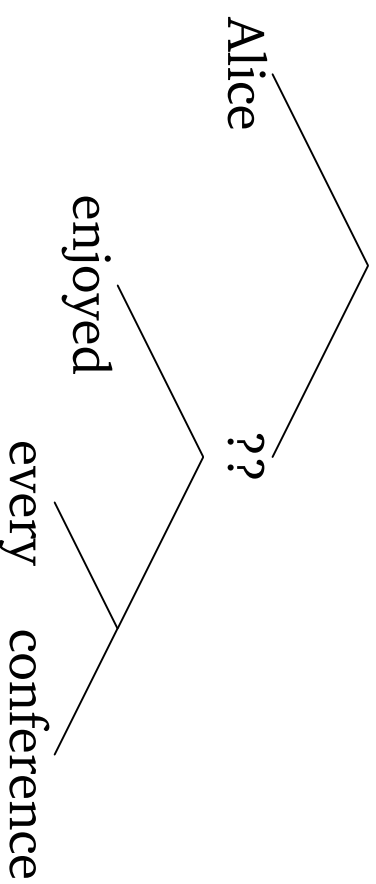


student

$: \text{Thing} \rightarrow \text{Bool}$

$\llbracket \text{every} \rrbracket = \lambda r. \lambda s. \forall x. r(x) \Rightarrow s(x) : (\text{Thing} \rightarrow \text{Bool}) \rightarrow (\text{Thing} \rightarrow \text{Bool}) \rightarrow \text{Bool}$

$\llbracket \text{some} \rrbracket = \lambda r. \lambda s. \exists x. r(x) \wedge s(x) : (\text{Thing} \rightarrow \text{Bool}) \rightarrow (\text{Thing} \rightarrow \text{Bool}) \rightarrow \text{Bool}$



Quantification with shift and reset

We want:

$$\llbracket \text{every conference} \rrbracket = \xi s. \forall x. \text{conf}(x) \Rightarrow s(x) : \mathbf{Thing}_{\text{Bool}}^{\text{Bool}}.$$

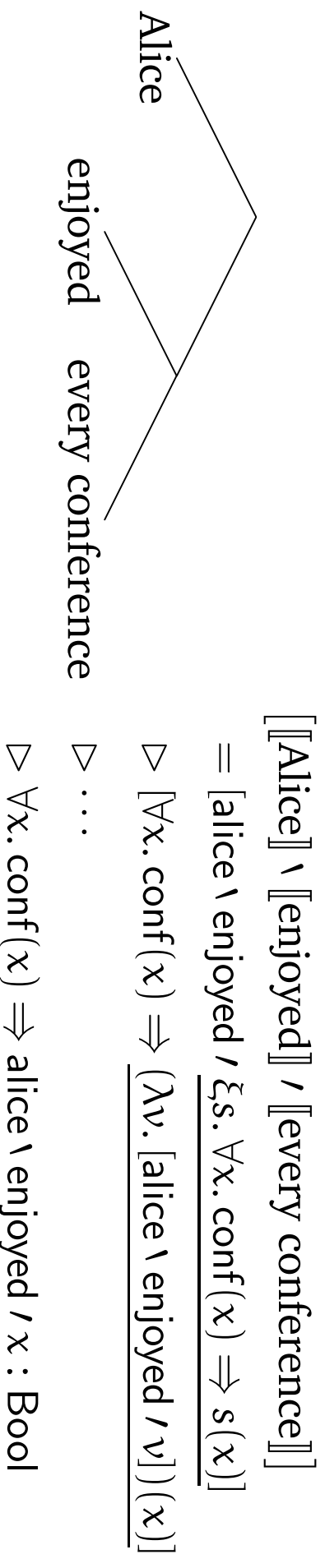
Here the type $\mathbf{Thing}_{\text{Bool}}^{\text{Bool}}$ has the CPS transform $(\mathbf{Thing} \rightarrow \text{Bool}) \rightarrow \text{Bool}$.

In general, α_{γ}^{δ} has the CPS transform $(\alpha \rightarrow \gamma) \rightarrow \delta$.

Reverse-engineer denotations for “every” and “some”:

$$\begin{aligned} \llbracket \text{every} \rrbracket &= \lambda r. \xi s. \forall x. r(x) \Rightarrow s(x) : (\mathbf{Thing} \rightarrow \text{Bool}) \rightarrow \mathbf{Thing}_{\text{Bool}}^{\text{Bool}} \\ \llbracket \text{some} \rrbracket &= \lambda r. \xi s. \exists x. r(x) \wedge s(x) : (\mathbf{Thing} \rightarrow \text{Bool}) \rightarrow \mathbf{Thing}_{\text{Bool}}^{\text{Bool}} \end{aligned}$$

Can now handle quantificational noun phrases in any position:



Notion of evaluation! Beginnings of psycholinguistics.

Quantifier scope ambiguity

Nondeterminism in natural language:

Some student enjoyed every conference.

$\exists x. \forall y. \text{enjoyed}(y)(x)$ ← linear scope

$\forall y. \exists x. \text{enjoyed}(y)(x)$ ← inverse scope

How to generate ambiguity?

Deterministic composition

+ Deterministic word meanings

Deterministic sentence meanings

Two approaches:

- *Nondeterministic evaluation order*

Quantifiers evaluated earlier scope wider.

- ▶ *Hierarchy of control operators*

Quantifiers at outer levels scope wider.

(cf. TDPE paper at this POPL by Balat, Di Cosmo, and Fiore)

People tend to process words in the order they are spoken.

Pronouns, questions, and polarity favor the second approach, but it needs staging—

Quantifier scope ambiguity with hierarchy & staging

$$\begin{aligned} & \llbracket \text{some student} \rrbracket \vee \llbracket \text{enjoyed} \rrbracket / \llbracket \text{every conference} \rrbracket \\ & = \llbracket (\xi^2 s. \exists x. \text{student}(x) \wedge s(x)) \vee \text{enjoyed} / (\xi^1 t. \forall y. \text{conf}(y) \Rightarrow t(y)) \rrbracket^0 \\ & \triangleright \llbracket \exists x. \text{student}(x) \wedge (\lambda v. [v \vee \text{enjoyed} / \xi^1 t. \forall y. \text{conf}(y) \Rightarrow t(y)]^2)(x) \rrbracket^0 \\ & \triangleright \dots \\ & \triangleright \llbracket \exists x. \text{student}(x) \wedge [x \vee \text{enjoyed} / \xi^1 t. \forall y. \text{conf}(y) \Rightarrow t(y)]^2 \rrbracket^0 \\ & \triangleright \llbracket \forall y. \text{conf}(y) \Rightarrow (\lambda v. [\exists x. \text{student}(x) \wedge [x \vee \text{enjoyed} / v]^2]^1)(y) \rrbracket^0 \\ & \triangleright \llbracket \forall y. \text{conf}(y) \Rightarrow [\exists x. \text{student}(x) \wedge [x \vee \text{enjoyed} / y]^2]^1 \rrbracket^0 \end{aligned}$$

What is \exists above, really?

- Is it *higher-order abstract syntax*: (Thing \rightarrow Bool) \rightarrow Bool? No, because then the body under \exists must be pure.
- Is it *gensym and first-order abstract syntax*: Bool \rightarrow Bool? Perhaps, but need to rule out unbound x in $\exists x. \xi^f. \text{student}(x) \triangleright \text{student}(x)$. (cf. “Some student enjoyed every conference s/he organized.”)
- Ideally, it is higher-order abstract syntax *staged* in a language with control.

Two kinds of control hierarchies

Danvy and Filinski:

- Post-CPS types look like $\alpha \begin{pmatrix} \delta_0 \\ \gamma_0 \delta_1 \\ \delta_2 \end{pmatrix}$
- Needs gensym for now

Barker and Shan:

- Post-CPS types look like $\begin{pmatrix} \gamma_0 \\ \alpha \gamma_1 \end{pmatrix} \delta_1$
- No direct-style terms yet

Both improve Hobbs and Shieber's and Lewin's quantifier scoping algorithms:

- ✓ Directly compositional, not a post-processing step after parsing
- ✓ Semantically motivated by delimited continuations
- Interacts properly with other linguistic side effects
 - ✓ (other) quantification
 - pronouns
 - questions

Outline

- ✓ Overview
- ✓ A simple grammatical formalism
- ✓ Quantification with shift and reset
- ✓ Quantifier scope ambiguity
- ▶ **Polarity sensitivity**

Computational side effects in the logical metalanguage ...

... handles “*linguistic side effects*”

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... handles pronouns and binding

Control operators in the logical metalanguage ...

... handles quantification and *polarity sensitivity*

Polarity sensitivity

The quantifiers “a”, “some”, and “any”[†] all look existential:

Did **a** student call?

Did **some** student call?

Did **any** student call?

$\exists x. \text{student}(x) \wedge \text{called}(x)$

But do not behave the same:

No student enjoyed some conference. (unambiguous $\exists \neg$)

No student enjoyed a conference. (ambiguous $\neg \exists, \exists \neg$)

No student enjoyed any conference. (unambiguous $\neg \exists$)

Some student enjoyed no conference. (unambiguous $\exists \neg$)

A student enjoyed no conference. (ambiguous $\neg \exists, \exists \neg$)

*Any student enjoyed no conference. (unacceptable)

“Any” is a *negative polarity item*:

Very roughly, it requires negative contexts, such as in the scope of “no”.

“Some” is a *positive polarity item*:

Very roughly, it is allergic to negative contexts.

Meaning affects ambiguity and acceptability! But linear order matters too.

Chaining answer types

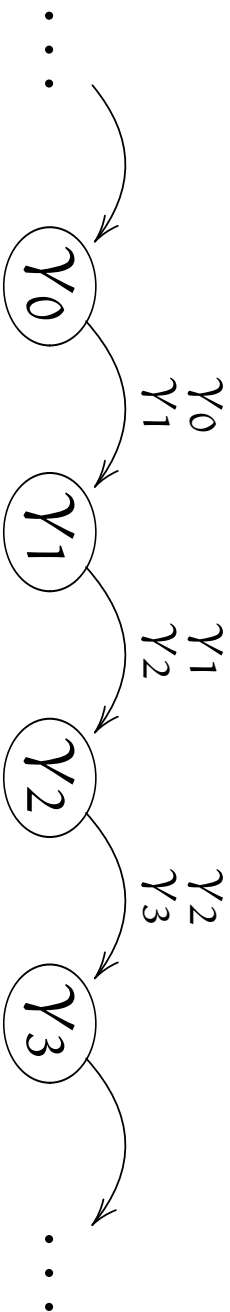
What an under-appreciated feature of shift and reset!

$$\frac{\Gamma \vdash F : (\alpha \rightarrow \beta_{\gamma_3}^{\gamma_2})^{\gamma_0} \quad \Gamma \vdash E : \alpha_{\gamma_2}^{\gamma_1}}{\Gamma \vdash F : (\alpha \rightarrow \beta_{\gamma_3}^{\gamma_2})^{\gamma_1} \rightarrow E}$$

$$\Gamma \vdash F \vee E : \beta_{\gamma_3}^{\gamma_0}$$

$$\frac{\Gamma \vdash E : \alpha_{\gamma_1}^{\gamma_0} \quad \Gamma \vdash F : (\alpha \rightarrow \beta_{\gamma_3}^{\gamma_2})^{\gamma_1}}{\Gamma \vdash E \vee F : \beta_{\gamma_3}^{\gamma_0} \rightarrow E}$$

$$\Gamma \vdash E \vee F : \beta_{\gamma_3}^{\gamma_0}$$



Polarity sensitivity with answer-type subtyping

A standard approach to modeling polarity sensitivity: split the answer type Bool into a family of subtypes.

$$\text{Bool} \leq \text{BoolPos} \quad \text{Bool} \leq \text{BoolNeg}$$

(in addition to the usual rules for subtyping)

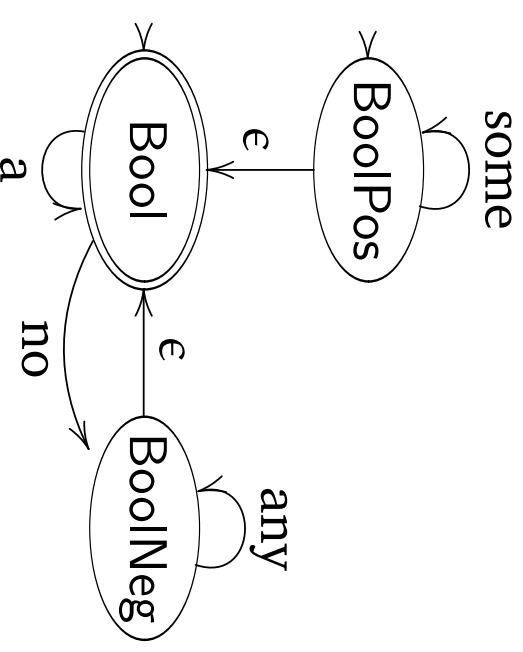
The return type of verbs like “enjoyed” remains Bool.

Also, restrict Reset to produce the answer type Bool or BoolPos, not BoolNeg.

$$\frac{\Gamma \vdash E : \alpha_\alpha^\beta}{\Gamma \vdash [E] : \beta} \text{Reset} \quad \text{where } \beta \leq \text{BoolPos}$$

Finally, refine the answer types for quantifiers.

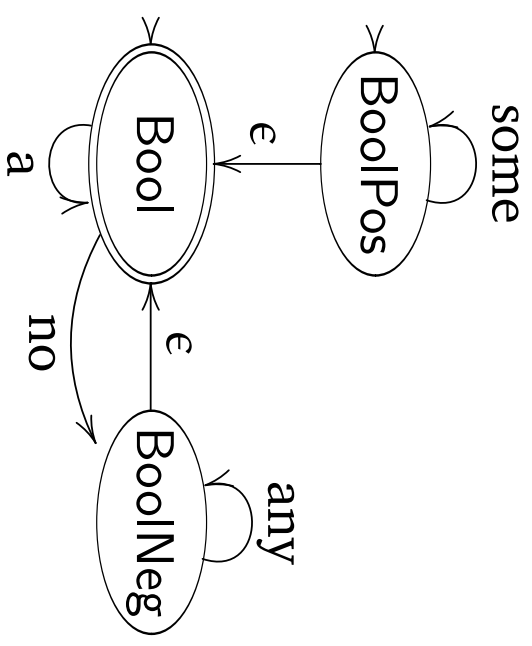
$$\begin{aligned} \llbracket \text{no} \rrbracket &: (\text{Thing} \rightarrow \text{Bool}) \rightarrow \text{Thing}_{\text{BoolNeg}}^{\text{Bool}}, \\ \llbracket \text{some} \rrbracket &: (\text{Thing} \rightarrow \text{Bool}) \rightarrow \text{Thing}_{\text{BoolPos}}^{\text{BoolPos}}, \\ \llbracket \text{a} \rrbracket &: (\text{Thing} \rightarrow \text{Bool}) \rightarrow \text{Thing}_{\text{Bool}}^{\text{Bool}}, \\ \llbracket \text{any} \rrbracket &: (\text{Thing} \rightarrow \text{Bool}) \rightarrow \text{Thing}_{\text{BoolNeg}}^{\text{BoolNeg}}. \end{aligned}$$



Polarity sensitivity: revisiting empirical data

Quantifiers with levels	Type	Reading
$no^1 \dots some^2$	$: Bool \begin{smallmatrix} BoolPos \\ BoolNeg \end{smallmatrix}$	\nrightarrow
$no^1 \dots some^1$	$:/$	
$no^2 \dots some^1$	$: Bool \begin{smallmatrix} BoolPos \\ BoolNeg \end{smallmatrix}$	$\rightarrow (\exists \neg)$
<hr/>		
$no^1 \dots a^2$	$: Bool \begin{smallmatrix} Bool \\ BoolNeg \end{smallmatrix}$	$\rightarrow (\neg \exists)$
$no^1 \dots a^1$	$: Bool \begin{smallmatrix} Bool \\ Bool \end{smallmatrix}$	$\rightarrow (\neg \exists)$
$no^2 \dots a^1$	$: Bool \begin{smallmatrix} Bool \\ BoolNeg \end{smallmatrix}$	$\rightarrow (\exists \neg)$
<hr/>		
$no^1 \dots any^2$	$: Bool \begin{smallmatrix} BoolNeg \\ BoolNeg \end{smallmatrix}$	\nrightarrow
$no^1 \dots any^1$	$: Bool \begin{smallmatrix} Bool \\ BoolNeg \end{smallmatrix}$	$\rightarrow (\neg \exists)$
$no^2 \dots any^1$	$: Bool \begin{smallmatrix} BoolNeg \\ BoolNeg \end{smallmatrix}$	\nrightarrow

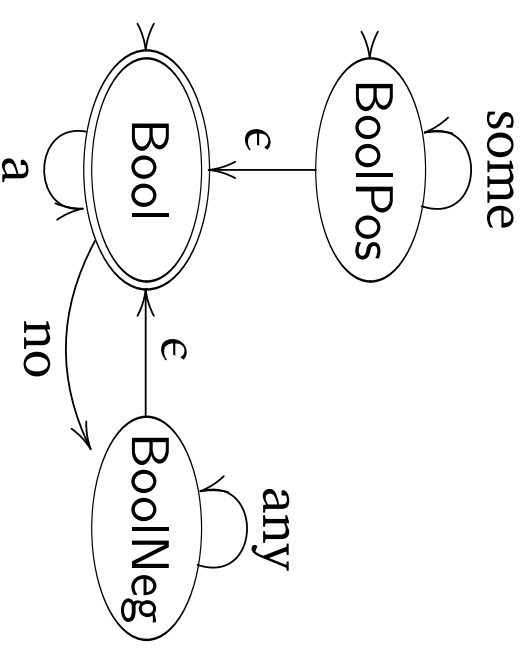
“no ... any” must be on the same level
 “no” must scope over “some”



Polarity sensitivity: revisiting empirical data

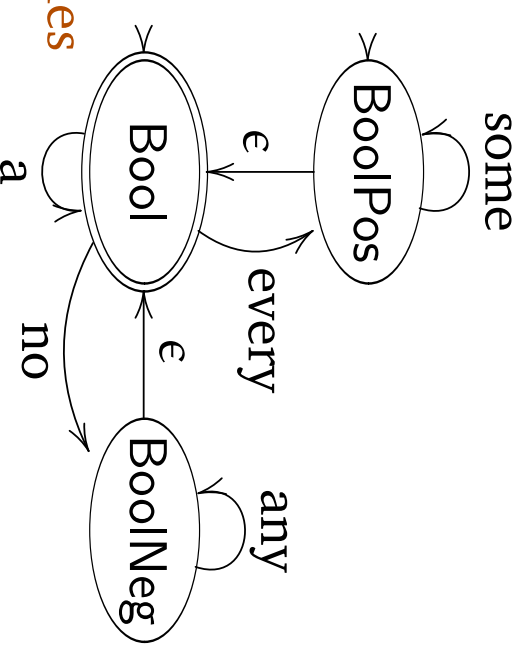
Quantifiers with levels	Type	Reading
$\text{some}^1 \dots \text{no}^2$	$\text{Bool}^{\text{BoolPos}} \mid \text{Bool}^{\text{BoolPos}} \mid \text{Bool}^{\text{Neg}}$	$\rightarrow (\exists \neg)$
$\text{some}^1 \dots \text{no}^1$	$\text{Bool}^{\text{Pos}} \mid \text{Bool}^{\text{Neg}}$	$\rightarrow (\exists \neg)$
$\text{some}^2 \dots \text{no}^1$	$\text{Bool}^{\text{Pos}} \mid \text{Bool}^{\text{Pos}} \mid \text{Bool}^{\text{Neg}}$	\nrightarrow
<hr/>		
$a^1 \dots \text{no}^2$	$\text{Bool}^{\text{Bool}} \mid \text{Bool}^{\text{Neg}}$	$\rightarrow (\exists \neg)$
$a^1 \dots \text{no}^1$	$\text{Bool}^{\text{Bool}} \mid \text{Bool}^{\text{Neg}}$	$\rightarrow (\exists \neg)$
$a^2 \dots \text{no}^1$	$\text{Bool}^{\text{Bool}} \mid \text{Bool}^{\text{Neg}}$	$\rightarrow (\neg \exists)$
<hr/>		
$\text{any}^1 \dots \text{no}^2$	$\text{Bool}^{\text{Neg}} \mid \text{Bool}^{\text{Neg}} \mid \text{Bool}^{\text{Neg}}$	\nrightarrow
$\text{any}^1 \dots \text{no}^1$	$\text{Bool}^{\text{Neg}} \mid \text{Bool}^{\text{Neg}}$	\nrightarrow
$\text{any}^2 \dots \text{no}^1$	$\text{Bool}^{\text{Neg}} \mid \text{Bool}^{\text{Neg}} \mid \text{Bool}^{\text{Neg}}$	\nrightarrow

“no ... any” must be on the same level, **in that order**
 “no” must scope over “some”



Polarity sensitivity: revisiting empirical data

Quantifiers with levels	Type	Reading
Every ¹ student enjoyed some ¹ conference	: Bool _{BoolPos} ^{Bool} → (∀∃)	
A ¹ student enjoyed every ¹ conference	: Bool _{BoolPos} ^{Bool} → (∃∀)	
$\llbracket \text{every} \rrbracket : (\text{Thing} \rightarrow \text{Bool}) \rightarrow \neg \text{Thing}_{\text{BoolPos}}^{\text{Bool}}$		
No ¹ professor gave every ¹ student some ¹ book	: Bool _{BoolPos} ^{Bool} → (¬∀∃)	



“no . . . any” must be on the same level, in that order

“no” must scope over “some”, **except if “every” intervenes**

Comparing approaches to scope ambiguity

Two approaches:

- Nondeterministic evaluation order: Quantifiers evaluated earlier scope wider.
- ▶ Hierarchy of control operators: Quantifiers at outer levels scope wider.
More complex perhaps, but captures more empirical data.

Polarity items (and pronouns, and questions) favor the second approach:

No student enjoyed any conference. (unambiguous $\neg\exists$)
*Any student enjoyed no conference. (unacceptable)

The first semantic analysis of polarity items to capture the long-observed sensitivity to linear order.

Implemented in a substructural logic of delimited continuations, using Richard Moot's theorem prover Grail for categorial grammar. (Joint work with Chris Barker.)

Summary

Control operators in the logical metalanguage handles quantification and polarity sensitivity.

Ingredients of this analysis include

- a control hierarchy,
- staged generation of logical formulas,
- left-to-right evaluation order, and
- changing and chaining answer types.

The first semantic analysis of polarity items to capture linear order.

Beyond the λ -calculus: operational semantics for natural language?